Backstepping Control of Density and Energy Profiles in a Burning Tokamak Plasma

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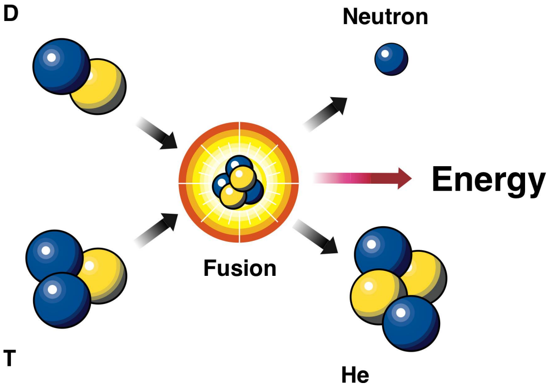
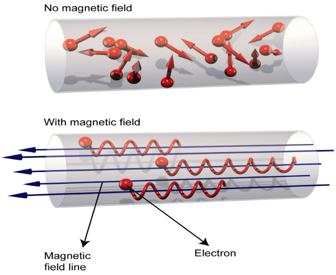
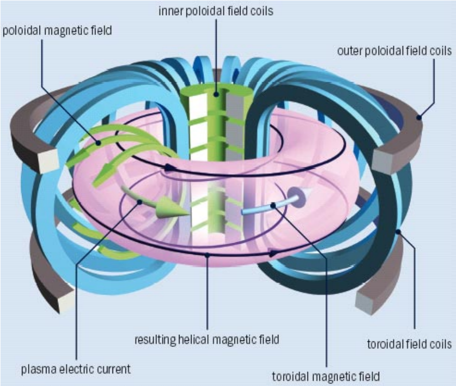
Nuclear fusion

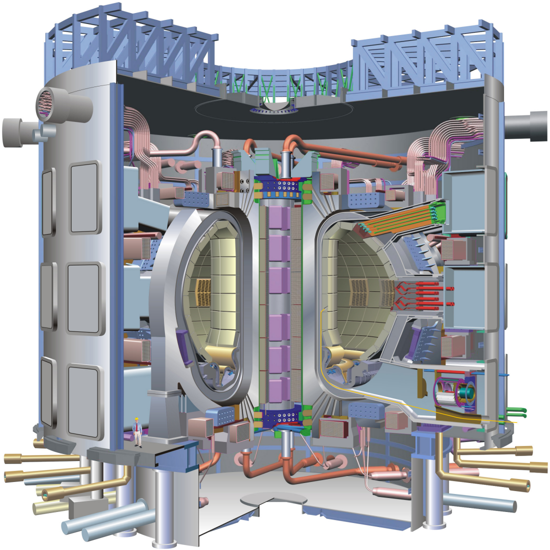


The tokamak


No magnetic field
With magnetic field
Magnetic
field line
Electron

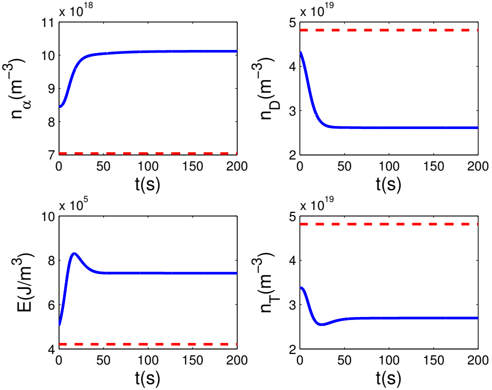

inner poloidal field coils
poloidal magnetic field
outer poloidal field coils
resulting helical magnetic field
toroidal field coils
plasma electric current
toroidal magnetic field

The next step: ITER



Burn control

- Regulation of plasma density, temperature, and fusion power
- Control needed for modification of burn condition during operation
- For certain operating conditions, the burn condition is unstable:

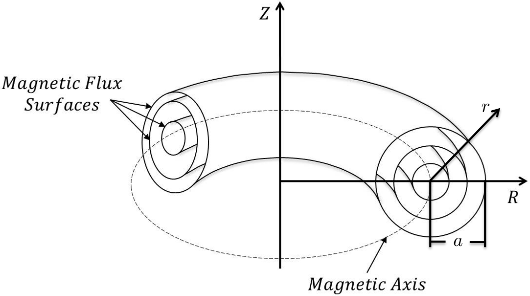


Previous approaches to burn control

- Previous work has focused on controlling spatial averages of density and temperature
- Most approaches consider only one of the available actuators (SISO) and design controllers based on linearized models
- In previous work, we have proposed a nonlinear controller design using the available actuators simultaneously
  - Much better performance results from this design
  - Still, the spatial distribution of parameters is not considered

