Multivariable Robust Control

Lecture 1 (Meeting 1)

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Book

MULTIVARIABLE FEEDBACK CONTROL
Analysis and design

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Lecture notes are based on material provided by Prof. Skogestad

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7. Performance limitations MIMO [Chapter 6]
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Another Books

- C. Chen, Linear System Theory and Design, Oxford University Press, 1999.
- G. Dullerud and F. Paganini, A Course in Robust Control Theory, Springer, 2000.
- M. Green and D. Limebeer, Linear Robust Control, Prentice Hall, 1995.
- J. Helton and O. Merino, Classical Control Using H-infinity Methods, SIAM, 1998.
- J. Maciejowski, Multivariable Feedback Design, Addison Wesley, 1989.
- R. Sanchez-Pena and M. Sznaier, Robust Systems Theory and Applications, John Wiley & Sons, 1998.
- K. Zhou, Robust and Optimal Control, Prentice Hall, 1996.
- K. Zhou, Essentials of Robust Control, Prentice Hall, 1998.

Web Page

https://www.lehigh.edu/~eus204/teaching/ME450_MRC/ME450_MRC.html

Introduction

1. Study the “plant” to be controlled and learn about control objectives
2. Model the system and simplify the model (if necessary)
3. Analyze the resulting model; determine its properties
4. Decide which variables are to be controlled (controlled outputs)
5. Decide on measurement (sensors)/manipulated (actuators) variables
6. Select the control configuration
7. Decide on the type of controller to be used
8. Decide on performance specifications (based on control objectives)
9. Design a controller
10. Analyze resulting controlled system (redesign if necessary)
11. Simulate the resulting controlled system
12. Repeat from Step 2, if necessary.
13. Choose hardware and software and implement the controller
14. Test and validate control system and tune on-line if necessary

Introduction

$\Rightarrow$ Much of the course will be spent on input-output "controllability analysis" of the plant/process.

Always keep in mind

- Power of control is limited.
- Control quality depends controller **AND** on plant/process.

Introduction

$$y = Gu + G_d d \tag{1}$$

y	:	output/controlled variable
u	:	input/manipulated variable
d	:	disturbance
r	:	reference/setpoint

Regulator problem	:	counteract d
Servo/Tracking problem	:	let y follow r

Goal of control: make control error $e = y - r$ “small”.

Introduction

Major difficulties:

Model (G, G_d) inaccurate
$\Rightarrow$ RealPlant: $G_p = G + E$;
$E =$ "uncertainty" or "perturbation" (unknown but bounded)

- **Nominal stability (NS)** : system is stable with no model uncertainty.
- **Nominal Performance (NP)** : system satisfies performance specifications with no model uncertainty.
- **Robust stability (RS)** : system stable for "all" perturbed plants
- **Robust performance (RP)** : system satisfies performance specifications for all perturbed plants

Introduction

The course will be make extensive use of the *transfer function* representation, i.e., we will work on the frequency domain.

Given a state-space representation

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx + Du, \end{aligned}$$

the transfer-function representation is given by

$$\frac{y(s)}{u(s)} = G(s) = C(sI - A)^{-1}B + D, \quad G(s) = \left[\begin{array}{cc} A & B \\ C & D \end{array} \right]$$

Introduction

$$G(s) = \frac{\beta_{n_z} s^{n_z} + \cdots + \beta_1 s + \beta_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \tag{2}$$

For multivariable systems, $G(s)$ is a matrix of transfer functions.
$n =$ order of denominator (or pole polynomial) or *order of the system*
$n_z =$ order of numerator (or zero polynomial)
$n - n_z =$ pole excess or *relative order*.

Introduction

Definition

- A system $G(s)$ is *strictly proper* if $G(s) \to 0$ as $s \to \infty$.
- A system $G(s)$ is *semi-proper* or *bi-proper* if $G(s) \to D \neq 0$ as $s \to \infty$.
- A system $G(s)$ which is strictly proper or semi-proper is *proper*.
- A system $G(s)$ is *improper* if $G(s) \to \infty$ as $s \to \infty$.

Introduction

Proper scaling simplifies controller design and performance analysis.

SISO :

unscaled:

$$\hat{y} = \hat{G}\hat{u} + \hat{G}_d\hat{d}; \quad \hat{e} = \hat{y} - \hat{r} \tag{3}$$

scaled:

$$d = \hat{d}/\hat{d}_{\max}, \quad u = \hat{u}/\hat{u}_{\max} \tag{4}$$

where:

- $\hat{d}_{\max}$ — largest expected change in disturbance
- $\hat{u}_{\max}$ — largest allowed input change

Introduction

Scale $\hat{y}$, $\hat{e}$ and $\hat{r}$ by:

- $\hat{e}_{\max}$ — largest allowed control error, or
- $\hat{r}_{\max}$ — largest expected change in reference value

Usually:

$$y = \hat{y}/\hat{e}_{\max}, \quad r = \hat{r}/\hat{e}_{\max}, \quad e = \hat{e}/\hat{e}_{\max} \tag{5}$$

