ME 433 – STATE SPACE CONTROL

Lecture 1

State Space Control

- Time/Place: Room 290, STEPS Building
  M/W 12:45-2:00 PM
- Instructor: Eugenio Schuster,
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  Office hours: By appointment
- Webpage:

  http://www.lehigh.edu/~eus204/Teaching/ME433/ME433.html
- E-mail list: Make sure to be in the mailing list!!!

State Space Control

State-space methods of feedback control system design and design optimization for invariant and time-varying deterministic, continuous systems; pole positioning, observability, controllability, modal control, observer design, the theory of optimal processes and Pontryagin's Maximum principle, the linear quadratic optimal regulator problem, Lyapunov functions and stability theorems, linear optimal open loop control; introduction to the calculus of variations. Intended for engineers with a variety of backgrounds. Examples will be drawn from mechanical, electrical and chemical engineering applications. MATLAB is used extensively during the course for the analysis, design and simulation.

State Space Control – Part I

- **Topics:**

- Course description, objectives, examples
- Review of Classical Control
- Transfer functions ↔ state-space representations
- Solution of linear differential equations, linearization
- Canonical systems, modes, modal signal-flow diagrams
- Observability & Controllability
- Observability & Controllability grammians; Rank tests
- Stability
- State feedback control; Accommodating reference inputs
- Linear observer design
- Separation principle

State Space Control – Part II

- **Topics:**

- Static Optimization
  - Optimization without/with constraints
  - Numerical solution methods
- Dynamic Optimization
  - Discrete-time and continuous-time systems
  - Open loop and closed loop control
  - Linear Quadratic Regulator (LQR)
  - Pontryagin's Minimum Principle
- Dynamic Programming
  - Bellman's Principle of Optimality
  - Discrete-time and continuous-time systems
  - Hamilton-Jacobi-Bellman Equation
- Optimal Estimation/Kalman Filtering
  - Discrete-time and continuous-time systems
  - Linear Quadratic Gaussian Control (LQG)

Modern Control

- **Books:**

– **B. Friedland, "Control System Design: An Introduction to State-Space Methods,"**
Dover Publications, 1986, ISBN: 0-486-44278-0.

– Kailath, "Linear Systems"
– Brogan, "Modern Control Theory"
– Rugh, "Linear System Theory"
– Dorf and Bishop, "Modern Control Systems"
– Antsaklis and Michel, "Linear Systems"
– Chen, "Linear system Theory and Design"

Optimal Control and Estimation

- **Books:**

– **D.S. Naidu, "Optimal Control Systems"**
CRC Press, 2002, ISBN: 0-849-30892-5.

– D.E. Kirk, "Optimal Control Theory: An Introduction"
– Bryson and Ho, "Applied Optimal Control"
– Lewis and Syrmos, "Optimal Control"

– Anderson and Moore, "Optimal Filtering"
– Gelb, "Applied Optimal Estimation"

– Stengel, "Optimal Control and Estimation"

Model Classification

Model Representation ⟷ **Control Technique**

Spatial Dependence	Lumped parameter system $f = f(t)$ Ordinary Diff. Eq. (ODE)	Distributed parameter system $f = f(t,x)$ Partial Diff. Eq. (PDE)
Linearity	Linear	Nonlinear
Temporal Representation	Continuous-time	Discrete-time
Domain Representation	Time	Frequency

Spatial Dependence

Distributed Parameter Systems

PDE $n_e = n_e(t,r)$

$$\frac{\partial n_e}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r} r\left(D_n \frac{\partial n_e}{\partial r} - n_e V_n\right) + S_n(t,r)$$

$$\left.\frac{\partial n_e}{\partial r}\right|_{r=0} = 0 \qquad \left.n_e\right|_{r=a} = n_e^a$$

Lumped Parameter Systems

ODE $n_e = n_e(t)$

$$\frac{dn_e}{dt} = -\frac{1}{\tau_e} n_e + S_n(t)$$

$$n_e(0) = n_{e0}$$

Control: **Interior** **Boundary**

1 Reduction

2 Keep the PDE representation (problem specific)

Linearity: **Nonlinear/Linear**

Linear/Nonlinear Distributed Parameter Control

Linear/Nonlinear Lumped Parameter Control

Linearity

Nonlinear (ODE) Systems

$$y^{(n)} = g\left(t, y^{(n-1)}, \cdots, y, u^{(m-1)}, \cdots, u\right)$$

Linear (ODE) Systems

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = b_{m-1}u^{(m-1)} + \cdots + b_0 u$$

