Nuclear Fusion and Radiation

Lecture 7 (Meetings 17, 18, 19 & 20)

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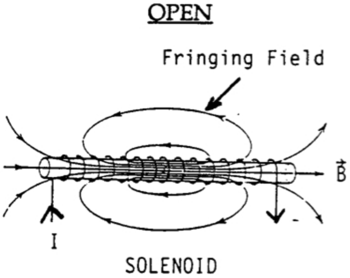
Magnetic Confinement

- We concluded in previous lectures that confinement by magnetic fields appears feasible.
- The magnetic confinement concepts can be divided into two general categories depending on field configuration:
  1. (1) "open/linear" configurations;
  2. (2) "closed" configurations.



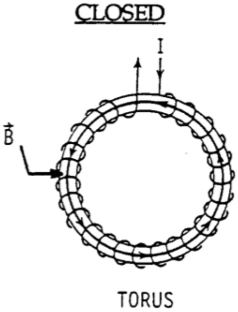


Figure 1: Open & closed magnetic configurations.

Closed Configurations

- We now turn to an examination of plasma confinement in closed configurations.
- Consider the simple toroidal field configuration shown below:

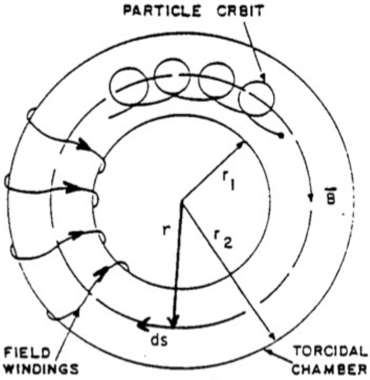


Figure 2: Closed toroidal configuration.

Closed Configurations

- We would expect that the toroidal magnetic field $\bar{B} \equiv \bar{B}_t$ would vary with radial position in such a geometry.

- We can determine this variation by applying Ampere's Law:

$$\oint \bar{B} \cdot \bar{ds} = \mu_o I \tag{1}$$

  where I is the total inclosed current.

- Note that at a given radial position, r, $\bar{B}$ is constant along $\bar{ds}$. Moreover, $\bar{B}$ and $\bar{ds}$ are parallel. Thus,

$$\oint \bar{B} \cdot \bar{ds} = \oint B ds = B \oint ds = 2\pi r B = \mu_o I \Rightarrow B = \frac{\mu_o I}{2\pi r} \tag{2}$$

- The above relationship must hold for any r between r_1 and r_2, since I is fixed in this range.

Closed Configurations

- Therefore, the toroidal magnetic field in the torus is given by

$$B_t(r) = \frac{\mu_o I}{2\pi r} \tag{3}$$

- The toroidal magnetic field in a torus varies as $1/r$.
- Schematically this variation is represented as follows:

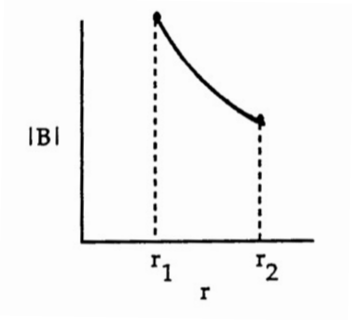


Figure 3: Magnetic field spatial variation.

Closed Configurations

- Thus, a gradient exists in the radial direction and we must examine the effect of a gradient field on the particle motion.
- Consider the motion of an ion in an idealized abrupt field gradient:

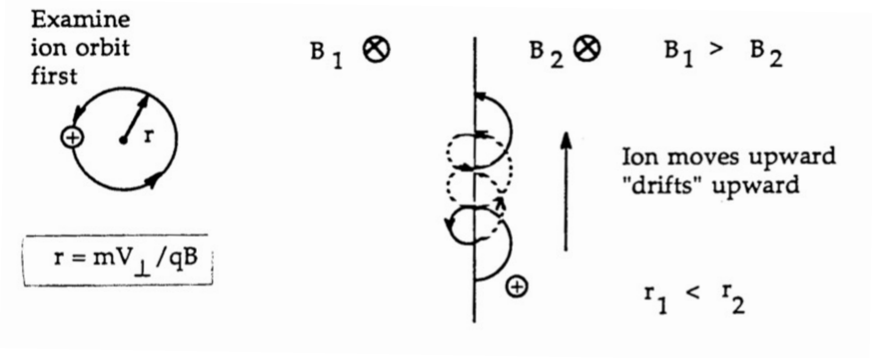


Figure 4: Ion vertical drift.

