Homework 8
Combinatorics (Math 446) Fall 2004 Lehigh University

Due Monday November 1

37. A rank k paving matroid is defined as follows: Given a ground set E and $A_1, A_2, \ldots, A_n$ all subsets of E with each $A_i \neq E$ and with $|A_i \cap A_j| \leq (k-2)$ for all $i \neq j$ let the bases of the matroid be all size k subsets of E that are not subsets of any of the A_i.

Prove that these are indeed matroids.

38. Let B_1, B_2 be bases of a matroid. Show that there exists a bijection $\alpha : (B_1 - B_2) \Rightarrow (B_2 - B_1)$ (that is a pairing of the elements of $B_1 - B_2$ with the elements of $B_2 - B_1$) such that for each $e \in B_1 - B_2$ the set $B_1 - e + \alpha(e)$ is also a base.

(Hint - Use Hall's Theorem to find the pairing.)

39. Do 18.1.23 parts (a) and (d)

40. Do 18.2.2 (Hint - consider matroid intersection on M_1 and M_2 and also intersection on M_1^* and M_2^*. Note also that in the intersection formula we get $\leq$ for arbitrary X. Recall also the rank formula for dual matroids.)

41. Do 18.2.3 (Hint - consider the head partition and tail partition matroids defined on page 933.)

42. Consider a graph G with the edges $E(G)$ colored (in any way) with the colors $\{1, 2, \ldots, t\}$. That is, each edge $e \in E$ is assigned a color $c(e) \in \{1, 2, \ldots, t\}$. A rainbow spanning tree is a spanning tree T of G such that no two edges in T have the same color. Prove that the following condition is necessary and sufficient for the G to have a rainbow spanning tree: For every k and every partition $V_1, V_2, \ldots, V_k$ of the vertex set of G into k parts the number of colors appearing on edges between parts is at least $k-1$.

(Hint - Use matroid intersection and consider the cycle matroid and a partition matroid (a partition of the edges here) based on the colors.)