Nuclear Fusion and Radiation

Lecture 7 (Meetings 17, 18, 19 & 20) - Problem 2

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Nuclear Fusion Reactor Design

Problem: Design a fusion reactor to operate in steady-state at a subignition point characterized by $kT_e = kT_i = kT = 10keV$ with $\beta = 4\%$, $Q = 10$ and a fusion power density $P_f = 10MW/m^3$. Assume that the reactor will operate based on a 50:50 mixture of deuterium and tritium, i.e., $n_D = n_T$. Assume that $f_p = 2$, and $Z_{eff} = 1$, which implies that only the hydrogenic population is present in the plasma. Stability considerations require that $q(a) = 4$ and $\beta_p = 4R/a$, where a and R denote the minor and major radii, respectively. Radiation losses cannot be neglected. If a design choice must be made, choose the option that maximizes the aspect ratio $A = R/a$. Provide the dimensions of the reactor in terms of the minor radius a and the major radius R, and the sizes of the toroidal and ohmic coils in terms of the toroidal magnetic field B_t and the induced plasma current I_p. Assume the energy confinement scaling given by the Alcator scaling

$$n_e \tau_E (m^{-3}s) = 6 \times 10^{-21} n_e^2 a^2.$$

If the thermal/electric power conversion efficiency is given by $\eta = 0.6$, will the reactor be able to satisfy the electricity demand of $80MW$ of a close-by town?

NOTE: Use $C = 5 \times 10^{-37} \frac{Wm^3}{keV^{1/2}}$ as Bremsstrahlung power constant, $< \sigma v >_{DT}^{10keV} = 1.1 \times 10^{-22} \frac{m^3}{s}$ and $E_\alpha = 3.5MeV$. Remember that $1eV = 1.60217733 \times 10^{-19} J$.

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As $Z_{eff} = 1$, only hydrogenic ions are considered present in the plasma. The quasi-neutrality condition is

$$n_e = n_D + n_T.$$

Since the reactor is operating with a 50:50 mixture of deuterium and tritium, i.e., $n_D = n_T \triangleq n_{DT}$, the quasi-neutrality condition implies $n_D = n_T \triangleq n_{DT} = n_e/2$. Moreover, $n = n_e + n_i = n_e + n_D + n_T = 2n_e$. The fusion power density is defined as

$$P_f = n_D n_T \langle \sigma v \rangle E_f f_p = \frac{n_e^2}{4} \langle \sigma v \rangle E_f f_p,$$

where $E_f = 17.6 MeV$ for the deuterium tritium fusion reaction. Therefore, the electron density is

$$n_e = \sqrt{\frac{4P_f}{\langle \sigma v \rangle E_f f_p}} = 2.5393 \times 10^{20} m^{-3} \quad \Rightarrow \quad n_{DT} = n_e/2 = 1.2696 \times 10^{20} m^{-3}.$$

At a steady-state subignition, i.e., $P_{aux} \neq 0$, operating point, the power balance in the reactor is

$$P_{aux} + P_\alpha = P_L + P_{rad},$$

where

$$Q = \frac{P_f}{P_{aux}} \quad \Rightarrow \quad P_{aux} = \frac{P_f}{Q} = 1MW/m^3$$

$$P_\alpha = \frac{1}{5} P_f = 2MW/m^3$$

$$P_L = \frac{3n_e kT}{\tau_E}$$

$$P_{rad} = C n_e^2 \sqrt{kT(keV)} Z_{eff} = 0.10195MW/m^3.$$

Therefore, the energy confinement time τ_E is given by

$$\tau_E = \frac{3n_e kT}{P_{aux} + P_\alpha - P_{rad}} = 0.4211s.$$

From the Alcator scaling

$$n_e \tau_E (m^{-3} s) = 6 \times 10^{-21} n_e^2 a^2,$$

we can obtain the minor radius of the plasma as

$$a = \sqrt{\frac{\tau_E}{6 \times 10^{-21} n_e}} = 0.5258m.$$

The total plasma beta is defined as

$$\beta = \frac{2n_e kT}{(B^2/2\mu_0)},$$

where $B^2 = B_t(R)^2 + B_p(a)^2$, and the poloidal plasma beta is defined as

$$\beta_p = \frac{2n_e kT}{(B_p(a)^2/2\mu_0)}.$$

Therefore,

$$\frac{\beta}{\beta_p} = \frac{\dfrac{2n_e kT}{\dfrac{B_t(R)^2 + B_p(a)^2}{2\mu_0}}}{\dfrac{2n_e kT}{\dfrac{B_p(a)^2}{2\mu_0}}} = \frac{B_p(a)^2}{B_t(R)^2 + B_p(a)^2} = \frac{1}{1 + \dfrac{B_t(R)^2}{B_p(a)^2}}.$$

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Recalling the definition of the safety factor at the plasma edge

$$q(a) = \frac{a}{R}\frac{B_t(R)}{B_p(a)} = \frac{1}{A}\frac{B_t(R)}{B_p(a)} \quad \Rightarrow \quad q(a)^2 A^2 = \frac{B_t(R)^2}{B_p(a)^2},$$

we can write

$$\frac{\beta}{\beta_p} = \frac{1}{1 + q(a)^2 A^2}.$$

As the stability considerations require that $q(a) = 4$ and $\beta_p = 4R/a = 4A$, we obtain

$$\frac{\beta}{4} q(a)^2 A^2 - A + \frac{\beta}{4} = 0. \quad (1)$$

When we solve this equation for the aspect ratio, we obtain two possible solutions, $A = 6.24$ and $A = 0.01$. Since the goal is the maximization of the aspect ratio, we take $A = 6.24.$ The major radius is then given by

$$R = aA = 3.2807m.$$

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From the definition of β_p and the constraint $\beta_p = 4A$, the poloidal magnetic field at the plasma edge is

$$B_p(a) = \sqrt{\frac{4\mu_0 n_e kT}{4A}} = 0.2862T,$$

and from the definition of the safety factor at the plasma edge, the toroidal magnetic field at the plasma major radius is

$$B_t(R) = q(a)AB_p(a) = 7.1444T.$$

The poloidal magnetic field is defined as

$$B_p(r) = \frac{\mu_0 I_p}{2\pi r},$$

therefore, the total induced plasma current I_p is given by

$$I_p = \frac{2\pi a B_p(a)}{\mu_0} = 0.7525MA.$$

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The volume of the torus is $V = 2\pi R \pi a^2 = 17.9009 m^3$. The total fusion power generated by the reactor is

$$P_f^{tot} = P_f V = 179.01 MW.$$

Assuming $P_{th}^{tot} = P_n^{tot} = 0.8 P_f^{tot} = 143.18 MW$, the available electrical power is

$$P_e^{tot} = \eta P_{th}^{tot} = 85.91 MW,$$

which is enough to satisfy the electricity needs of the close-by town.