Profile control in burning plasmas

- For optimal reactor performance, the spatial profiles, not just average values, of many of the plasma parameters must be controlled
- Density and current profile control can improve:
  - energy confinement
  - fusion power
  - plasma stability
  - the fraction of non-inductive plasma current



Nonlinear processes effecting the burn condition

Reaction rate

$$S_\alpha = \left(\frac{n_{DT}}{2}\right)^2 \langle\sigma\nu\rangle \tag{1}$$

where
$$\langle\sigma\nu\rangle = exp\left(\frac{a_1}{T^r} + a_2 + a_3T + a_4T^2 + a_5T^3 + a_6T^4\right) \tag{2}$$

$$T = \frac{2}{3}\frac{E}{n} \tag{3}$$

$$n = 2n_{DT} + 3n_\alpha \tag{4}$$

Radiation losses

$$P_{rad} = A_b Z_{eff} n_e^2 \sqrt{T} \tag{5}$$

where
$$n_e = n_{DT} + 2n_\alpha \tag{6}$$

$$Z_{eff} = \sum_i \frac{n_i Z_i^2}{n_e} = \frac{n_{DT} + 4n_\alpha}{n_e} \tag{7}$$

1-D burning plasma model

Energy and density transport

$$\frac{\partial n_\alpha}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left(D \frac{\partial n_\alpha}{\partial r} \right) + S_\alpha \quad (8)$$

$$\frac{\partial n_{DT}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left(D \frac{\partial n_{DT}}{\partial r} \right) - 2S_\alpha + S_{DT}(r) \quad (9)$$

$$\frac{\partial E}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \left(D \frac{\partial E}{\partial r} \right) + Q_\alpha S_\alpha - P_{rad} + P_{aux}(r) \quad (10)$$

Boundary conditions

$$\left.\frac{\partial n_\alpha}{\partial r}\right|_{r=a} = - k_\alpha n_\alpha(a) + \Delta(\tilde{n}_\alpha)_r \quad (11)$$

$$\left.\frac{\partial n_{DT}}{\partial r}\right|_{r=a} = - k_{DT} n_{DT}(a) + \Delta(\tilde{n}_{DT})_r \quad (12)$$

$$\left.\frac{\partial E}{\partial r}\right|_{r=a} = - k_E E(a) + \Delta\tilde{E}_r \quad (13)$$

Desired equilibrium

Energy and density transport

$$0 = \frac{1}{r}\frac{\partial}{\partial r} r \left(D \frac{\partial \bar{n}_\alpha}{\partial r} \right) + \bar{S}_\alpha \tag{14}$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r} r \left(D \frac{\partial \bar{n}_{DT}}{\partial r} \right) - 2\bar{S}_\alpha + \bar{S}_{DT}(r) \tag{15}$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r} r \left(D \frac{\partial \bar{E}}{\partial r} \right) + Q_\alpha \bar{S}_\alpha - \bar{P}_{rad} + \bar{P}_{aux}(r) \tag{16}$$

Boundary conditions

$$\left. \frac{\partial \bar{n}_\alpha}{\partial r} \right|_{r=a} = - k_\alpha \bar{n}_\alpha(a) \tag{17}$$

$$\left. \frac{\partial \bar{n}_{DT}}{\partial r} \right|_{r=a} = - k_{DT} \bar{n}_{DT}(a) \tag{18}$$

$$\left. \frac{\partial \bar{E}}{\partial r} \right|_{r=a} = - k_E \bar{E}(a) \tag{19}$$

Deviations from equilibrium

Energy and density transport

$$\frac{\partial \tilde{n}_\alpha}{\partial t} = D\frac{\partial^2 \tilde{n}_\alpha}{\partial r^2} + \frac{1}{r}D\frac{\partial \tilde{n}_\alpha}{\partial r} + \tilde{S}_\alpha \tag{20}$$

$$\frac{\partial \tilde{n}_{DT}}{\partial t} = D\frac{\partial^2 \tilde{n}_{DT}}{\partial r^2} + \frac{1}{r}D\frac{\partial \tilde{n}_{DT}}{\partial r} - 2\tilde{S}_\alpha \tag{21}$$

$$\frac{\partial \tilde{E}}{\partial t} = D\frac{\partial^2 \tilde{E}}{\partial r^2} + \frac{1}{r}D\frac{\partial \tilde{E}}{\partial r} + Q_\alpha \tilde{S}_\alpha - \tilde{P}_{rad} \tag{22}$$

