Homework 5: Due Monday 9-29-08

Turn in:
Section 2.2 # 2,6,22,24
problem 5.1 below

Do (but do not turn in):
Section 2.1 # 2
Section 2.2 # 1, 13

Comments: 2.2.2 You should give a very brief explanation for each case. Do not write too much. However you do need to justify your answer. If it is a subspace show closure of addition and scalar multiplication (or show both together using $c\boldsymbol{v}+d\boldsymbol{d}$). If it is not a subspace either show that closure of addition or scalar multiplication fails or show that $\mathbf{0}$ is not in the set.
2.2.6 For (a) consider the union of the x-axis and y-axis (vectors in $\mathbb{R}^2$ with either first coordinate 0 or second coordinate 0). For (b) consider the non-negative quadrant (vectors in $\mathbb{R}^2$ with nonnegative coordinates).
2.2.22(c) If $\boldsymbol{w}+\boldsymbol{z}$ is in the union it is in one of W or Z. If its in W, show that $\boldsymbol{z} \in W$ by considering an appropriate linear combination of $\boldsymbol{w}+\boldsymbol{z}$ and $\boldsymbol{w}$.
2.2.24 For (a),(b),(c) just show that only $\mathbf{0}$ is in both sets. For (d), if $\boldsymbol{w}+\boldsymbol{z} = \hat{\boldsymbol{v}} + \hat{\boldsymbol{z}}$, observe that $\boldsymbol{w} - \hat{\boldsymbol{w}} = \boldsymbol{z} - \hat{\boldsymbol{z}}$. Explain why this new vectors is in W and Z and use that to to show $\boldsymbol{w} = \hat{\boldsymbol{w}}$ and $\boldsymbol{z} = \hat{\boldsymbol{z}}$.
hw5.1 (below) - This is really just a matter of rearranging terms to see that the definition coincide. Stated more formally, show the following: Let V be a vector space and $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k \in V$. Prove that $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$ is linearly dependent if and only if for some i, v_i is a linear combination of $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{i-1}, \boldsymbol{v}_{i+1}, \boldsymbol{v}_k\}$.

hw5.1 Prove that a set of vectors in a vector space is linearly dependent if and only if one of them is a linear combination of the others. Let V be a vector space and $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k \in V$. Prove that $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$ is linearly dependent if and only if for some i, v_i is a linear combination of $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{i-1}, \boldsymbol{v}_{i+1}, \boldsymbol{v}_k\}$

Homework 6: Due Friday 10-10-08

Turn in:
Section 2.3 # 3a, 4bd, 8ac, 21ace, 31
Section 2.4 # 2bc, 10, 21
Section 2.5 # 8, 22, 31bd, 39, 46bc
problem 6.1 below

Do (but do not turn in):
Section 2.3 # 3b, 8b, 17, 19, 21df, 29
Section 2.4 # 9ab, 20, 22, 26
Section 2.5 # 20, 31ac, 38, 42, 46a

hw6.1 Let $A = LU$ with

$$A = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 2 & 2 & 4 & 4 & 4 & 5 & 7 \\ -1 & -1 & 1 & -2 & -2 & 2 & 4 \\ 0 & 0 & -4 & 0 & 0 & 7 & 3 \\ 3 & 3 & 5 & 6 & 6 & 13 & 15 \end{bmatrix} L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 3 & 1 & 0 \\ 3 & 1 & 2 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 0 & 0 & 2 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} L^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 \\ 3 & -1 & 1 & 0 & 0 \\ -13 & 5 & -3 & 1 & 0 \\ -7 & 1 & -2 & 0 & 1 \end{bmatrix}.$$

Find a basis for each of the four fundamental subspaces; the kernel, cokernel, range and corange (i.e., nullspace, left nullspace, column space, row space). For all but one you should be able to write down the answer directly and the remaining should require very little computation.

Homework 7: Due Wednesday 10-22-08

Turn in:
Section 3.1 # 1
Section 3.2 # 6,8,15
Section 3.3 # 13
Section 3.4 # 7, 10, 31
Section 3.5 # 11, 19a
problem 7.1 below

Do (but do not turn in):
Section 3.1 # 2,3, 7,11
Section 3.2 # 5,16
Section 3.4 # 4,8
Section 3.5 # 1, 19bd

hw7.1 We discussed a proof of the Cauchy-Schwarz inequality in $\mathbb{R}^N$ by first proving by induction the inequality $\frac{(a_1+a_2+\cdots+a_n)^2}{u_1+u_2+\cdots+u_n} \leq \frac{a_1^2}{u_1} + \frac{a_2^2}{u_2} + \cdots + \frac{a_n^2}{u_n}$ when each $u_i > 0$ and then making an appropriate substitution for the a_i and u_i. Write up this proof neatly and concisely.