Burn Control in Fusion Reactors Using Simultaneous Boundary and Distributed Actuation

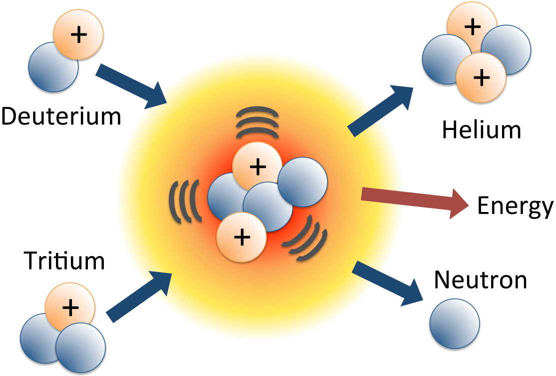
Mark D. Boyer and **Eugenio Schuster**

LEHIGH UNIVERSITY

Department of Mechanical Engineering & Mechanics

December 12, 2013
52nd IEEE Conference on Decision and Control

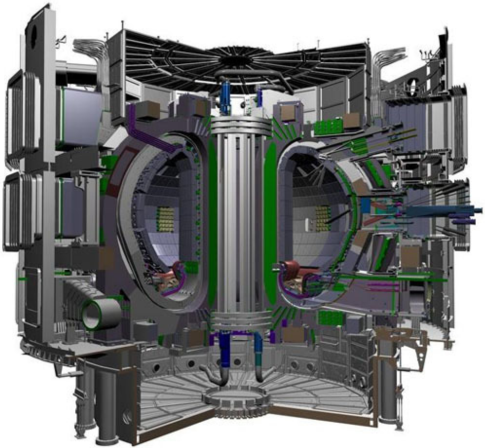
Nuclear Fusion



- In magnetic confinement fusion:
  - neutron escapes to the walls and energy can be captured to create electricity
  - energetic **alpha particle** remains in the plasma, creating a **'self-heating' source**.
- **Reactor efficiency** is characterized by

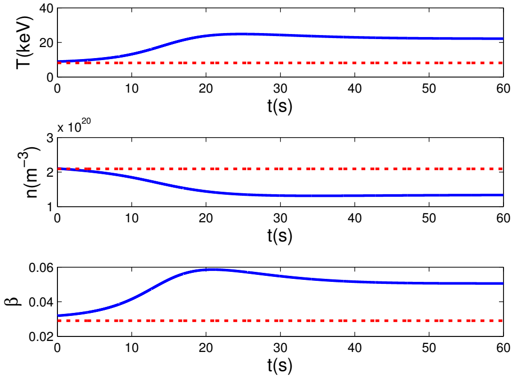
$$Q = \frac{P_{fusion}}{P_{aux}}$$

The ITER Tokamak



- A multi-billion dollar international collaboration.
- The first tokamak to explore the **burning plasma regime**.
- Designed to achieve
  - Q>5 for 1000 s long discharges,
  - Q=10 for certain operating scenarios.

Burn Control Challenges



- For experiments like ITER, burn control scheme should be flexible → **model-based**.
- Coupled, nonlinear dynamics → **nonlinear control**.
- Multiple actuators, each subject to saturation limits, and many plasma parameters must be regulated → **multi-variable control**.
- Potential for **thermal instability**.

- Even when operating at stable equilibria, system **performance during transients and disturbances** could be **undesirable without control.**
- **Active burn control could enable operation at unstable operating points and could improve overall reactor performance.**

Previous Approaches to Burn Control

- Previous work has focused on controlling bulk parameters like the **spatial averages of density and temperature**.
- Most approaches consider only one of the available actuators **(SISO)** and design controllers based on **linearized models**.
- In previous work, we have proposed a **nonlinear controller design using the available actuators simultaneously**.
  - Much better performance results from this design
  - Still, the **spatial distribution of parameters was not considered**.

