Math 242 fall 2008 comments/hints for section 1.3 and 1.4 homework.

1. 1.3.13(c) Give a proof similar to that we did for upper triangular in class. You will need an extra case to show that the diagonal elements in the product are 1.

2. 1.3.30 Recall my hint to prove that in general $a_{ij}^k = 0$ for $i > j - k$ by induction (where a_{ij}^K indicates the i, j entry of A^k). Observe that the case $k = 1$ follows from the definition of strictly upper triangular. Then for the induction what you really need to do is prove that $a_{ij}^{k+1} = 0$ for $i > j - (k + 1)$ using the assumption $a_{ij}^k = 0$ for $i > j - k$ (this is called the induction hypothesis) and the assumption $a_{ij} = 0$ for $i > j - 1$ (since A is strictly upper triangular). This will be similar to the proof we did for upper triangular (and thus also the proof you did for 1.3.13). Write $A^{k+1} = A\,A^K$ and write down using sigma notation what a_{ij}^{k+1} is in terms of a sum whose are a_{it} and a_{tj}^k. Figure out perhaps using small examples where to split the sum so that for the first part the terms a_{it} are 0 and for the second part the terms a_{tj}^k are 0. Here you may use the assumption just stated and also the fact that you are proving this new statement for $i > j - (k + 1)$.

   Do your best to write the details carefully and succinctly. If you are not familiar with induction at least carry out the steps I just described and come see me for an explanation of induction. If you are familiar with induction try to put the who thing together as a well written induction proof.

3. For 1.4.19(f). There are a few fractions in the computations but if you are careful with the arithmetic the final answer is quite simple. The arithmetic is slightly easier if permute rows at first so the the fourth row becomes the new third row instead of just switching the first and third rows.