Backstepping control of the current profile in the DIII-D Tokamak

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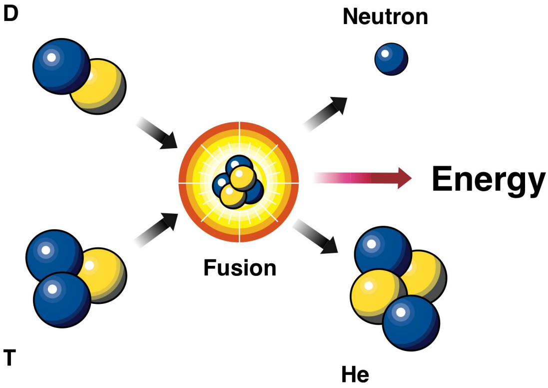
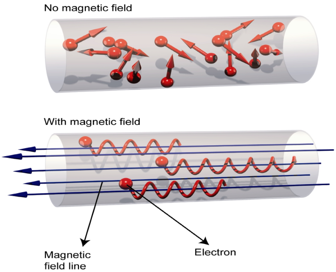
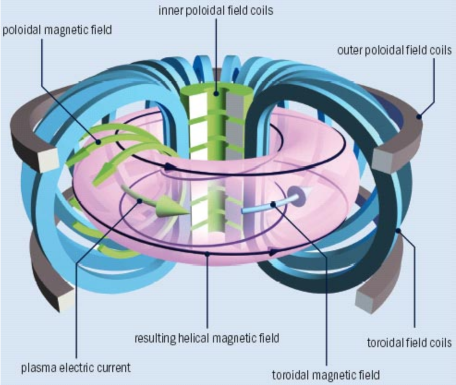
Nuclear Fusion



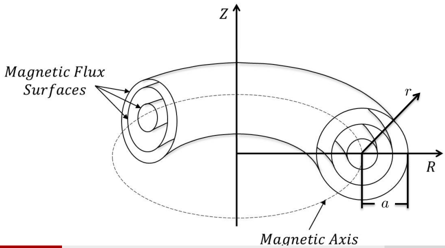
The Tokamak


No magnetic field
With magnetic field
Magnetic field line
Electron


inner poloidal field coils
poloidal magnetic field
outer poloidal field coils
resulting helical magnetic field
toroidal field coils
plasma electric current
toroidal magnetic field

The Need for Current Profile Control

- One of the challenges of tokamak fusion reactors is achieving operation with sufficiently long plasma discharges.
- Non-inductive sources of current are required for steady state-operation
- Setting up a suitable toroidal current profile can lead to self-generated, non-inductive current (bootstrap current)
- Controlling the current profile will therefore be important to achieving steady-state reactor operation



Overview

- A control oriented, first principles based model for the current profile evolution in DIII-D was developed
- We utilized a backstepping technique to design a feedback law for controlling the current profile
- The current profile control algorithm was implemented in the DIII-D Plasma Control System
- Simulations were performed to test the implementation code and tune the controller design
- Experimental tests of the controller were also done, showing results that are very similar to the simulations

Current Profile Evolution Model

- Derived from Gauss's law, Ampere's law, Faraday's law, Ohm's law, and an equilibrium momentum balance

$$\frac{\partial \psi}{\partial t} = f_1(\hat{\rho})u_1(t)\frac{1}{\hat{\rho}}\frac{\partial}{\partial \hat{\rho}}\left(\hat{\rho} f_4(\hat{\rho})\frac{\partial \psi}{\partial \hat{\rho}}\right) + f_2(\hat{\rho})u_2(t) \tag{1}$$

with boundary conditions

$$\left.\frac{\partial \psi}{\partial \hat{\rho}}\right|_{\hat{\rho}=0} = 0 \qquad \left.\frac{\partial \psi}{\partial \hat{\rho}}\right|_{\hat{\rho}=1} = -k_3 u_3(t). \tag{2}$$

Note: $u_1(t)$, $u_2(t)$, and $u_3(t)$ are the control actuators which are nonlinear functions of:

  - Total plasma current.
  - Total non-inductive power (neutral beams).
  - Line-averaged plasma density.