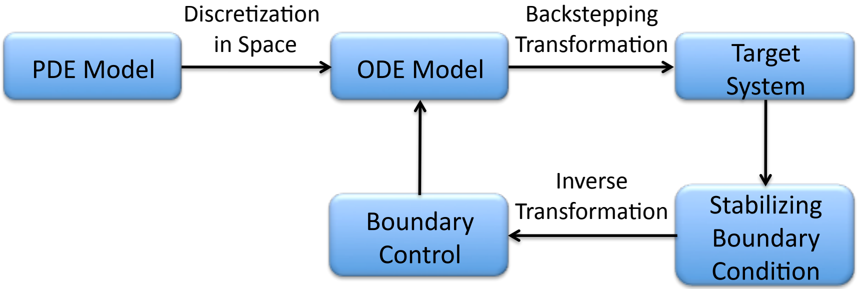
Boundary conditions

$$\left.\frac{\partial \tilde{n}_\alpha}{\partial r}\right|_{r=a} = -k_\alpha \tilde{n}_\alpha(a) + \Delta(\tilde{n}_\alpha)_r \tag{23}$$

$$\left.\frac{\partial \tilde{n}_{DT}}{\partial r}\right|_{r=a} = -k_{DT} \tilde{n}_{DT}(a) + \Delta(\tilde{n}_{DT})_r \tag{24}$$

$$\left.\frac{\partial \tilde{E}}{\partial r}\right|_{r=a} = -k_E \tilde{E}(a) + \Delta\tilde{E}_r \tag{25}$$

Backstepping technique



Controller design: discretization

First, we discretize each subsystem and desired target system:

System

$$\dot{\tilde{n}}^{i}_{DT} = D\frac{\tilde{n}^{i+1}_{DT} - 2\tilde{n}^{i}_{DT} + \tilde{n}^{i-1}_{DT}}{h^2} + \frac{1}{ih}D\frac{\tilde{n}^{i+1}_{DT} - \tilde{n}^{i}_{DT}}{h} - 2\tilde{S}^{i}_{\alpha},$$

$$\frac{\tilde{n}^{N}_{DT} - \tilde{n}^{N-1}_{DT}}{h} = -k_{DT}\tilde{n}^{N}_{DT} + \Delta(\tilde{n}_{DT})_r$$

$\rightarrow$

Target System

$$\dot{\tilde{m}}^{i} = D\frac{\tilde{m}^{i+1} - 2\tilde{m}^{i} + \tilde{m}^{i-1}}{h^2} + \frac{1}{ih}D\frac{\tilde{m}^{i+1} - \tilde{m}^{i}}{h} - C_m\tilde{m}$$

$$\frac{\tilde{m}^{N} - \tilde{m}^{N-1}}{h} = -G\tilde{m}^{N}$$

Controller design: backstepping transformation

We then seek a backstepping transformation of the form

$$\tilde{m}^i = \tilde{n}_{DT}^i - \beta^{i-1}(\tilde{E}^0, \ldots, \tilde{E}^{i-1}, \tilde{n}_{DT}^0, \ldots, \tilde{n}_{DT}^{i-1}, \tilde{n}_\alpha^0, \ldots, \tilde{n}_\alpha^{i-1}) \tag{26}$$

This can be done taking the time derivative of the transformation:

$$\dot{\beta}^{i-1} = \dot{\tilde{n}}_{DT}^i - \dot{\tilde{m}}^i \tag{27}$$

and solving the resulting expression for β^i, yielding:

$$\beta^i = \frac{1}{D + D/i}\left[\left(2D + \frac{D}{i} + C_m h^2\right)\beta^{i-1} - D\beta^{i-2}\right. \tag{28}$$

$$\left. -h^2 C_m \tilde{n}_{DT}^i + h^2 \dot{\beta}^{i-1} + 2h^2 \tilde{S}_\alpha^i\right] \tag{29}$$

This expression can then be recursively evaluated starting with $\beta^0 = 0$.