Non-Autonomous

$$\dot{x} = f(t,x,u) \qquad y = h(t,x,u)$$

Autonomous

$$\dot{x} = f(x,u) \qquad y = h(x,u)$$

$x = [x_1 \ \cdots \ x_n]^T$ states

u input

y output

$$\dot{x} = A(t)x + B(t)u \qquad y = C(t)x + D(t)u$$

LTV

$$\dot{x} = Ax + Bu \qquad y = Cx + Du$$

LTI

1 Linearization

2 Keep the nonlinearities

Nonlinear Control

Linear Control

Output/State Feedback

Estimation: How to estimate states from input/output?

Linearity

Particular type of nonlinearities: Constraints

LTI $\dot{x} = Ax + Bu$ $\quad u, y \rightarrow sat(u), sat(y)$ input/output constraints

$y = Cx + Du$ $\quad \underline{x}_i < x_i < \overline{x}_i$ state constraints

1. A priori → Constraint is considered for control design
2. A posteriori → Constraint is NOT considered for control design

Anti-windup Techniques

Temporal Representation

Continuous-time Systems **Discrete-time Systems**

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = b_{m-1}u^{(m-1)} + \cdots + b_0 u$$

Sampled-Data Systems

$$y[kT] + a_{k-1}y[(k-1)T] + \cdots + a_n y[(k-n)T] = b_o u[kT] + \cdots + b_m u[(k-m)T]$$

Sampling Time

LTI
$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$$x[k+1] = Ax[k] + Bu[k]$$
$$y[k] = Cx[k] + Du[k]$$
LTI

System Identification: How to create models from data?

Fault Detection and Isolation: How to detect faults from data?

System Identification

Domain Representation

Continuous-time Systems

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = b_{m-1}u^{(m-1)} + \cdots + b_0 u$$

Laplace Transform

$$\left(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0\right)Y(s) = \left(b_{m-1}s^{m-1} + \cdots + b_1 s + b_0\right)U(s)$$

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

$$= K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$$s = j\omega$$

$$T(j\omega) = |T(j\omega)|e^{\angle T(j\omega)}$$

Discrete-time Systems

$$y[k] + a_{k-1}y[k-1] + \cdots + a_n y[k-n] = b_o u[k] + \cdots + b_m u[k-m]$$

Modern Control

Classical Control

Z Transform

$$\left(1 + a_1 z + \cdots + a_{n-1}z^{n-1} + a_n z^{-n}\right)Y(s) = \left(b_o + \cdots + b_m z^{-m}\right)U(s)$$

$$T(z) = \frac{Y(z)}{U(z)} = \frac{b_o + \cdots + b_m z^{-m}}{1 + a_1 z + \cdots + a_{n-1}z^{n-1} + a_n z^{-n}}$$

$$= K\frac{(z-z_1)(z-z_2)\cdots(z-z_m)}{(z-p_1)(z-p_2)\cdots(z-p_n)}$$

$$z = e^{j\omega T}$$

$$T(e^{j\omega T}) = |T(e^{j\omega T})|e^{\angle T(e^{j\omega T})}$$

T	TF
p_i	poles
z_i	zeros
K	gain

Frequency Response

Optimality

Continuous-time Systems

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = b_{m-1}u^{(m-1)} + \cdots + b_0 u$$

$$\dot{x} = A(t)x + B(t)u$$
$$y = C(t)x + D(t)u$$

Discrete-time Systems

$$y[k] + a_{k-1}y[k-1] + \cdots + a_n y[k-n] = b_o u[k] + \cdots + b_m u[k-m]$$

$$x_{k+1} = A_k x_k + B_k u_k$$
$$y_k = C_k x_k + D_k u_k$$

$$\min_{u_k} \frac{1}{2}x_N^T S_N x_N + \frac{1}{2}\sum_{k=0}^{N-1}\left(x_k^T Q_k x_k + u_k^T R_k u_k\right)$$

$$\min_{u(t)} \frac{1}{2}x^T(T)S_T x(T) + \frac{1}{2}\int_0^T \left(x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)\right)dt$$

Optimal Control

Robustness

How to deal with uncertainties in the model?