Current Profile Evolution Model - Flux Gradient

- The development of the bootstrap current is related to the shape of the q profile, which is proportional to $\theta = \partial\psi/\partial\hat{\rho}$. We can find:

$$\frac{\partial\theta}{\partial t} = h_0(\hat{\rho})u_1\theta'' + h_1(\hat{\rho})u_1\theta' + h_2(\hat{\rho})u_1\theta + h_3(\hat{\rho})u_2 \tag{3}$$

with boundary conditions:

$$\theta\Big|_{\hat{\rho}=0} = 0 \qquad \theta\Big|_{\hat{\rho}=1} = -k_3u_3(t) \tag{4}$$

- Using feedforward inputs and nominal initial conditions, the system would satisfy

$$\frac{\partial\theta_{ff}}{\partial t} = h_0u_{1_{ff}}\theta''_{ff} + h_1u_1\theta' + h_2u_{1_{ff}}\theta + h_3u_{2_{ff}} \tag{5}$$

with boundary conditions:

$$\theta\Big|_{\hat{\rho}=0} = 0 \qquad \theta\Big|_{\hat{\rho}=1} = -k_3u_{3_{ff}} \tag{6}$$

Current Profile Evolution Model - Deviations

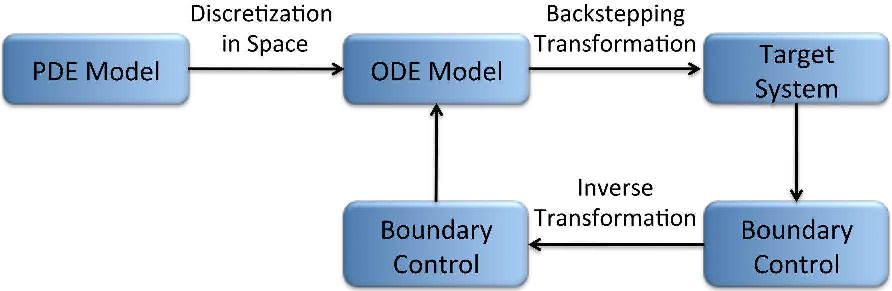
- In the presence of disturbances or perturbed initial conditions, an unwanted deviation $\tilde{\theta} = \theta - \theta_{ff}$ will exist.
- We can write the evolution of the deviations as

$$\frac{\partial \tilde{\theta}}{\partial t} = h_0 u_{1_{ff}} \tilde{\theta}'' + h_1 u_{1_{ff}} \tilde{\theta}' + h_2 u_{1_{ff}} \tilde{\theta} \tag{7}$$

with boundary conditions

$$\tilde{\theta}\Big|_{\hat{\rho}=0} = 0 \qquad \tilde{\theta}\Big|_{\hat{\rho}=1} = -k_3 u_{3_{fb}} \tag{8}$$

Backstepping Controller Design



- The backstepping technique provides a recursive method for finding a boundary condition control law that transforms the original system into a chosen target system.
- The stability and performance of the closed loop system can be altered through the choice of the target system.

Backstepping Transformation

ODE Model

$$\dot{\tilde{\theta}}^i = h_0^i u_{1\,ff} \frac{\tilde{\theta}^{i+1} - 2\tilde{\theta}^i + \tilde{\theta}^{i-1}}{h^2} + h_1^i u_{1\,ff} \frac{\tilde{\theta}^{i+1} - \tilde{\theta}^{i-1}}{2h} + h_2^i u_{1\,ff} \tilde{\theta}^i \qquad (9)$$

$$\tilde{\theta}^N = -k_3 u_{3\,fb} \qquad (10)$$

$\rightarrow$

Target System

$$\dot{\tilde{w}}^i = h_0^i u_{1\,ff} \frac{\tilde{w}^{i+1} - 2\tilde{w}^i + \tilde{w}^{i-1}}{h^2} + h_1^i u_{1\,ff} \frac{\tilde{w}^{i+1} - \tilde{w}^{i-1}}{2h} + h_2^i u_{1\,ff} \tilde{w}^i - C_w^i u_{1\,ff} \tilde{w}^i \qquad (11)$$

$$\tilde{w}^N = 0 \qquad (12)$$

- We find a transformation of the form

$$\tilde{w}^i = \tilde{\theta}^i - \alpha^{i-1}(\tilde{\theta}^0, \ldots, \tilde{\theta}^i - 1)$$

by subtracting (11) from (9) and solving for α^i.