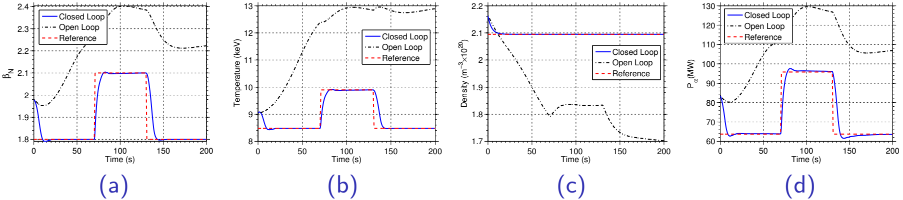
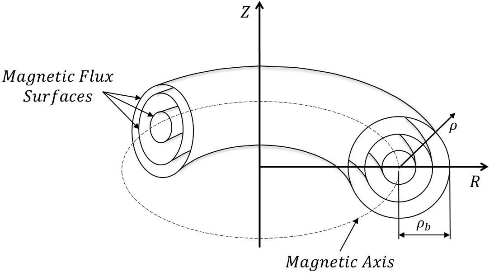


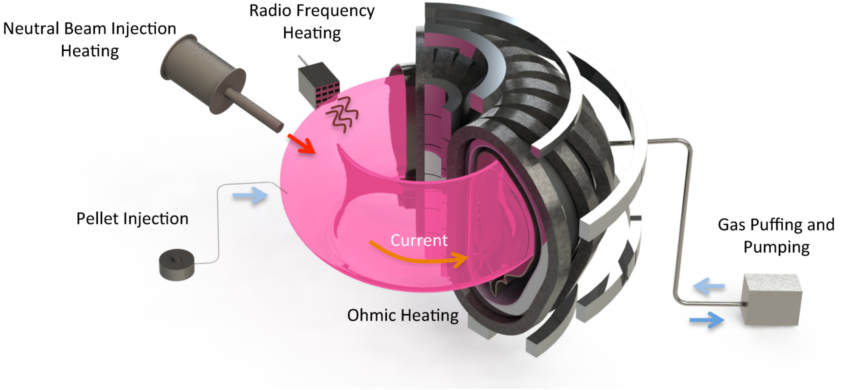
Figure: Closed loop simulation with nonlinear controller, open loop simulation, and desired operating points.

Kinetic Profile Control Importance and Challenges

- Importance of Profile Control
  - Profiles affect the **burn condition** and plasma parameters.
  - Profile shapes affect c**onfinement**, **MHD stability**, and **non-inductive current drive**.
  - Tailoring the shape of profiles could lead to **high-performance, steady-state plasmas**.
- Profile Control Challenges
  - **Infinite dimensional system** with finite number of actuators.
  - **Nonlinear**, coupled dynamics with the potential of **instability**.
  - **Boundary** actuation and **interior** actuation.



Actuators Used To Control Kinetic Variables



- Plasma current contributes to heating through **Ohmic heating**.
- **Neutral beam injectors** and **radio frequency waves** heat the plasma and drive non-inductive current.
- Refueling at the plasma boundary is achieved through **gas puffing**.
- **Pellet injection** refuels the plasma in the core.
- **Gas pumping** removes exhausted fuel, alpha particles, and impurities.

1D Burning Plasma Model

We consider a simplified model of the 1D dynamics

$$\begin{aligned}
\frac{\partial n_\alpha}{\partial t} &= \frac{1}{r}\frac{\partial}{\partial r} r\left(D\frac{\partial n_\alpha}{\partial r}\right) + S_\alpha, \\
\frac{\partial n_{DT}}{\partial t} &= \frac{1}{r}\frac{\partial}{\partial r} r\left(D\frac{\partial n_{DT}}{\partial r}\right) - 2S_\alpha + u_{fuel}(t)\hat{S}_{DT}(r), \\
\frac{\partial E}{\partial t} &= \frac{1}{r}\frac{\partial}{\partial r} r\left(D\frac{\partial E}{\partial r}\right) + Q_\alpha S_\alpha - P_{rad} + u_{aux}(t)\hat{P}_{aux}(r),
\end{aligned}$$

with the boundary conditions

$$\left.\frac{\partial n_\alpha}{\partial r}\right|_{r=0} = \left.\frac{\partial n_{DT}}{\partial r}\right|_{r=0} = \left.\frac{\partial E}{\partial r}\right|_{r=0} = 0,$$

$$\begin{aligned}
n_\alpha(a) &= u_\alpha(t), \\
n_{DT}(a) &= u_{DT}(t), \\
E(a) &= u_E(t).
\end{aligned}$$