Closed Configurations

- Consider now the motion of an electron in an idealized abrupt field gradient:

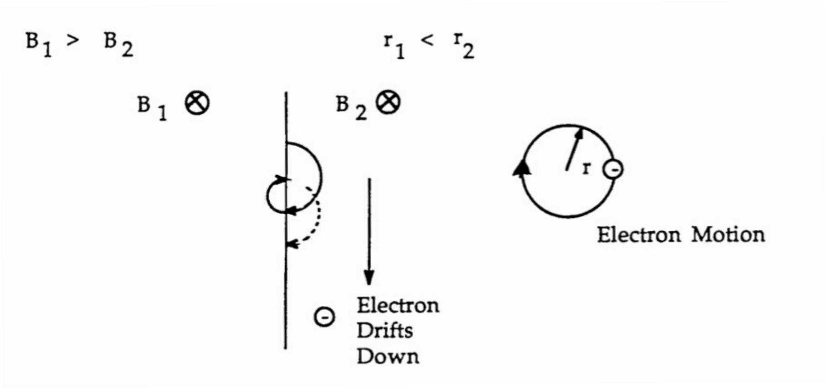


Figure 5: Electron vertical drift.

Closed Configurations

- Thus, in the gradient field considered above, ions drift upward and electrons drift downward.
- In a torus, the gradient in the toroidal field, B_t, leads to a separation of ions and electrons (by virtue of the drifts) and this results in an electric field E.

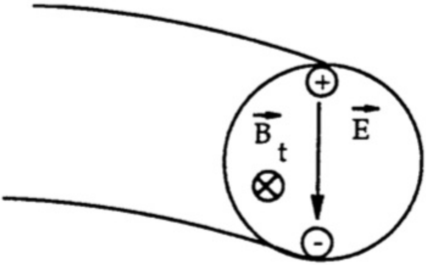


Figure 6: Induced electric field

- Now consider the ion motion in the presence of an electric field and a magnetic field but no magnetic field gradient.

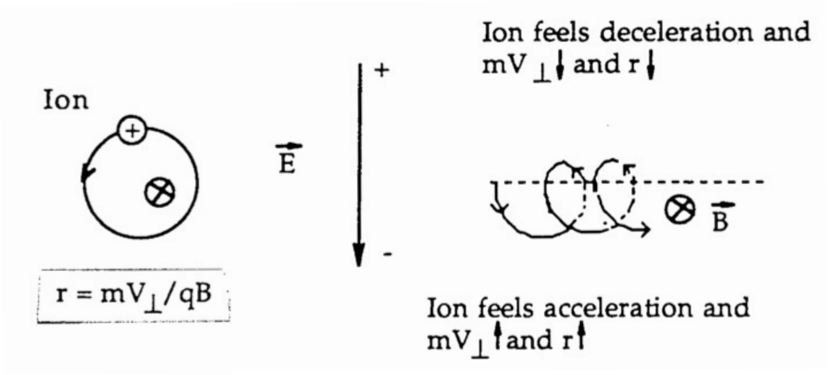


Figure 7: Ion horizontal drift

- Thus the ion drifts perpendicular to both E and B (called $E \times B$ Drift).

Closed Configurations

- The electron motion in the presence of an electric field and a magnetic field but no magnetic field gradient is shown below:

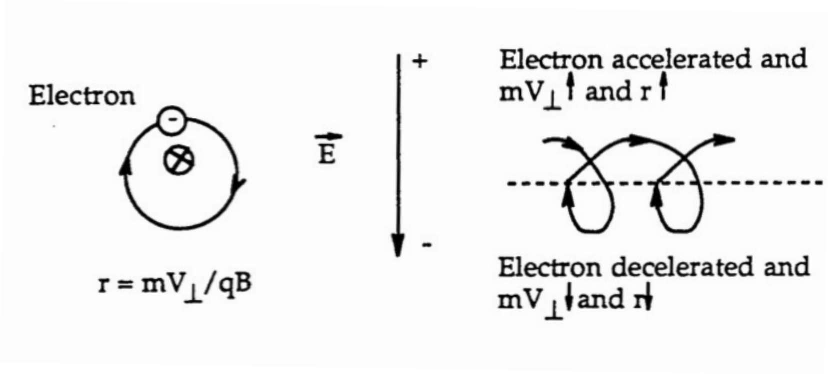


Figure 8: Electron horizontal drift

- Therefore, the electron drifts in the same direction as the ion ($E \times B$ Drift).

Closed Configurations

- Thus, in a torus, the drifts associated with the gradient of B_t set up an electric field and this electric field results in $E \times B$ drifts, which drive plasma (electron and ions) to the chamber wall, destroying confinement. Recall that in open geometry end losses are the fundamental confinement problem.

- The various closed configurations differ primarily in the manner in which they deal with the drift problem.

- The preferred approach is to prevent the charge separation resulting from gradient-B drifts and thereby avoid the $E \times B$ problem.

- The gradient-B drifts and resulting charge separation can be cancelled out by twisting the toroidal field lines to form helices - the twisting of the field lines is called providing "rotational transform", and is depicted below:

- We add now a poloidal component to the magnetic field $\bar{B}$, i.e. $\bar{B} \equiv \bar{B}_t + \bar{B}_p$.