- We obtain the formula

$$\alpha^i = -\left[\frac{1}{\frac{h_0^i}{h^2} + \frac{h_1^i}{2h}}\right]\left[\left(\frac{-2h_0^i}{h^2} + h_2^i - C_w^i\right)\alpha^{i-1} + \left(\frac{h_0^i}{h^2} - \frac{h_1^i}{2h}\right)\alpha^{i-2} - \frac{1}{u_{1_{ff}}}\dot{\alpha}^{i-1} + C_w^i\tilde{\theta}^i\right] \quad (13)$$

where $\dot{\alpha}^{i-1}$ is calculated as

$$\dot{\alpha}^{i-1} = \sum_{k=1}^{i-1} \frac{\partial \alpha^{i-1}}{\partial \tilde{\theta}^k}\dot{\tilde{\theta}}^k \quad (14)$$

- Expression (13) can be recursively evaluated at each node of the discretization scheme, starting with $\alpha^0 = 0$.

Boundary Condition Control Law

- The boundary condition control law is found by subtracting (12) from (10) and solving the resulting expression for $u_{3_{fb}}$:

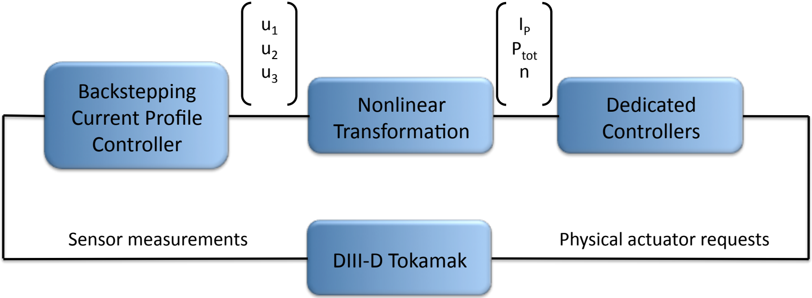
$$u_{3_{fb}} = -\frac{1}{k_3}\alpha^{N-1} \tag{15}$$

- The expression α^{N-1} is a linear function of the measurements of $\tilde{\theta}$ at each of the interior nodes that can be evaluated offline to obtain a static state feedback control law.

$$u_{3_{fb}} = -K\Theta \tag{16}$$

where K is a row of controller gain values and Θ is a vector of error measurements.

Nonlinear Transformation of Inputs



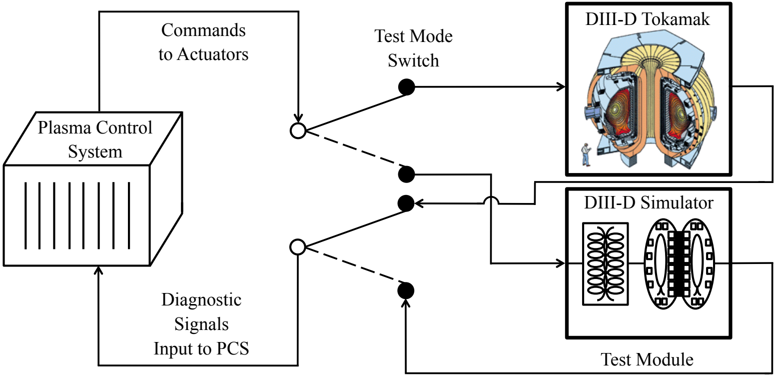
$$I_p = u_3 \tag{17}$$

$$P_{tot} = u_3^2 u_{2_{ff}}^2 \tag{18}$$

$$\bar{n} = u_{1_{ff}}^{2/3} u_3^2 u_{2_{ff}} \tag{19}$$

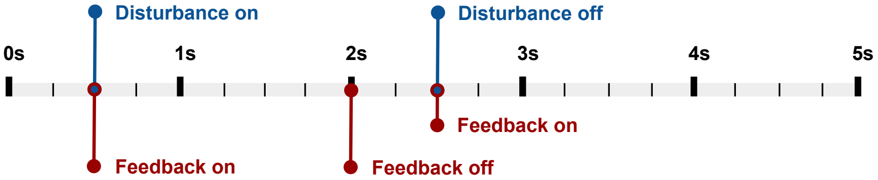
Simserver Simulation

- Prior to experimental testing, a simserver simulation study was done
- The simserver architecture allows the PCS to receive simulated data and provide control commands to a simulation model
- This enabled us to tune the controller design and debug the implementation of the algorithm