(A) Non-model-based control

- PID
- Extremum Seeking

(B) Model-based control

1. Robust Control → Design for a family of plants
2. Adaptive Control → Update model (controller) in real time

Robust & Adaptive Control

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Model Classification

Model Representation ↔ **Control Technique**

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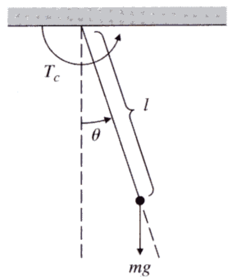
Dynamic Model

MECHANICAL SYSTEM: $F = I\alpha$ Newton's law



damping coefficient

$$I\alpha = -lmg\sin\theta - b\omega + T_c$$

$\omega = \dot{\theta}$ angular velocity

$\alpha = \dot{\omega} = \ddot{\theta}$ angular acceleration

$I = ml^2$ moment of inertia

$$\ddot{\theta} = -\frac{b}{ml^2}\dot{\theta} - \frac{g}{l}\sin\theta + \frac{T_c}{ml^2}$$

Which are the equilibrium points when $T_c=0$?

At equilibrium: $\ddot{\theta} = \dot{\theta} = 0 \Rightarrow 0 = -\frac{g}{l}\sin\theta \Rightarrow \theta = 0,\pi$

Stable ($\theta = 0$), Unstable ($\theta = \pi$)

Open loop simulations: pend_par.m, pendol01.mdl

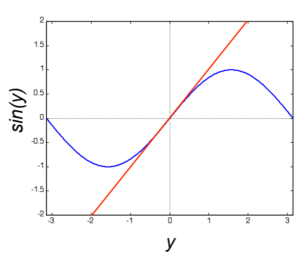
Linearization

What happens around $\theta=0$?

$$\theta = y \Rightarrow \quad \ddot{y} = -\frac{b}{ml^2}\dot{y} - \frac{g}{l}\sin(y) + \frac{T_c}{ml^2}$$

By Taylor Expansion:

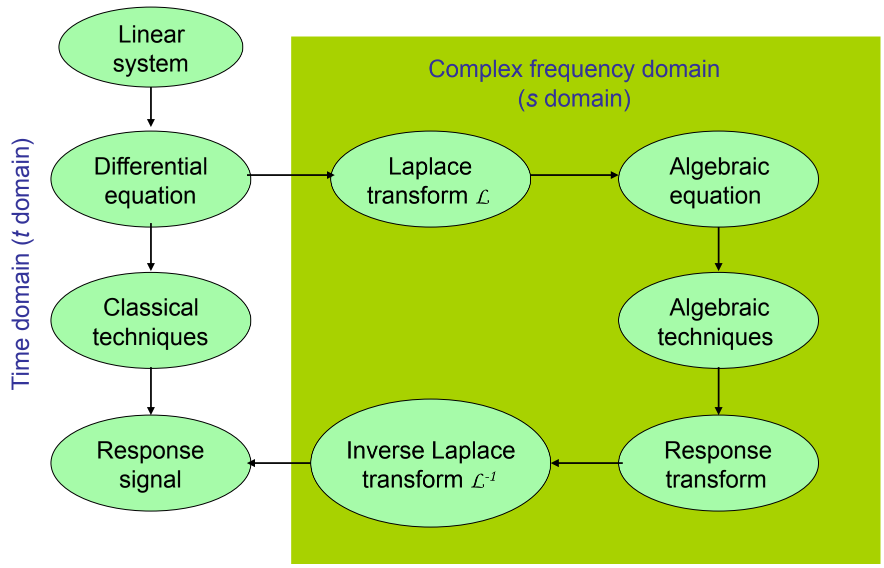
$$\sin(y) = y + h.o.t. \Rightarrow \sin(y) \approx y$$



Linearized Equation:

$$\ddot{y} = -\frac{b}{ml^2}\dot{y} - \frac{g}{l}y + \frac{T_c}{ml^2}$$

Laplace Transform



Transfer Function

$$u \equiv T_c \Rightarrow \quad \ddot{y} = -\frac{b}{ml^2}\dot{y} - \frac{g}{l}y + \frac{T_c}{ml^2}$$

$$\ddot{y} + \frac{b}{ml^2}\dot{y} + \frac{g}{l}y = \frac{u}{ml^2}$$

Laplace Transform

$$\mathcal{L}\left\{\frac{d^m f(t)}{dt^m}\right\} = s^m F(s),\ U(s) = \mathcal{L}\{u\},\ Y(s) = \mathcal{L}\{y\}$$