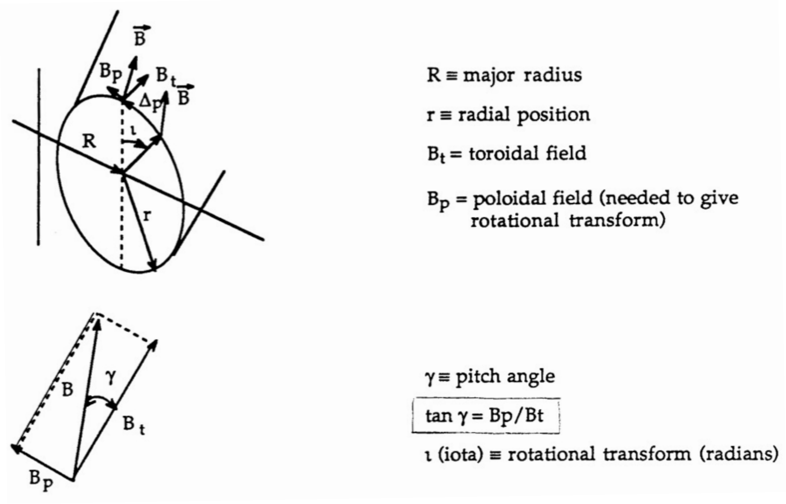


Figure 9: Rotational transform.

Closed Configurations

- During one trip around the torus ($2\pi R$ in the toroidal direction) a field line also moves in the poloidal direction a distance, Δp, which can be expressed as

$$\frac{\Delta p}{2\pi R} = \tan\gamma = \frac{B_p}{B_t} \iff \Delta p = 2\pi R \tan\gamma \tag{4}$$

- This distance can also be expressed in terms of the rotational transform, ι.

$$\Delta p = \frac{\iota(\text{radians})}{2\pi} 2\pi r = \iota r \tag{5}$$

- Equating the two expressions for Δp yields

$$2\pi R \tan\gamma = \iota r \iff \iota = 2\pi \frac{R}{r} \tan\gamma \tag{6}$$

Closed Configurations

- Since $\tan\gamma = \frac{B_p}{B_t}$, we obtain the following expression for the rotational transform
$$\iota = 2\pi \frac{R}{r} \frac{B_p}{B_t} \tag{7}$$

- In the literature the rotational transform is also discussed in terms of another parameter called the safety factor q – which is related to the fluid stability of the plasma-field configuration.

- The safety factor is defined as the number of rotations a field line makes in the toroidal direction per rotation in the poloidal direction.

- Since $\iota/2\pi$ is the number of rotations in the poloidal direction per rotation in the toroidal direction,
$$q = \frac{1}{\iota/2\pi} = \frac{2\pi}{\iota} = \frac{2\pi}{2\pi \frac{R}{r} \frac{B_p}{B_t}} = \frac{r}{R} \frac{B_t}{B_p} \tag{8}$$

Closed Configurations

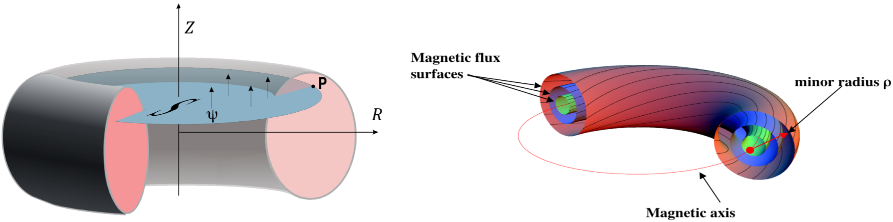


Figure 10: Poloidal flux in a tokamak at point P (left). Flux surfaces (right)

- The poloidal component of the helical magnetic lines defines nested toroidal surfaces corresponding to constant values of the poloidal magnetic flux.
- The poloidal flux ψ at a point P in the (R, Z) cross section of the plasma (i.e., poloidal cross section) is the total flux through the surface S bounded by the toroidal ring passing through P, i.e., $\psi = \frac{1}{2\pi} \int B_{pol} dS$.
- The poloidal flux ψ can be used as spatial coordinate in the 2D cross section.

Closed Configurations

- The rotational transform's effects can be explained through this schematic:

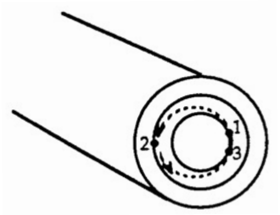


Figure 11: Drift averaging.

- An ion at position 1 will experience upward drift due to the field gradient and will eventually move outward to the next flux surface at position 2.
- If there were no more upward drift, the particle would stay on this surface, and it would have stepped away from center, leading to charge separation.
- However, as the particle at 2 moves into the lower portion of the torus, it continues to experience an upward drift which moves it back to the original flux surface, position 3.

Closed Configurations

- Thus, averaged over many transits around the torus, the particles tend to stay on given flux surfaces and do not tend to separate (i.e. do not experience net upward/ net downward drifts).

- The rotational transform provides an averaging of the drifts such that the net drift is almost exactly zero and no E is produced.

How do we produce rotational transform?