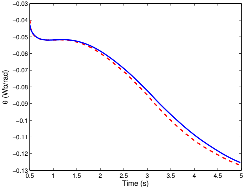


Controller Test Shots

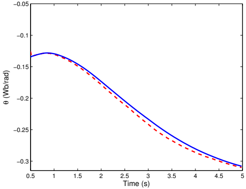
- In one shot, a particular set of feedforward inputs was used to to generate a target θ evolution
- In a second shot, an input bias was added to the feedforward inputs to artificially create profile perturbations and disturbances
- As part of testing, the controller and disturbances were turned on and off according to the timeline below



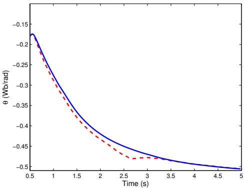
Simulation Results - Time Traces



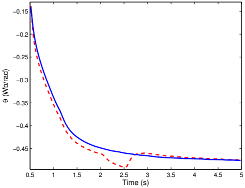
(a) $\hat{\rho} = 0.1$



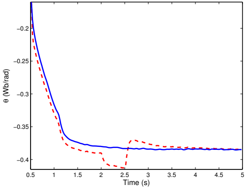
(b) $\hat{\rho} = 0.25$



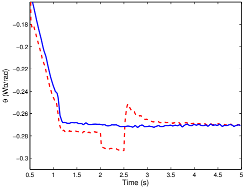
(c) $\hat{\rho} = 0.5$



(d) $\hat{\rho} = 0.65$



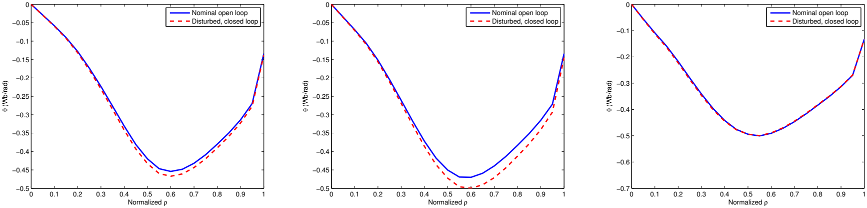
(e) $\hat{\rho} = 0.8$



(f) $\hat{\rho} = 0.95$

Figure: Time trace of θ at various points comparing the target (blue-solid) and the closed loop, disturbed simulation (red-dashed).

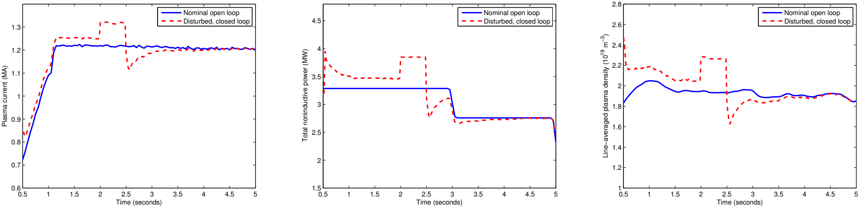
Simulation Results - Profiles



(a) $t = 2.00$s **(control on)** (b) $t = 2.5$s **(control off)** (c) $t = 4.00$s **(control on)**

Figure: Comparison of θ profiles at various times for the target (blue-solid) and the closed loop, disturbed case (red-dashed). Partial disturbance rejection is seen in (a), the effect of the uncontrolled disturbance can be noted in (b), and the recovery of the target profile after the disturbance is removed and the controller is turned back on can be observed in (c).