Error Dynamics

We consider a set of **equilibrium inputs** $\bar{u}$ and the presence of **feedback** terms $\tilde{u}$ and **input disturbances** d. Estimating the unknown disturbance with $\hat{d}$, we define the **feedback law** $\tilde{u} = v - \hat{d}$. This results in the error dynamics

$$\frac{\partial \tilde{n}_\alpha}{\partial t} = D\frac{\partial^2 \tilde{n}_\alpha}{\partial r^2} + \frac{1}{r}D\frac{\partial \tilde{n}_\alpha}{\partial r} + \tilde{S}_\alpha \quad (1)$$

$$\frac{\partial \tilde{n}_{DT}}{\partial t} = D\frac{\partial^2 \tilde{n}_{DT}}{\partial r^2} + \frac{1}{r}D\frac{\partial \tilde{n}_{DT}}{\partial r} - 2\tilde{S}_\alpha + \left(v_{fuel} + \tilde{d}_{DT}\right)\hat{S}_{DT} \quad (2)$$

$$\frac{\partial \tilde{E}}{\partial t} = D\frac{\partial^2 \tilde{E}}{\partial r^2} + \frac{1}{r}D\frac{\partial \tilde{E}}{\partial r} + Q_\alpha \tilde{S}_\alpha - \tilde{P}_{rad} + \left(v_{aux} + \tilde{d}_{aux}\right)\hat{P}_{aux} \quad (3)$$

The **boundary conditions** are written as

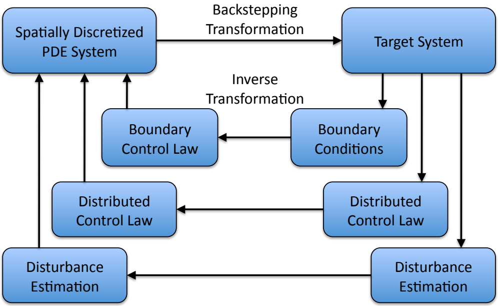
$$\left.\frac{\partial \tilde{n}_\alpha}{\partial r}\right|_{r=0} = \left.\frac{\partial \tilde{n}_{DT}}{\partial r}\right|_{r=0} = \left.\frac{\partial \tilde{E}}{\partial r}\right|_{r=0} = 0 \quad (4)$$

$$\tilde{n}_\alpha(a) = v_\alpha(t) + \tilde{d}_\alpha \quad (5)$$

$$\tilde{n}_{DT}(a) = v_{DT}(t) + \tilde{d}_{DT} \quad (6)$$

$$\tilde{E}(a) = v_E(t) + \tilde{d}_E \quad (7)$$

Backstepping Technique



- The backstepping technique provides a **recursive method for finding a boundary condition control law** that transforms the original system into a chosen target system.
- The **stability and performance** of the closed loop system can be altered through the **choice of the target system**.
- In this work, the method is extended to include **distributed interior control and online disturbance estimation**.

Backstepping Transformation

ODE Model

$$\dot{\tilde{E}}^i = D\frac{\tilde{E}^{i+1} - 2\tilde{E}^i + \tilde{E}^{i-1}}{h^2} + \frac{D}{ih}\frac{\tilde{E}^{i+1} - \tilde{E}^i}{h} + Q_\alpha \tilde{S}^i_\alpha - \tilde{P}^i_{rad} + \left(v_{aux} + \tilde{d}_{aux}\right)\hat{P}^i_{aux},$$

$$\tilde{E}^N = v_E(t).$$

Target System

$$\dot{\tilde{f}}^i = D\frac{\tilde{f}^{i+1} - 2\tilde{f}^i + \tilde{f}^{i-1}}{h^2} + \frac{1}{ih}D\frac{\tilde{f}^{i+1} - \tilde{f}^i}{h} - C_f\tilde{f}^i + \left(v_{fuel} + \tilde{d}_{fuel}\right)A^i_{fuel} + \left(v_{aux} + \tilde{d}_{aux}\right)A^i_{aux},$$