Magnetic Confinement

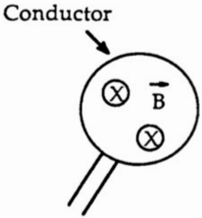
- The major confinement problem in a torus is associated with particle drifts.
- A gradient B drift, associated with the $1/r$ variation of B_t, results in charge separation.
- Charge separation creates an electrical field, E. The electric field, E, gives rise to $E \times B$ drifts which drive plasma (ions and electrons) radially outward – destroying magnetic confinement.
- The basic method for dealing with this problem is to give the toroidal field a helical pitch – by adding a field component in the poloidal direction. This technique is referred to as "rotational transform."
- With rotational transform a particle exhibits an average drift approaching zero – drifting upward from a flux surface in upper portion of the torus, but drifting upward to the original flux surface in bottom portion of the torus.

Tokamak

- The tokamak scheme has been the most successful confinement approach since the early 1970s and we will focus our attention on this scheme.
- Tokamak is an acronym developed from the Russian words TOroidalnaya KAmera ee MAgnitaya Katushka which means "toroidal chamber with magnetic coils."
- The tokamak employs an induced current in the plasma and the associated poloidal field to provide rotational transform.

Tokamak

- Consider Faraday's Law of Induction applied to a conductor linking a changing B field:



The induced electromotive force, ε^{ind}, is given by:

$$\varepsilon^{ind} = -\frac{d\phi}{dt}$$

where,

$\phi \equiv BA \equiv$ Magnetic flux

and A is the area.

Figure 12: Induced electromotive force.

- If B is changing with time (A fixed), the direction of ϵ_{ind} ($= IR$) is such that the induced current, I, flows in a direction to oppose $dB/dt \propto d\Phi/dt$.

Tokamak

- Consider dB/dt decreasing:

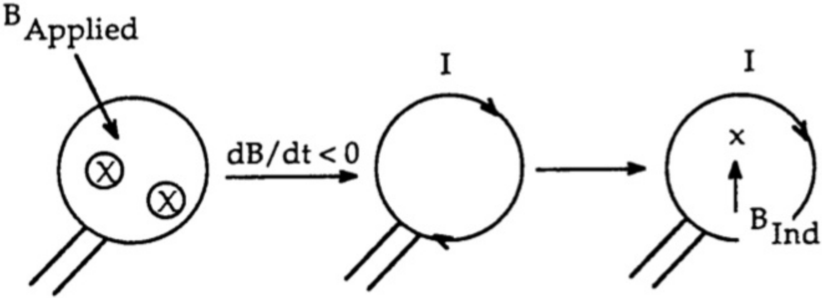


Figure 13: Induced magnetic field.

- Thus, B_{ind} tries to compensate for $dB/dt < 0$, that is, B_{ind} negates to some extent the decreasing B applied.

Tokamak

- Consider a transformer analog of the tokamak:

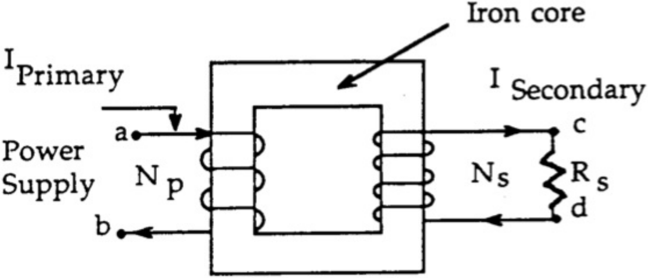


Figure 14: Transformer.

- Driving a current in the primary, p, causes a flux change on the primary side, $d\Phi_p/dt$ ($\propto dB/dt$) and this flux change also links the secondary side, s. The EMF on the secondary side, ϵ_s , is given by:

$$\begin{aligned} \epsilon_s &= I_s R_s = -N_s d\Phi/dt \\ \epsilon_p &= -N_p d\Phi/dt \end{aligned}$$

Tokamak

- The tokamak is a "transformer" with a single turn secondary – the plasma. We can see the plasma as a wire in the shape of a big fat ring.
- The magnetic flux change in the tokamak transformer induces a plasma current, I_p, given by:

$$I_p R_p = -\frac{d\Phi}{dt} \tag{9}$$

  where R_p is the resistivity of the plasma.
- We can get a reasonable estimate of the associated poloidal field, B_p, in a tokamak using a cylindrical approximation:

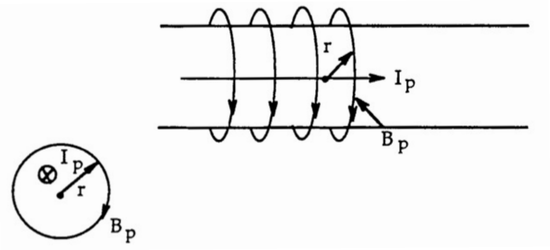


Figure 15: Induced poloidal magnetic field.