Transfer Function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\frac{1}{ml^2}}{s^2 + \frac{b}{ml^2}s + \frac{g}{l}}$$

Characteristic Equation

Solution of the ODE

$$T_c = 0 \Rightarrow \ddot{y} + \frac{b}{ml^2}\dot{y} + \frac{g}{l}y = 0$$ What is the solutions *y(t)*?

Characteristic Equation

$$\lambda^2 + \frac{b}{ml^2}\lambda + \frac{g}{l} = 0 \Rightarrow \lambda_{1,2} = \frac{-\frac{b}{ml^2} \pm \sqrt{\left(\frac{b}{ml^2}\right)^2 - 4\frac{g}{l}}}{2}$$

$$G(s) = \frac{1/ml^2}{s^2 + \frac{b}{ml^2}s + \frac{g}{l}}$$

$$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

The dynamics of the system is given by the roots of the denominator (poles) of the trasfer function

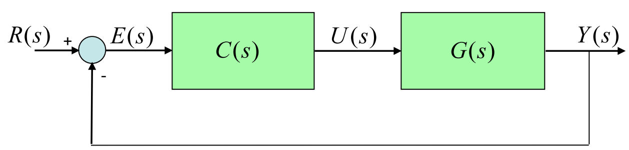
real(λ_1,λ_2)<0 $\Rightarrow$ STABLE SYSTEM

We use feedback control for PERFORMANCE

Closed-loop Control

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)},$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + C(s)G(s)}.$$



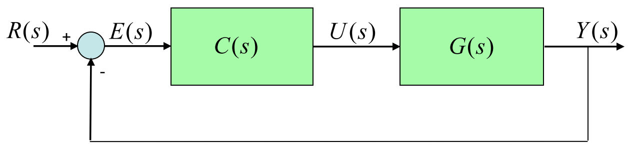
PID: Proportional – Integral – Derivative

$$C(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_I}{s} + K_D s$$

$$u(t) = K_p e(t) + K_I \int_0^d e(\tau)d\tau + K_D \frac{de(t)}{dt}$$

Closed loop simulations: pid.m

Closed-loop Control



$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1+C(s)G(s)} = \frac{(1/ml^2)(K_I + K_P s + K_D s^2)}{s^3 + \frac{b+K_D}{ml^2}s^2 + \left(\frac{g}{l} + \frac{K_P}{ml^2}\right)s + \frac{K_I}{ml^2}}$$

We can place the poles at the desired location to obtain the desired dynamics

CLASSICAL CONTROL (ME 343)

Linearization

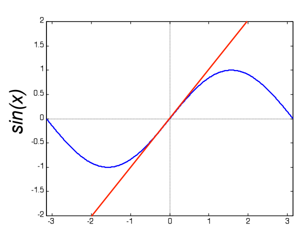
What happens around $\theta=\pi$?

$$\theta = \pi + x \Rightarrow \quad \ddot{x} = -\frac{b}{ml^2}\dot{x} - \frac{g}{l}\sin(\pi + x) + \frac{T_c}{ml^2}$$

$$\ddot{x} = -\frac{b}{ml^2}\dot{x} + \frac{g}{l}\sin(x) + \frac{T_c}{ml^2}$$

By Taylor Expansion:

$$\sin(x) = x + h.o.t. \Rightarrow \sin(x) \approx x$$



Linearized Equation:

$$\ddot{x} = -\frac{b}{ml^2}\dot{x} + \frac{g}{l}x + \frac{T_c}{ml^2}$$

State-variable Representation

$$\ddot{x} = -\frac{b}{ml^2}\dot{x} + \frac{g}{l}x + \frac{T_c}{ml^2}$$ ⇨ Reduce to first order equations:

State Variable Representation

$$\begin{aligned} x_1 &= x \\ x_2 &= \dot{x} \end{aligned} \Rightarrow \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{b}{ml^2}x_2 + \frac{g}{l}x_1 + \frac{T_c}{ml^2} \end{aligned}$$

$$x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, u \equiv T_c \Rightarrow \dot{x} = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} & -\frac{b}{ml^2} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} u = Ax + Bu$$

$$eig(A) = \{\lambda : |\lambda I - A| = 0\} = \left\{\lambda : \lambda^2 + \frac{b}{ml^2}\lambda - \frac{g}{l} = 0\right\}$$

Characteristic Equation

Solution of the ODE

$$T_c = 0 \Rightarrow \ddot{x} + \frac{b}{ml^2}\dot{x} - \frac{g}{l}x = 0$$ What is the solution *x(t)*?