$$\tilde{f}^N = 0.$$

- By subtracting the system equations, a formula is found for a transformation of the form

$$\tilde{f}^i = \tilde{E}^i - \alpha^{i-1}(\tilde{E}^0, \ldots, \tilde{E}^{i-1}, \tilde{n}^0_{DT}, \ldots, \tilde{n}^{i-1}_{DT}, \tilde{n}^0_\alpha, \ldots, \tilde{n}^{i-1}_\alpha)$$

- A boundary control law is then found by subtracting the boundary conditions, i.e., $v_E(t) = \alpha^{N-1}$.

Backstepping Transformation

$$\alpha^i = \frac{1}{D + D/i}\left[\left(2D + \frac{D}{i} + C_f h^2\right)\alpha^{i-1} - D\alpha^{i-2} - h^2 C_f \tilde{E}^i \right.$$
$$+ h^2 \dot{\alpha}^{i-1} - h^2 Q_\alpha \tilde{S}_\alpha^i + h^2 \tilde{P}_{rad}^i - h^2\left(v_{aux} + \tilde{d}_{aux}\right)\hat{P}_{aux}^i$$
$$\left. + h^2\left(v_{fuel} + \tilde{d}_{fuel}\right) A_{fuel}^i + h^2\left(v_{aux} + \tilde{d}_{aux}\right) A_{aux}^i \right] \quad (8)$$

where $\alpha^0 = 0$ and $\dot{\alpha}^{i-1}$ is given by

$$\dot{\alpha}^{i-1} = \sum_{k=1}^{i-1} \frac{\partial \alpha^{i-1}}{\partial \tilde{n}_{DT}^k} \dot{\tilde{n}}_{DT}^k + \sum_{k=1}^{i-1} \frac{\partial \alpha^{i-1}}{\partial \tilde{E}^k} \dot{\tilde{E}}^k + \sum_{k=1}^{i-1} \frac{\partial \alpha^{i-1}}{\partial \tilde{n}_\alpha^k} \dot{\tilde{n}}_\alpha^k \quad (9)$$

By choosing

$$A_{fuel}^i = -\sum_{k=1}^{i-1} \frac{\partial \alpha^{i-1}}{\partial \tilde{n}_{DT}^k} \hat{S}_{DT}^k \qquad A_{aux}^i = \hat{P}_{aux}^i - \sum_{k=1}^{i-1} \frac{\partial \alpha^{i-1}}{\partial \tilde{E}^k} \hat{P}_{aux}^k$$

the non-spatially-causal terms of the transformation are eliminated, allowing it to be calculated recursively and recovering the strict- feedback structure required for backstepping.

Backstepping Transformation

The distributed control laws v_{fuel} and v_{aux}, and the disturbance estimation update laws are designed by considering the control Lyapunov function

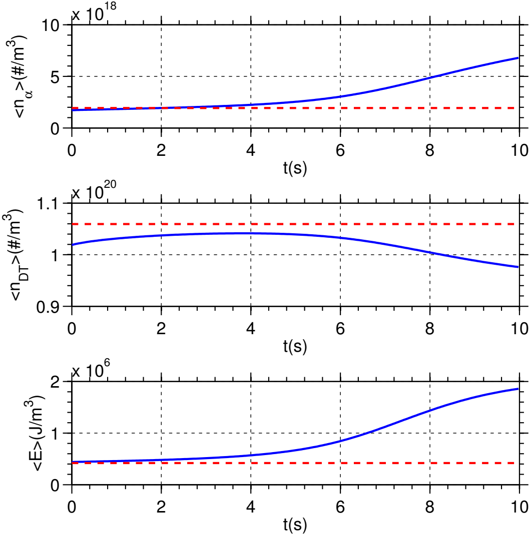
$$V = \frac{1}{2}\sum_{i=1}^{N-1} Q_w^i \left(\tilde{w}^i\right)^2 + \frac{1}{2}\sum_{i=1}^{N-1} Q_m^i \left(\tilde{m}^i\right)^2 + \frac{1}{2}\sum_{i=1}^{N-1} Q_f^i \left(\tilde{f}^i\right)^2 + \frac{\tilde{d}_\alpha^2}{2k_\alpha} + \frac{\tilde{d}_{DT}^2}{2k_{DT}} + \frac{\tilde{d}_E^2}{2k_E} + \frac{\tilde{d}_{fuel}^2}{2k_{fuel}} + \frac{\tilde{d}_{aux}^2}{2k_{aux}}$$