Tokamak

- Using Ampere's Law:

$$\oint \bar{B}_p \cdot \bar{ds} = \mu_o I_p \tag{10}$$

- Since B_p is fixed at a given r, and $\bar{B}_p$ and $\bar{ds}$ are parallel,

$$\oint \bar{B}_p \cdot \bar{ds} = \oint B_p ds = B_p \oint ds = \mu_o I_p \quad \Longleftrightarrow \quad B_p 2\pi r = \mu_o I_p \tag{11}$$

$$\Longleftrightarrow \quad B_p = \frac{\mu_o I_p}{2\pi r} \tag{12}$$

- The induced current, I_p, gives rise to the poloidal field, $\bar{B}_p$, and the interaction of this field with the applied toroidal field provides the rotational transform.

- Typical tokamak configurations are shown below.

Tokamak

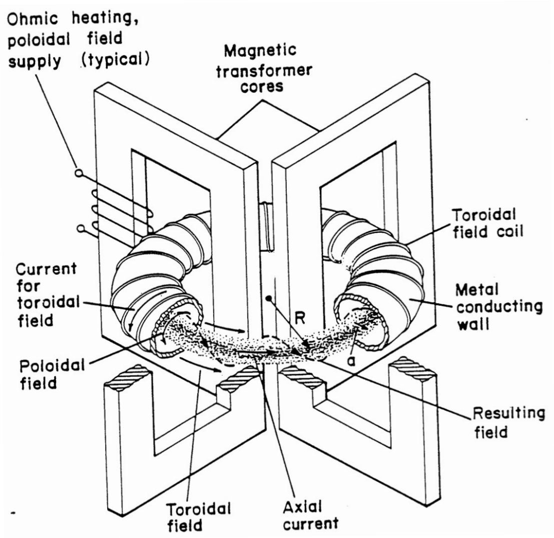


Figure 16: Tokamak coil configuration.

Tokamak

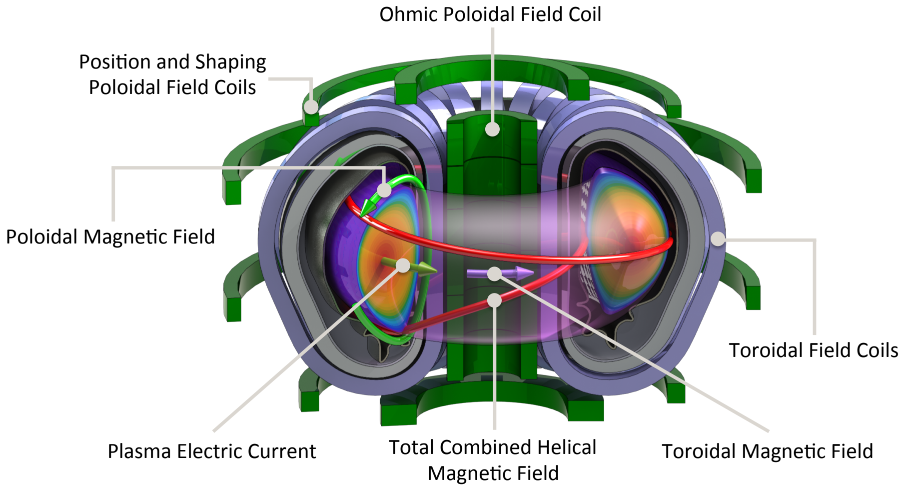


Figure 17: Tokamak coil configuration.

Tokamak

- The toroidal field on axis, $B_t(R)$ is given by

$$B_t(R) = \frac{\mu_o N I_c}{2\pi R} \qquad (13)$$

where NI_c is the total number of ampere turns in the toroidal coils and R is the major radius.

- Using a cylindrical approximation for the torus and Ampere's Law, we obtain the following expression for the poloidal field, B_p:

$$\oint \bar{B}_p \cdot \bar{ds} = \mu_o I_p \iff B_p(r) = \frac{\mu_o I_p}{2\pi r} \qquad (14)$$

where I_p is the induced plasma current and r is the plasma minor radius.

Tokamak - Limits on β

- Since $P_f \propto \beta^2 B^4$, the achievable value of beta, β, has a profound effect on the feasibility of fusion power.

- Recall
$$\beta = \frac{nkT}{\frac{B^2}{2\mu_o}}. \tag{15}$$

- Here,
$$B^2 = B_t^2 + B_p^2. \tag{16}$$

- We define beta poloidal, β_p, as
$$\beta_p = \frac{nkT}{\frac{B_p^2}{2\mu_o}}. \tag{17}$$

Tokamak - Limits on β

- Thus,

$$\frac{\beta}{\beta_p} = \frac{\frac{nkT}{\frac{B_t^2}{2\mu_o}+\frac{B_p^2}{2\mu_o}}}{\frac{nkT}{\frac{B_p^2}{2\mu_o}}} \quad (18)$$

- After simplifications,

$$\frac{\beta}{\beta_p} = \frac{B_p^2}{B_t^2 + B_p^2} = \frac{1}{1 + \frac{B_t^2}{B_p^2}} \quad (19)$$

- Finally,

$$\beta = \frac{\beta_p}{1 + \frac{B_t^2}{B_p^2}} \quad (20)$$

Tokamak - Limits on β

- Recall the safety factor, q, equals the number of rotations a field line makes in the toroidal direction per rotation in the poloidal direction $(2\pi/\iota)$.