Characteristic Equation

$$\lambda^2 + \frac{b}{ml^2}\lambda - \frac{g}{l} = 0 \Rightarrow \lambda_{1,2} = \frac{-\frac{b}{ml^2} \pm \sqrt{\left(\frac{b}{ml^2}\right)^2 + 4\frac{g}{l}}}{2}$$

$$eig(A) = \{\lambda : |\lambda I - A| = 0\} = \left\{\lambda : \lambda^2 + \frac{b}{ml^2}\lambda - \frac{g}{l} = 0\right\} \qquad x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

The dynamics of the system is given by the eigenvalues of the system matrix

real(eig(A))>0 (real(λ_1,λ_2)>0) ⇒ INSTABILITY

We use feedback control for STABILIZATION

Linear State Feedback

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} & -\frac{b}{ml^2} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} u = Ax + Bu$$

$$u = -Kx = -\begin{bmatrix} K_1 & K_2 \end{bmatrix} x$$

$$\dot{x} = (A - BK)x = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} - \frac{1}{ml^2}K_1 & -\frac{b}{ml^2} - \frac{1}{ml^2}K_2 \end{bmatrix}$$

How do we choose K_1 and K_2 to make real(eig(A-BK))<0? Always possible?

How do we choose K_1 and K_2 to satisfy optimality condition?

How do we proceed when states are not measurable?

MODERN CONTROL (ME 433)

Closed loop simulations: pend_par.m, statevar_control_lin.m
pendcllin01.mdl

Nonlinear State Feedback

$$\dot{x} = \begin{bmatrix} 0 & x_2 \\ \frac{g}{l}\sin(x_1) & -\frac{b}{ml^2}x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{u}{ml^2} \end{bmatrix}$$

$$u = -mgl\sin(x_1) + ml^2 v$$

Feedback Linearization

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{ml^2} \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v = A^* x + B^* u$$

$$v = -Kx = -\begin{bmatrix} K_1 & K_2 \end{bmatrix} x \quad \Rightarrow \dot{x} = (A^* - B^* K)x = \begin{bmatrix} 0 & 1 \\ -K_1 & -\frac{b}{ml^2} - K_2 \end{bmatrix}$$

We choose K_1 and K_2 to make real(eig(A^*-B^*K))<0

$$u = -mgl\sin(\theta - \pi) - ml^2\left[K_1(\theta - \pi) + K_2\dot{\theta}\right]$$

NONLINEAR CONTROL (ME 350/450)

Closed loop simulations: pend_par.m, statevar_control_nolin.m
pendclnolin01.mdl

Nonlinear State Feedback

$$\dot{x} = \begin{bmatrix} 0 & x_2 \\ \frac{g}{l}\sin(x_1) & -\frac{b}{ml^2}x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{u}{ml^2} \end{bmatrix}$$

Parameters (*m, l, b*) are not well known:

MULTIVARIABLE ROBUST CONTROL (ME 350/450)

SYSTEM IDENTIFICATION AND ADAPTIVE CONTROL (ME 350/450)

Flexible pendulum ⇒ ODE → PDE:

DISTRIBUTED PARAMETER SYSTEMS (ME 350/450)

Controls Education at Lehigh

ME 343: CLASSICAL CONTROL	FALL
ME 389: CONTROLS LAB	SPRING
ME 433: MODERN & OPTIMAL CONTROL	FALL
ME 350: ADVANCED TOPICS IN CONTROL	SPRING

- NONLINEAR CONTROL
- MULTIVARIABLE ROBUST CONTROL
- SYSTEM IDENTIFICATION AND ADAPTIVE CONTROL
- DISTRIBUTED PARAMETER SYSTEMS