Nuclear Fusion and Radiation

Lecture 6 (Meetings 13 & 14)

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Fusion Approaches: How to confine the hot plasma?

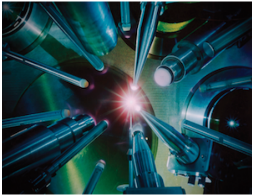
Wall	Energy confinement time $\sim 10^6$ too small; Heat fluxes to wall too intense.
Size	Physically very large, $\sim 2km$; Power output $\sim 10^7\, 1000MWe$ plants.
Underground explosions	Energetically feasible; Socially and politically appears impractical.
Electrostatic fields	Field only "confines" $+$ or - charges, not both; Useful combined with other approach?
Inertial confinement	Energetically appears feasible.
Magnetic fields	Energetically appears feasible.

Table 1: Possible approaches to fusion.

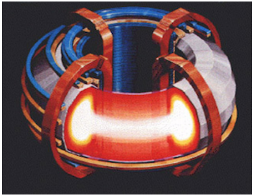
Fusion Approaches: How to confine the hot plasma?

- **Gravitational Confinement** $(300W/m^3)$
  - In a deep gravitational well, even fast particles are trapped.
  - This is the method used by the Sun.

- **Inertial Confinement** $(10^{28}W/m^3)$
  - Imploding the hydrogen gases together with inertia, then holding them together long enough for fusion reactions to occur.



- **Magnetic Confinement** $(10^7W/m^3)$
  - Using magnetic fields acting on hydrogen atoms which have been ionized.



Inertial Confinement

- Inertial confinement is based on microexplosions of fuel pellets triggered by a suitable driver.

- Consider a fuel pellet of solid DT fuel and an appropriate driver:

Figure 1: Inertial confinement approach to fusion.

- The pellet is heated to fusion temperatures in the time τ_H. The pellet "burns" (undergoes fusion) for a time, τ_B, before the pellet disassembles in the time, τ_D. In general, we need $\tau_D \sim \tau_B > \tau_H$.

Inertial Confinement

- To estimate τ_D, the thermal disassembly time, we assume

$$\tau_D \sim \frac{r}{< V_i >}$$

- In this time, the pellet will have expanded a distance r at which point the fusion burn is assumed to terminate.
- Note that $< V_i >$ is used since it refers to the fuel; if the electrons leave/expand, the burn would not be directly affected. For the deuteron, $< V_D > \sim 10^6 m/s$ at $10keV$.
- We can write the $n\tau_E$ (Lawson) criterion for inertial confinement systems as:

$$n\tau_E \sim n\tau_D \sim \frac{nr}{< V_D >}$$

- We recall that the value for $n\tau_E$ for DT at $10keV$ was $\sim 10^{20} m^{-3} s$. Then,

$$n\tau_E \sim n\tau_D \sim \frac{nr}{< V_D >} \sim 10^{20} m^{-3} s \quad (DT@10keV)$$

Inertial Confinement

- Therefore, the $n\tau_E$ (Lawson) criterion for inertial fusion can be rewritten as

$$(nr)_L \sim 10^{20}(m^{-3}s)10^6(m/s) \sim 10^{26}\frac{\#}{m^2}$$

- In the literature this criterion is usually expressed as the product, ρr, where ρ is the density and the units are in cgs.

- To arrive at this form, we define an average mass for the DT mixture, $\bar{M}_i$, as

$$\bar{M}_i = \frac{M_D + M_T}{2} = \frac{(3.34 + 5.01)10^{-27}kg}{2} = 4.18 \times 10^{-24}g$$

Multiplying $(nr)_L$ by $\bar{M}_i$ yields

$$\bar{M}_i nr = 4.18 \times 10^{-24}(g)10^{22}\left(\frac{1}{cm^2}\right) \sim 0.042\frac{g}{cm^2}$$

Inertial Confinement

- Noting that the product, $\bar{M}_i n$ is simply the density, ρ, we obtain

$$(\rho r)_L \sim 0.042 \frac{g}{cm^2}$$

- A more accurate estimate yields $(\rho r)_{BE} \sim 0.1 g/cm^2$ and this is the value usually seen in the literature.

- If the fuel pellets are assumed to be of dimension $r \sim 0.5cm$, then

$$\tau_D \sim \frac{0.5cm}{10^8 cm/s} \sim 5 \times 10^{-9} s \quad (5 \text{ nanoseconds})$$

Therefore, τ_H should be $< 10^{-9} s$.

Inertial Confinement

- Several drivers have been proposed to heat the pellet to fusion conditions on this time scale. These include:
  - Lasers (solid and gas)
  - Light ion beams (LIB) – e.g., p
  - Heavy ion beams (HIB) – e.g., $Hg \to U$
- Inertial fusion research is a large world wide effort since it appears to be energetically feasible.

