Math 242 Exam 1 Lehigh University 9-22-2008

This is closed book, closed notes etc. You have 50 minutes to take this exam.

Points for each problem are indicated as $[\cdot]$.

1: [12] Find a $PA = LU$ factorization for $A = \begin{pmatrix} 2 & 4 & 6 \\ -2 & -4 & -5 \\ 4 & 7 & 9 \end{pmatrix}$.

2: [16] Let $A = LU$ with

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 & -1 \\ 4 & 2 & 3 & 1 & -3 \\ -4 & -2 & 1 & 3 & -1 \end{bmatrix} L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & 3 & 1 \end{bmatrix} U = \begin{bmatrix} 2 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 8 & -3 & 1 \end{bmatrix}.$$

For

$$\boldsymbol{x}^T = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{pmatrix} \quad \boldsymbol{b'} = \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} \quad \boldsymbol{b''} = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}.$$

Consider both $A\boldsymbol{x} = \boldsymbol{b'}$ and $A\boldsymbol{x} = \boldsymbol{b''}$. For each either solve the system (making use of LU, not Gaussian elimination) or give a certificate (relating to A and $\boldsymbol{b}$ not L or U) showing that there is no solution.

3: [12] Prove that matrix multiplication is associative: If A is $m \times n$, B is $n \times p$ and C is $p \times q$ then $A(BC) = (AB)C$.

4: [12] If L and M are $n \times n$ lower triangular matrices, prove that the product LM is lower triangular. Make sure that you clearly state a condition for a matrix to be lower triangular and prove that it holds for LM.

5: [10] Assume that A, B are invertible matrices of the same size. Prove that $(AB)^{-1}$ exists.

6: [18] Assume that A is an 6×6 invertible matrix.
Let B be obtained from A by multiplying the 4^{th} row of A by 42.
Let C be obtained from A by adding 13 times the second row of A to the 5^{th} row.
Let D be obtained from A by rearranging the rows so that the new row 2 is the old row 6, the new row 4 is the old row 2 and the new row 6 is the old row 4.
For each of B, C, D describe how to obtain its inverse from A^{-1} using words. In addition, justify your answer by describing and using appropriate elementary matrices.

7: [10] Prove that $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -2 & -2 \end{pmatrix}$ has a left inverse but no right inverse.

8: [10] One of the statements below is true and the other false. Give a proof for the one that is true and a counterexample for the one that is false.
(i) If the first and third columns of A are the same, so are the first and third columns of AB.
(ii) If the first and third rows of A are the same, so are the first and third rows of AB.