where Q_w^i, Q_m^i, Q_f^i for $i \in [1, N-1]$ are positive definite weights, and k_α, k_{DT}, k_E, k_{fuel}, and k_{aux} are positive constants.

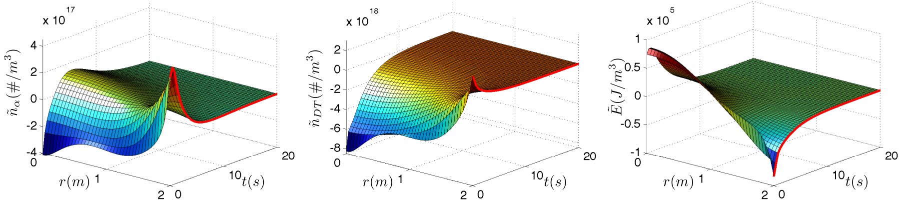
Interior control laws and disturbance estimation update laws are chosen to **render the target system asymptotically stable**.

Simulation Results

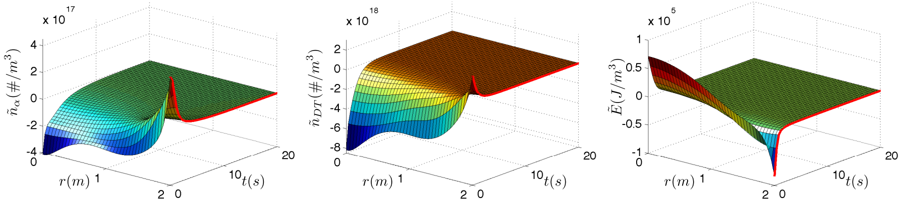
- Desired equilibrium is **unstable in open loop**.
- First set of simulations compare results for **boundary+interior control with boundary control only**.
- Second set of simulations compare results with and without **disturbance estimation**.



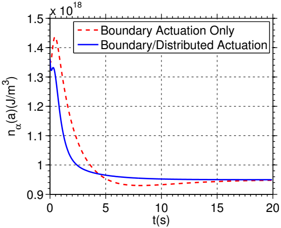
Simulation Results: Boundary vs Boundary+Interior



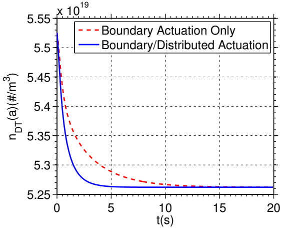
(a) Boundary feedback only.



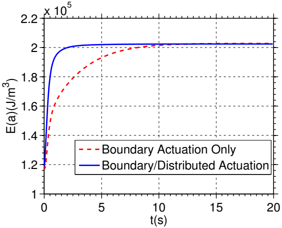
(b) Simultaneous boundary and distributed feedback.


x 10^18
Boundary Actuation Only
Boundary/Distributed Actuation
$n_\alpha(a)(J/m^3)$
t(s)

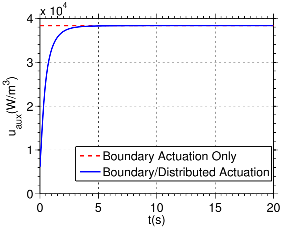
(a)


x 10^19
Boundary Actuation Only
Boundary/Distributed Actuation
$n_{DT}(a)(\#/m^3)$
t(s)