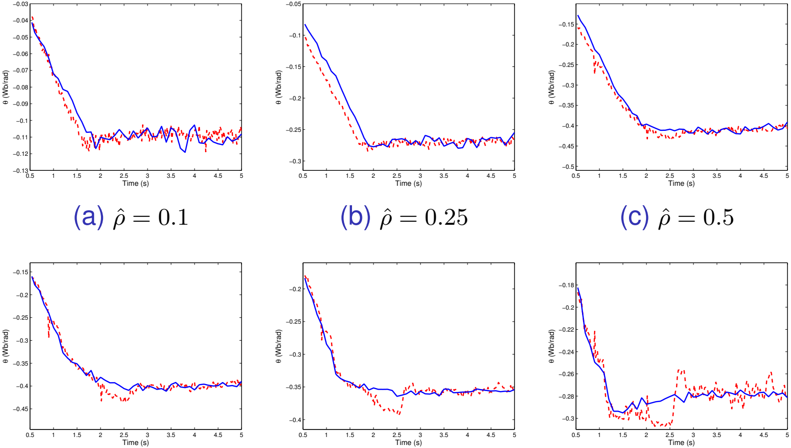
Simulation Results - Actuators



(a) Plasma current I_p (b) Power P_{tot} (c) Line averaged density $\bar{n}$

Figure: Comparison of actuators during the nominal simulation and the closed loop, disturbed simulation.

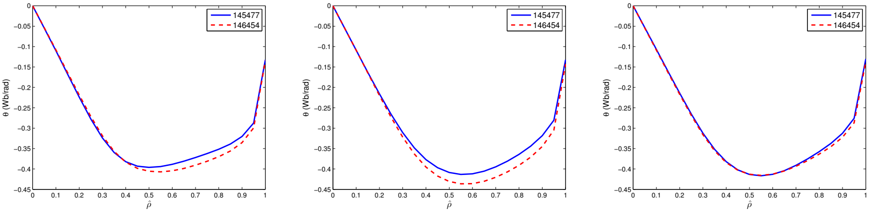
Experimental Results - Time Traces



(a) $\hat{\rho} = 0.1$ (b) $\hat{\rho} = 0.25$ (c) $\hat{\rho} = 0.5$

(d) $\hat{\rho} = 0.65$ (e) $\hat{\rho} = 0.8$ (f) $\hat{\rho} = 0.95$

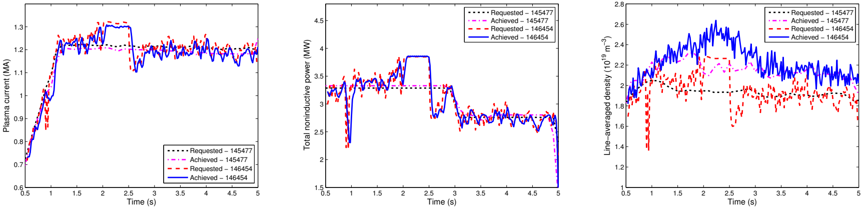
Figure: Time trace of θ at various points comparing the reference shot 145477 (blue-solid) and the closed loop, disturbed shot 146454 (red-dashed).



(a) $t = 2.00$s **(control on)** (b) $t = 2.50$s **(control off)** (c) $t = 4.00$s **(control on)**

Figure: Comparison of θ profiles at various times for reference shot 145477 (blue-solid) and the closed loop, disturbed shot 146454 (red-dashed). Partial disturbance rejection is seen in (a), the effect of the uncontrolled disturbance can be noted in (b), and the recovery of the target profile after the disturbance is removed and the controller is turned back on can be observed in (c).

Experimental Results - Actuators



(a) Plasma current I_p (b) Non-inductive power P_{tot} (c) Line averaged density $\bar{n}$

Figure: Comparison of requested and achieved actuator values during the reference shot 145477 and the closed loop, disturbed shot 146454.

Conclusions and Future Work

- Current profile control will be an important part of achieving steady-state fusion reactor operation.
- We have proposed an approach based on a first-principles model and a backstepping control design technique.
- Implemented in both simulations and experiments with promising results.
- We plan to add more feedback terms (u_{1fb}, u_{2fb}) for further performance improvement
- Integral action will be added to improve disturbance rejection
- The technique will eventually be applied to an H-mode discharge