Nuclear Fusion and Radiation

Lecture 7 (Meetings 17, 18, 19 & 20) - Problem 1

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Problem: A conceptual tokamak design has major radius $R = 3m$, minor radius $a = 1m$, toroidal magnetic at major radius $B_t(R) = 5T$, and profile factor $f_p = 2$. Assume a purely hidrogenic (no impurities, negligible α population) DT plasma with $50 - 50$ DT mixture ($n_D = n_T \triangleq n_{DT}$), $T \triangleq T_i = T_e = 10keV$. The energy confinement scaling is given by Alcator scaling

$$n_e \tau_E (m^{-3}s) = 6 \times 10^{-21} n_e^2 a^2.$$

Stability considerations require $q(a) = 3$. Performance considerations require steady-state operation at $Q = 5$ (radiation losses are negligible). Complete the table below. Show your work!

A	$n(10^{20}m^{-3})$	$\tau_E(sec)$	$I_p(MA)$	$\beta(\%)$	$B_p(T)$	$P_f(MW/m^3)$	$P_{th}^{tot}(MW)$	$L_n(MW/m^2)$

NOTE: Remember that $<\sigma v>_{DT}^{10keV} = 1.09 \times 10^{-22} \frac{m^3}{s}$, $\mu_o = 4\pi 10^{-7} \frac{H}{m}$, $E_\alpha = 3.5MeV$, $E_f = 17.6MeV$, and $1eV = 1.602 \times 10^{-19} J$. $T = \frac{kg}{Cs}$, $H = \frac{m^2 kg}{C} = \frac{Tm^2}{A}$, $J = \frac{mkg}{s^2}$.

The aspect ration can be computed as

$$A = \frac{R}{a} = 3. \quad (1)$$

The safety factor at the minor radius can be written as

$$q(a) = \frac{1}{A}\frac{B_t(R)}{B_p(a)}, \quad (2)$$

from where we can compute

$$B_p(a) = \frac{1}{A}\frac{B_t(R)}{q(a)} = 0.5556T. \quad (3)$$

Since

$$B_p(a) = \frac{\mu_o I_p}{2\pi a}, \quad (4)$$

we can compute

$$I_p = \frac{2\pi a B_p(a)}{\mu_o} = 2.7778MA. \quad (5)$$

The quasi-stationary condition is given by

$$n_e = n_D + n_T. \tag{6}$$

Since $n_D = n_T$, we can write

$$n_D = n_T = n_e/2. \tag{7}$$

Moreover, since $n_i = n_D + n_T$, we can write the total plasma density as

$$n = n_e + n_i = n_e + n_D + n_T = n_e + n_e = 2n_e. \tag{8}$$

The power balance is given by

$$P_\alpha + P_{aux} = P_L, \tag{9}$$

where

$$P_\alpha = n_D n_T < \sigma v >_{DT}^{10keV} E_\alpha f_p = \frac{n_e^2}{4} < \sigma v >_{DT}^{10keV} E_\alpha f_p, \tag{10}$$

$$P_{aux} = \frac{P_f}{Q} = \frac{5P_\alpha}{Q}, \tag{11}$$

$$P_L = \frac{\frac{3}{2}nkT}{\tau_E} = \frac{3n_e kT}{\tau_E}. \tag{12}$$

Nuclear Fusion Reactor Design

Therefore, we can state the power balance as

$$\frac{n_e^2}{4} < \sigma v >_{DT}^{10keV} \; E_\alpha f_p \left(1 + \frac{5}{Q}\right) = \frac{3 n_e k T}{\tau_E}. \tag{13}$$

From the Alcator scaling we can write

$$\tau_E(m^{-3}s) = 6 \times 10^{-21} n_e a^2, \tag{14}$$

which allows us to rewrite the power balance as

$$\frac{n_e^2}{4} < \sigma v >_{DT}^{10keV} \; E_\alpha f_p \left(1 + \frac{5}{Q}\right) = \frac{3kT}{6 \times 10^{-21} a^2}, \tag{15}$$

and solve for the electron density

$$n_e = \sqrt{\frac{12kT}{6 \times 10^{-21} a^2 < \sigma v >_{DT}^{10keV} \; E_\alpha f_p \left(1 + \frac{5}{Q}\right)}} = 1.1448 \times 10^{20} m^{-3}, \tag{16}$$

which in turn yields

$$n = 2 n_e = 2.2896 \times 10^{20} m^{-3}. \tag{17}$$

From (14) we can compute

$$\tau_E(m^{-3}s) = 6 \times 10^{-21} n_e a^2 = 0.6869s. \quad (18)$$

We can compute β as

$$\beta = \beta = \frac{nkT}{\frac{B_t^2(R)+B_p^2(a)}{\mu_o}} = \frac{2n_e kT}{\frac{B_t^2(R)+B_p^2(a)}{\mu_o}} = 0.0364, \quad (19)$$

and the fusion power density as

$$P_f = \frac{n_e^2}{4} < \sigma v >_{DT}^{10keV} E_f f_p = 2.0139 \frac{MW}{m^3}. \quad (20)$$

Knowing the geometry of the reactor we can finally compute

$$P_f^{tot} = P_f V = P_f(\pi a^2)(2\pi R) = 119.26 MW, \quad (21)$$

and

$$L_n = \frac{P_f^{tot} \frac{4}{5}}{S} = \frac{P_f^{tot} \frac{4}{5}}{(2\pi a)(2\pi R)} = 0.8067 \frac{MW}{m^2}, \quad (22)$$

where $P_{th}^{tot} = P_n^{tot} = \frac{4}{5} P_f^{tot} = 95.41 MW$.

Summarizing:

A	$n(10^{20}m^{-3})$	$\tau_E(sec)$	$I_p(MA)$	$\beta(\%)$	$B_p(T)$	$P_f(MW/m^3)$	$P_{th}^{tot}(MW)$	$L_n(MW/m^2)$
3	2.2896	0.6869	2.7778	0.0364	0.5556	2.0139	95.41	0.8067