Controller design: boundary control law

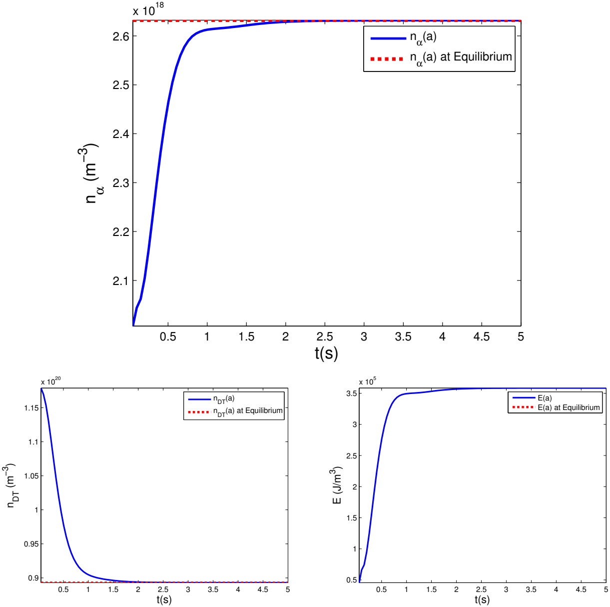
By subtracting the boundary condition of the target system from the boundary condition of the original system and solving for the input, we obtain:

$$\Delta(\tilde{n}_{DT})_r = \frac{\beta^{N-1} - \beta^{N-2}}{h} + k_{DT}\tilde{n}_{DT}^N - G(\tilde{n}_{DT}^N - \beta^{N-1}) \tag{30}$$

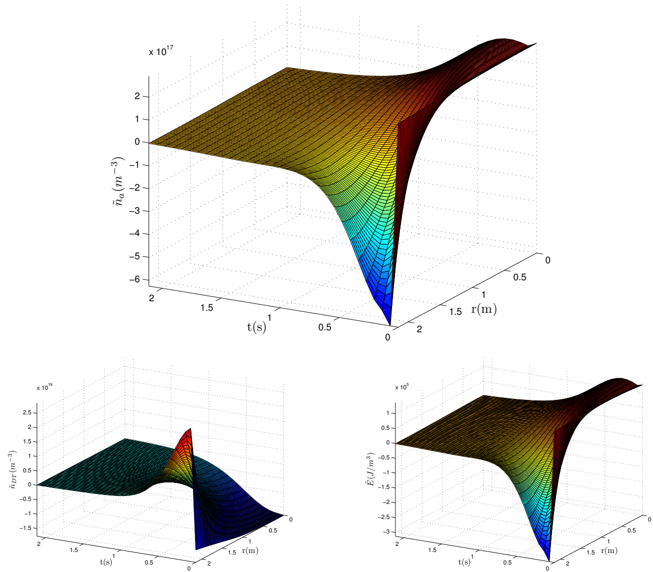
which is the desired boundary control law. Since we have discretized the system, we can put this in terms of the value of the density at the plasma's edge:

$$\tilde{n}_{DT}^N = \beta^{N-1} + \frac{1}{(1+Gh)}\left[\tilde{n}_{DT}^{N-1} - \beta^{N-2}\right] \tag{31}$$

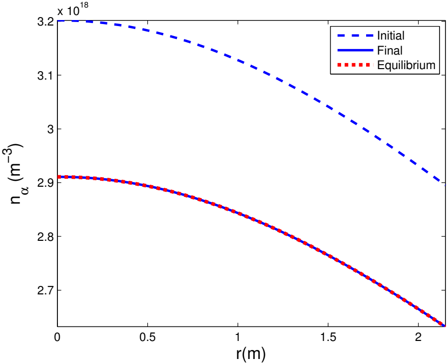
Simulation results - actuation at the plasma's edge

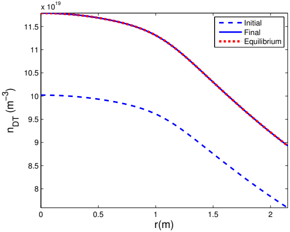


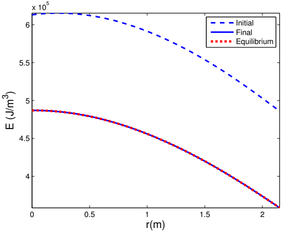
Simulation results - profile error evolution



Simulation results - Initial, final, and desired profiles







Conclusions & Future work

- A nonlinear boundary feedback controller for kinetic profiles within a burning plasma has been developed.
- Simulations show that a controller designed with just one step of backstepping is able to stabilize a particular unstable equilibrium
- Future work:
  - Develop a model of the plasma scrape-off layer to create more realistic boundary conditions
  - Include interior actuation through pellet injection and auxiliary heating
  - Include ways to deal with uncertainty in the model