- At the plasma edge, the safety factor $q(a)$ is given by

$$q(a) = \frac{a}{R}\frac{B_t(R)}{B_p(a)} \tag{21}$$

- The ratio, R/a, is called the aspect ratio A. Thus,

$$q(a) = \frac{1}{A}\frac{B_t(R)}{B_p(a)} \tag{22}$$

- Thus,

$$\left(\frac{B_t}{B_p}\right)^2 = q^2 A^2 \iff \beta = \frac{\beta_p}{1 + q^2 A^2} \tag{23}$$

Tokamak - Limits on β

- Stability considerations place limits on β_p and q while A is determined by engineering.
- Stability requires that $q(r) > 1$.

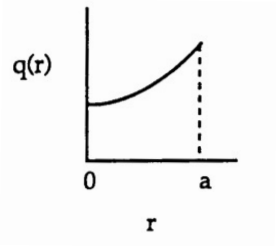


Figure 18: Typical q profile.

- If $q \leq 1$, major "disruptions" of the plasma is observed. Typical parameters for a tokamak might be:

$$q(0) > 1 \text{ and } q(a) \gtrapprox 3 \tag{24}$$

Tokamak - Limits on β

- With regard to β_p, it has been observed that gross confinement deteriorates for
$$\beta_p \geq 0.6A \tag{25}$$

- Consider a tokamak with the following parameters: $A = 4$, $q(a) = 3$ and $\beta_p = 0.5A$. For this case,
$$\beta = \frac{(0.5)(4)}{1 + (3)^2(4)^2} = \frac{2}{145} \sim 0.0138 \tag{26}$$

- The inherently low beta values of tokamaks must be increased in order to make tokamaks economically attractive.

- The tokamak program has examined several techniques for higher β: circular cross sections, $\beta \sim 3 - 5\%$, elongated plasma, $\beta \sim 10\%$, bean shaped plasma, $\beta \sim 10 - 15\%$, low aspect ratio, $\beta \sim 10 - 15\%$.

Tokamak

- The induced current in the tokamak leads to ohmic heating of the plasma via $I_p^2 R_p$ where R_p is the resistance of the plasma.

- However, $R_p \propto 1/T^{3/2}$ – as the plasma gets hotter R_p decreases and P_{ohmic} decreases.

- Thus, it appears unlikely that ohmic heating in tokamaks will be sufficient to raise the temperature to the ignition point and tokamaks will require auxiliary heating.

Tokamak

- Also, recall that $I_p R_p = -d\Phi/dt$, where $\Phi = BA$. Thus, the induced current persists only as long as $d\Phi/dt$ persists.

- For fixed area, A, $d\Phi/dt \propto dB/dt$, and Δt is limited by ΔB which is limited by technology.

- Therefore, if I_p is to be sustained by the transformer action, it will be limited in duration by technology. Thus, in the transformer mode, a tokamak is a pulsed device.

- A large effort in tokamak research is aimed at non-inductive current drive – delivering momentum in a preferred direction to the plasma particles (usually electrons) – RF and beam techniques are being considered for this application.

Tokamak

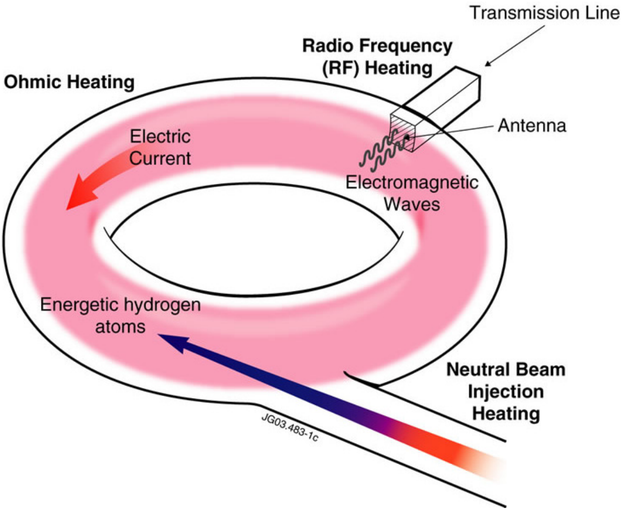


Figure 19: Plasma Heating and Current Drives.

