Nonlinear Systems and Control

Homework 5

Due on Thursday, May 9, 2024

READING ASSIGNMENT:

- Khalil: Chapters 6, 13 & 14.

PROBLEMS:

1. Khalil, Exercise 13.2
   Hint: Take $\phi_1(x) = x_1$
2. Khalil, Exercise 13.14
3. Consider the model of the"kinematic" car,

$$
\begin{aligned}
\dot{x} &= u_1 \cos(\theta) \\
\dot{y} &= u_1 \sin(\theta) \\
\dot{\theta} &= u_1 \tan(\phi) \\
\dot{\phi} &= u_2
\end{aligned}
$$

   where u_1 and u_2 are control inputs.

   (a) Show that the linearization at the origin is not completely controllable.
   (b) By computing $[g_1, g_2]$ and $[g_1, [g_1, g_2]]$ show that the nonlinear system is completely controllable.
   (c) Show that the system fails Brockett's condition, i.e., the origin is not C^1-stabilizable.

4. Extend Lemma 14.2 in Khalil to systems of the form

$$
\begin{aligned}
\dot{\eta} &= f(\eta, \xi), \qquad f(0,0) = 0 \\
\dot{\xi} &= u
\end{aligned}
$$

   where $f(\eta, \xi)$ is continuously differentiable. Assume $\phi(\eta)$ and $V(\eta)$ are available such that

$$
\frac{\partial V}{\partial \eta} f(\eta, \phi(\eta)) \leq -W(\eta)
$$

   where $W(\eta)$ is a positive definite function.

   Hint: Note that by the mean-value theorem, the continuous differentiability of $f(\eta, \xi)$ implies that there exists a continuous function $\psi(\eta, \xi)$ such that

$$
f(\eta, \xi) - f(\eta, \phi(\eta)) = (\xi - \phi(\eta))\psi(\eta, \xi)
$$

5. Apply the result from the previous problem to stabilize the system

$$
\begin{aligned}
\dot{\eta} &= \eta^2 - \xi - \xi^2 \\
\dot{\xi} &= u
\end{aligned}
$$

6. Compute the Sontag formula for the system from the previous problem using the backstepping CLF.
7. Khalil, Exercise 14.40
8. Khalil, Exercise 6.3
9. Khalil, Exercise 6.11
10. Khalil, Exercise 14.46
    Hint: Use Theorem 14.5.