National Ignition Facility and Photon Science

https://lasers.llnl.gov

- Take a hollow, spherical plastic capsule about two millimeters in diameter (about the size of a small pea)
- Fill it with 150 micrograms (less than one-millionth of a pound) of a mixture of deuterium and tritium, the two heavy isotopes of hydrogen.
- Take a laser that for about 20 billionths of a second can generate 500 trillion watts – the equivalent of five million million 100-watt light bulbs.
- Focus all that laser power onto the capsule surface.
- Wait ten billionths of a second.
- Result: one miniature star.

Inertial Confinement

- In this process the capsule and its deuterium-tritium (DT) fuel will be compressed to a density 100 times that of solid lead, and heated to more than 100 million degrees Celsius – hotter than the center of the sun.
- These conditions are just those required to initiate thermonuclear fusion, the energy source of stars.
- By following our recipe, we would make a miniature star that lasts for a tiny fraction of a second.
- During its brief lifetime, it will produce energy the way the stars and the sun do, by nuclear fusion.
- Our little star will produce ten to 100 times more energy than we used to ignite it.

Inertial Confinement

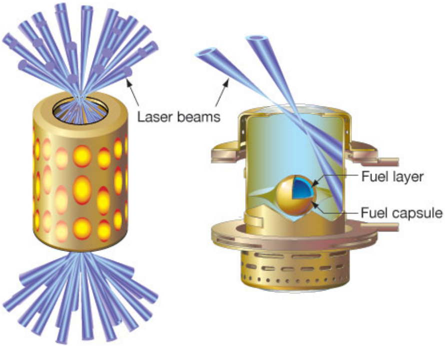
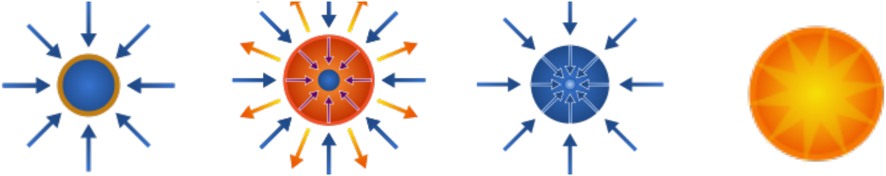


Figure 2: Hohlraum

- All of the energy of NIF's 192 beams is directed inside a gold cylinder called a hohlraum, which is about the size of a dime.
- A tiny capsule inside the hohlraum contains atoms of deuterium (hydrogen with one neutron) and tritium (hydrogen with two neutrons) that fuel the ignition process.

Inertial Confinement



Laser beams rapidly heat the surface of the fusion target, forming a surrounding plasma envelope.

Fuel is compressed by the rocketlike blowoff of the hot surface material.

During the final part of the capsule implosion, the fuel core reaches 20 times the density of lead and ignites at 100 million degrees.

Thermonuclear burn spreads rapidly through the compressed fuel, yielding many times the input energy.

Figure 3: Inertial confinement.

Inertial Confinement

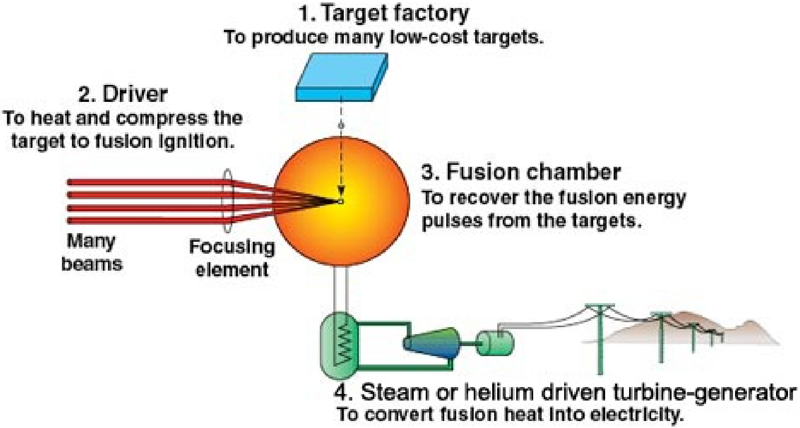


Figure 4: Inertial confinement power plant.

Magnetic Confinement

- Let us consider the motion of a charged particle in a magnetic field:

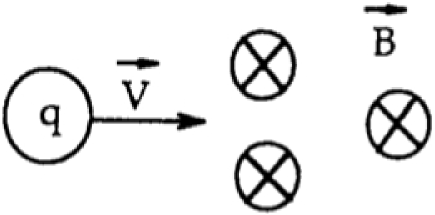


Figure 5: Moving particle in magnetic field.