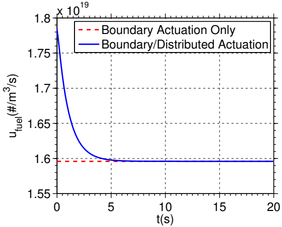
(b)


x 10^5
Boundary Actuation Only
Boundary/Distributed Actuation
$E(a)(J/m^3)$
t(s)

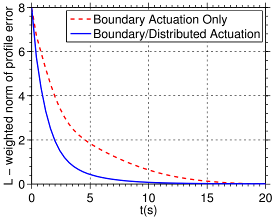
(c)


x 10^4
Boundary Actuation Only
Boundary/Distributed Actuation
$u_{aux}(W/m^3)$
t(s)

(d)

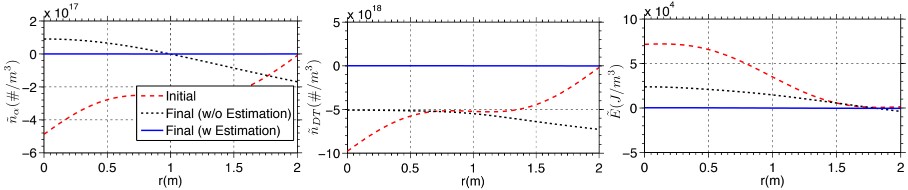

x 10^19
Boundary Actuation Only
Boundary/Distributed Actuation
$u_{fuel}(\#/m^3/s)$
t(s)

(e)


Boundary Actuation Only
Boundary/Distributed Actuation
L – weighted norm of profile error
t(s)

(f)

Simulation Results: Disturbance Estimation

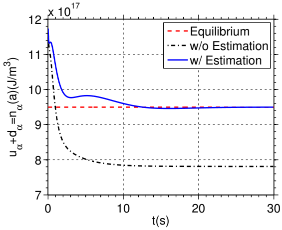


(a) (b) (c)

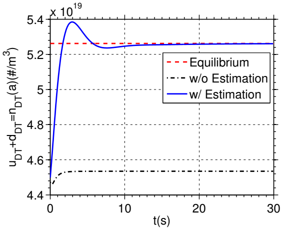
The weighted norm of the profile error is introduced for performance comparison

$$L = \sqrt{\sum_{i=1}^{N-1} \left[\left(10^{-5}\tilde{E}^i\right)^2 + (10^{-18}\tilde{n}_\alpha^i)^2 + \left(10^{-19}\tilde{n}_{DT}^i\right)^2 \right] h}$$

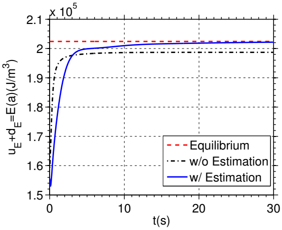
Simulation Results: Disturbance Estimation



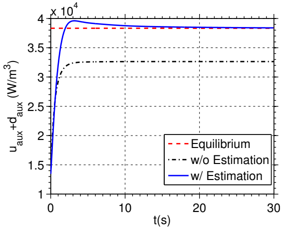
(d)



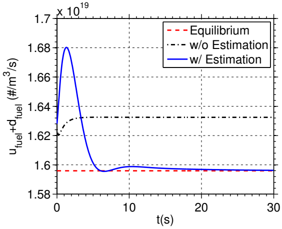
(e)



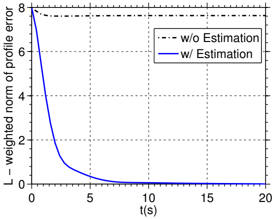
(f)



(g)



(h)



(i)

Conclusions and Future Work

- A **nonlinear feedback controller** for kinetic profiles in a burning plasma has been proposed.
- The **backstepping boundary control** technique was extended to include **interior feedback actuation** and **estimation of input disturbances**.
- Simulations show that a **controller designed with a small number of steps** is able to stabilize an unstable equilibrium.
- Interior control and disturbance estimation are both shown to **improve closed loop performance**.
- Future work:
  - Develop a model of the plasma **scrape-off layer** to create more realistic boundary conditions.
  - Assess and deal with **uncertainty** in the model (diffusivity modeling).