Tokamak - Plasma System

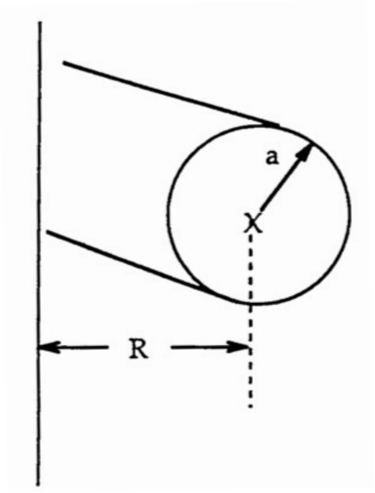


Figure 20: Plasma system.

$$0 \leq r \leq a \equiv \text{Minor Radius}, R \equiv \text{Major Radius}, A = \frac{R}{a} \equiv \text{Aspect Ratio}. \quad (27)$$

Tokamak - Volume-Averaged Quantities

- It is emphasized that plasma density, n, and temperature, T, vary with position as indicated schematically below:

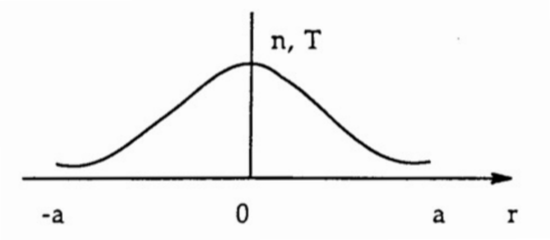


Figure 21: Spatial dependence.

- Experimentally we can measure n and T versus position and we can calculate average quantities such as

$$\bar{n} = \frac{\int n dV}{V} \quad \bar{T} = \frac{\int T dV}{V} \tag{28}$$

- Also $<\sigma v>$ depends on T and therefore, is also a function of position. We can thus define the following average:

$$<\bar{\sigma v}> = <\sigma v>_{\bar{T}} \tag{29}$$

- Consider the volume averaged power density defined as

$$\bar{P}_f = \frac{\int n_D n_T <\sigma v> E_f dV}{V} \tag{30}$$

- Note that, in general,

$$\bar{P}_f \neq \bar{n}_D \bar{n}_T <\bar{\sigma v}> E_f \tag{31}$$

- However,

$$\bar{P}_f = \bar{n}_D \bar{n}_T <\bar{\sigma v}> E_f f_p \tag{32}$$

where f_p is a profile factor defined as

$$f_p = \frac{\frac{\int n_D n_T <\sigma v> E_f dV}{V}}{\bar{n}_D \bar{n}_T <\bar{\sigma v}> E_f} \approx 1.5 - 2 \text{ (typically)} \tag{33}$$

Tokamak - Reactor Design

A DT tokamak plasma has the following fixed parameters:

- $\bar{n}_e = 10^{20} m^{-3}$, $\bar{T}_i = \bar{T}_e = 10 keV$, $\bar{n}_D = \bar{n}_T$, $Z_{eff} = 1$ (purely hydrogenic)
- $P_{aux} = 0$ (ignition)
- $R = 6.6m$
- $\beta = 5\%$
- $f_p = 2$

These are questions we are typically interested in answering:

1. What is the necessary confinement time for ignition or given Q?
2. What is the necessary tokamak size?
3. What are the required magnetic fields (currents) to confine the plasma?
4. What is the resulting fusion power?
5. What are the resulting wall loadings?

NOTE:

- Bremsstrahlung is not negligible ($C = 5 \times 10^{-37} \frac{Wm^3}{\sqrt{keV}}$)
- Assume $B_t >> B_p$ ($B^2 = B_t^2 + B_p^2 \approx B_t^2$)

Tokamak - Confinement for Ignition

What is the needed confinement time to operate at ignition ($P_{aux} \equiv 0$)?

Tokamak - Confinement for Ignition

What is the needed confinement time to operate at ignition ($P_{aux} \equiv 0$)?

Consider the power balance averaged over the volume ($P_\alpha = P_L + P_B$),

$$\overbrace{\frac{\bar{n}_e^2}{4} < \bar{\sigma v} > E_\alpha f_p}^{P_\alpha = \bar{n}_D \bar{n}_T < \bar{\sigma v} > E_\alpha f_p} = \overbrace{\frac{3\bar{n}_e k\bar{T}}{\tau_E}}^{P_L = (\frac{3}{2}\bar{n}k\bar{T})/\tau_E} + \overbrace{C\bar{n}_e^2 \bar{T}^{1/2}}^{P_B = C\bar{n}_e^2 \bar{T}^{1/2} Z_{eff}}, \qquad (34)$$

where we have used $\bar{n}_e = \bar{n}_D + \bar{n}_T \triangleq \bar{n}_i$ (quasineutrality), $\bar{n}_D \bar{n}_T = \bar{n}_e/4$, $\bar{n} = \bar{n}_e + \bar{n}_i = 2\bar{n}_e$. Solving for $\bar{n}_e \tau_E$ yields (Lawson Criterion)

$$\bar{n}_e \tau_E = \frac{3k\bar{T}}{\frac{1}{4} < \bar{\sigma v} > E_\alpha f_p - C\bar{T}^{1/2}} \qquad (35)$$

Since $k\bar{T} = 10keV \Rightarrow < \bar{\sigma v} > = 1.1 \times 10^{-22} m^3 s^{-1}$, we obtain

$$(\bar{n}_e \tau_E)_{ign}^{req} = \frac{3 \times 10 \times 1.6 \times 10^{-16}}{\frac{1.1 \times 10^{-22}}{4} \times 3.5 \times 1.6 \times 10^{-13} \times 2 - 5 \times 10^{-37} \times (10)^{1/2}}$$
$$= 1.64 \times 10^{20} m^{-3} s$$

Since $\bar{n}_e = 10^{20} m^{-3}$, $\tau_E|_{ign}^{req} = 1.64s$.