MIMO :

$$d = D_d^{-1}\hat{d}, \quad u = D_u^{-1}\hat{u}, \quad y = D_e^{-1}\hat{y} \tag{6}$$

$$e = D_e^{-1}\hat{e}, \quad r = D_e^{-1}\hat{r} \tag{7}$$

where $D_e = \hat{e}_{\max}$, $D_u = \hat{u}_{\max}$, $D_d = \hat{d}_{\max}$ and $D_r = \hat{r}_{\max}$ are diagonal scaling *matrices*

Introduction

Substituting (6) and (7) into (3):

$$D_e y = \hat{G} D_u u + \hat{G}_d D_d d; \quad D_e e = D_e y - D_e r$$

and introducing the scaled transfer functions

$$G = D_e^{-1} \hat{G} D_u, \quad G_d = D_e^{-1} \hat{G}_d D_d \tag{8}$$

Model in terms of scaled variables:

$$y = Gu + G_d d; \quad e = y - r \tag{9}$$

Often also:

$$\tilde{r} = \hat{r}/\hat{r}_{\max} = D_r^{-1} \hat{r} \tag{10}$$

so that:

$$r = R\tilde{r} \quad \text{where} \quad R \stackrel{\Delta}{=} D_e^{-1} D_r = \hat{r}_{\max}/\hat{e}_{\max} \tag{11}$$

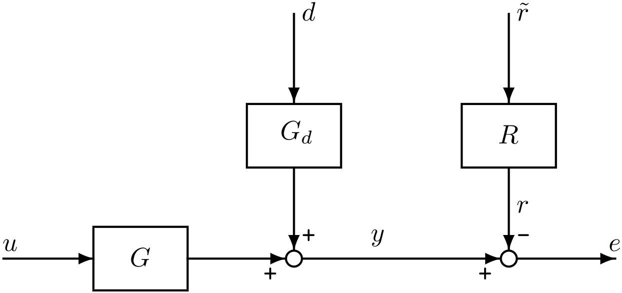


Figure 1: Model in terms of scaled variables

Objective:
for $|d(t)| \leq 1$ and $|\tilde{r}(t)| \leq 1$,
manipulate u with $|u(t)| \leq 1$
such that $|e(t)| = |y(t) - r(t)| \leq 1$.

Introduction

Given the general nonlinear model

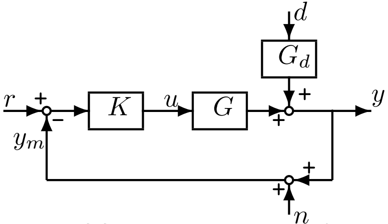
$$\dot{x} = f(x, u) \tag{12}$$

where (x^*, u^*) is an equilibrium, i.e., $f(x^*, u^*) = 0$. The linearization around this equilibrium is given by

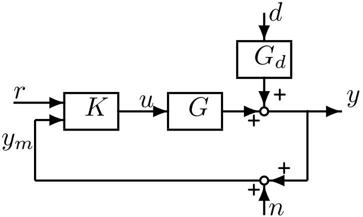
$$\dot{\tilde{x}} \approx \left(\frac{\partial f}{\partial x}\right)^* \tilde{x} + \left(\frac{\partial f}{\partial u}\right)^* \tilde{u} \triangleq A\tilde{x} + B\tilde{u} \tag{13}$$

where $\tilde{x} \triangleq x - x^*$ and $\tilde{u} \triangleq u - u^*$. Therefore,

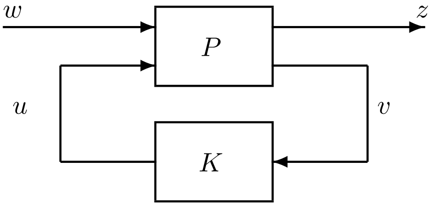
$$\tilde{x}(s) = (sI - A)^{-1} B\tilde{u} \tag{14}$$



(a) One degree-of-freedom control configuration



(b) Two degrees-of-freedom control configuration



(c) General control configuration

Figure 2: Control configurations

Table 1: Nomenclature

K	controller, in whatever configuration. Sometimes broken down into parts. For example, in Figure 2(b), $K = [K_r \quad K_y]$ where K_r is a prefilter and K_y is the feedback controller.
Conventional configurations (Fig 2(a), 2(b)):	
G	plant model
G_d	disturbance model
r	reference inputs (commands, setpoints)
d	disturbances (process noise)
n	measurement noise
y	plant outputs. (include the variables to be controlled ("primary" outputs with reference values r) and possibly additional "secondary" measurements to improve control)
y_m	measured y
u	control signals (manipulated plant inputs)

Introduction

General configuration (Fig 2(c)):

P generalized plant model. Includes G and G_d and the interconnection structure between the plant and the controller.
May also include weighting functions.

w exogenous inputs: commands, disturbances and noise

z exogenous outputs; "error" signals to be minimized, e.g. $y - r$

v controller inputs for the general configuration, e.g. commands, measured plant outputs, measured disturbances, etc. For the special case of a one degree-of-freedom controller with perfect measurements we have $v = r - y$.

u control signals