Tokamak - Plasma Size for Ignition

What size of tokamak is needed for ignition ($P_{aux} \equiv 0$)?

Tokamak - Plasma Size for Ignition

What size of tokamak is needed for ignition ($P_{aux} \equiv 0$)?

A commonly used empirical scaling is that first observed by the MIT group on the Alcator Tokamak experiment and now called "Alcator Scaling." By this scaling

$$\bar{n}_e \tau_E (m^{-3}s) = 6 \times 10^{-21} \bar{n}_e^2 a^2 \tag{36}$$

Where the volume average density, $\bar{n}_e$, is in $\#/m^3$ and the plasma radius, a, is in m. For ignition,

$$(\bar{n}_e \tau_E)_{ALC} = 6 \times 10^{-21} \bar{n}_e^2 a^2 = (\bar{n}_e \tau_E)_{ign}^{req} \tag{37}$$

Thus,

$$6 \times 10^{-21} \bar{n}_e^2 a^2 = 1.64 \times 10^{20} \Rightarrow a^2 = \frac{1.64 \times 10^{20}}{(6 \times 10^{-21})(10^{20})^2} = 2.733 \tag{38}$$

Therefore,

$$a^{req} = 1.65m \tag{39}$$

Tokamak - Magnetic Field at Ignition

What is the required field on axis, $B_t(R)$?

Tokamak - Magnetic Field at Ignition

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Recall,

$$\beta = \frac{\bar{n}k\bar{T}}{\frac{B^2}{2\mu_o}} \iff B_t^2 = \frac{2\mu_o(2\bar{n}_e k\bar{T})}{\beta} \tag{40}$$

where we have used $B_t >> B_p$ ($B^2 = B_t^2 + B_p^2 \approx B_t^2$) and $\bar{n} = 2\bar{n}_e$.

Given $k\bar{T} = 10keV$, $\bar{n}_e = 10^{20}m^{-3}$, and $\beta = 5\%$,

$$B_t^2 = \frac{2(4\pi \times 10^{-7})(2 \times 10^{20} \times 10 \times 1.6 \times 10^{-16})}{0.05} = 16.07 \tag{41}$$

Therefore,

$$B_t^{req}(R) \approx 4T \tag{42}$$

Tokamak - Fusion Power at Ignition

What is the resulting fusion power?

Tokamak - Fusion Power at Ignition

What is the resulting fusion power?

We next calculate the fusion power density at ignition:

$$\begin{aligned}\bar{P}_f &= \bar{n}_D \bar{n}_T < \bar{\sigma v} > E_f f_p = \frac{\bar{n}_e^2}{4} < \bar{\sigma v} > E_f f_p \\ &= \frac{(10^{20})^2}{4}(1.1 \times 10^{-22})(17.6 \times 1.6 \times 10^{-13})(2) \\ &= 1.55 \frac{MW}{m^3}\end{aligned}$$

where we have used $\bar{n}_D = \bar{n}_T = \bar{n}_e/2$ and negligible $\bar{n}_\alpha$ and $\bar{n}_I$ $(Z_{eff} = 1)$. The total fusion power, P_f^T, is calculated from the volume and the power density. The volume of the torus is given by $V = \pi a^2 \times 2\pi R = 2\pi^2 a^2 R$. Since $R = 6.6m$,

$$P_f^T = P_f \times V = P_f \times 2\pi^2 a^2 R = 1.55 \times 2\pi^2 (6.6)(1.65)^2 = 549MW \qquad (43)$$

Note that $P_n^T = 0.8 P_f^T = 439MW$ and $P_\alpha = 0.2 P_f^T = 110MW$.

Tokamak - Wall Loadings

What are the wall loadings?

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What are the wall loadings?

First we define the neutron wall loading as $L_n = \frac{P_n}{S}$, where S is the area of the torus surface, which is given by $S = 2\pi a \times 2\pi R = 4\pi^2 Ra$. Therefore,

$$S = 4\pi^2 Ra = 4\pi^2 (6.6)(1.65) \approx 430m^2 \tag{44}$$

Note $V/S = 2\pi^2 Ra^2 / 4\pi^2 Ra = a/2$. Thus,

$$L_n = \frac{439}{430} \approx 1 \frac{MW}{m^2} = 10^6 \frac{J}{m^2 s} \tag{45}$$

One neutron carries $\sim 14.1 MeV$ or $14.1 \times 1.6 \times 10^{-13} J$ of energy. Thus, the wall loading of $1MW/m^2$ can also be expressed as

$$L_n = \frac{10^6 \frac{J}{m^2 s}}{14.1 \times 1.6 \times 10^{-13} \frac{J}{\#n}} \approx 4.43 \times 10^{17} \frac{\#n}{m^2 s} \tag{46}$$