- Notation: The magnetic field, $\bar{B}$, is down into the paper.

Magnetic Confinement

- A force, F_m, is observed and is given by $F_m = qv \times B$. The direction of the force depends on the sign of the charge, q.

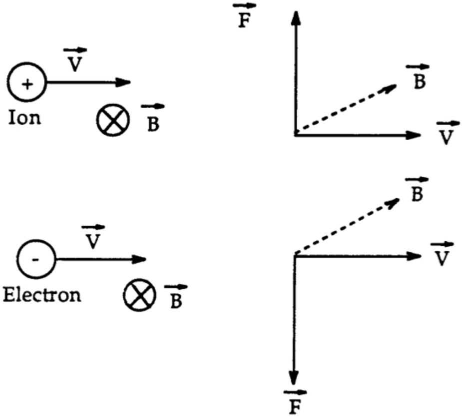


Figure 6: Magnetic force.

Magnetic Confinement

- The equation of motion for the charged particle is given by
$$F_m = m\frac{dv}{dt} = ma = qv \times B.$$
Therefore,
$$F_m \perp v \text{ and } \frac{dv}{dt} \perp v.$$
- Thus, F_m does not work on the charged particle and v changes in orientation but not in magnitude. This situation describes circular motion. Schematically,

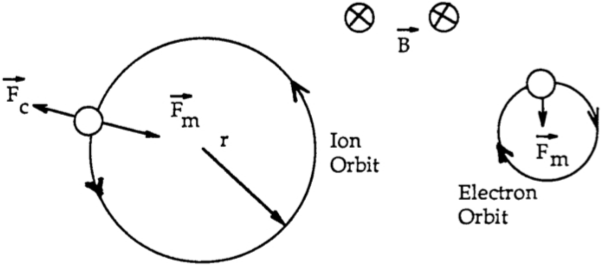


Figure 7: Circular motion of the moving particle.

Magnetic Confinement

- At any point of the orbit the inward magnetic force $\bar{F}_m$ is balanced by an outward centrifugal force $\bar{F}_c$. Thus,

$$F_m \triangleq |q|v \times B = m\frac{v^2}{r} \triangleq F_c$$

- If $v \perp B$,

$$|q|v \times B = |q|vB = m\frac{v^2}{r}$$

  and

$$r = \frac{mv}{|q|B}$$

  is the gyromagnetic radius (Larmor radius).

- The magnetic field is given in "Tesla", where

$$1Tesla = 10^4 Gauss = 1Weber/m^2$$

  and $1Weber = 1volt - sec$ is a unit of magnetic flux.

Magnetic Confinement

- The field strength represents a field line density, illustrated schematically below:

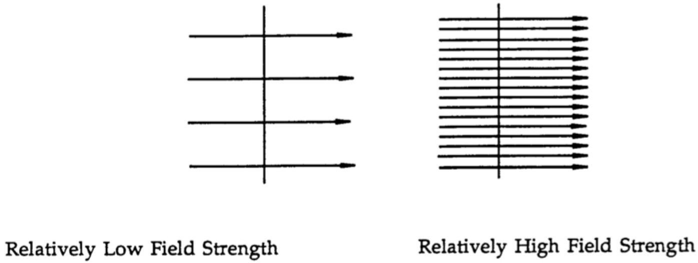


Figure 8: Magnetic field strength.

Magnetic Confinement

- For particles in a plasma it is convenient to write

$$mv = \sqrt{2mE} \propto \sqrt{mkT}$$

and therefore

$$r \propto \frac{\sqrt{mkT}}{|q|B}$$

- At the same kT and B, and $q_i = q_e$,

$$r_i \propto \sqrt{M_i} \text{ and } r_e \propto \sqrt{M_e}$$

and

$$r_e = \left(\frac{M_e}{M_i}\right)^{1/2} r_i$$

For deuterons, $\left(\frac{M_e}{M_D}\right)^{1/2} \sim 1/60 \Rightarrow r_e \sim r_D/60$.

Magnetic Confinement

- Consider now the case when v is not perpendicular to B:

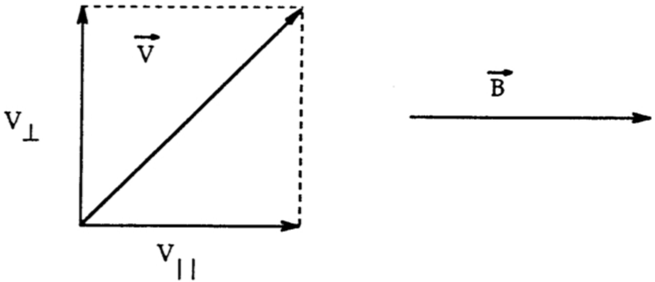


Figure 9: Not perpendicular magnetic and velocity fields.

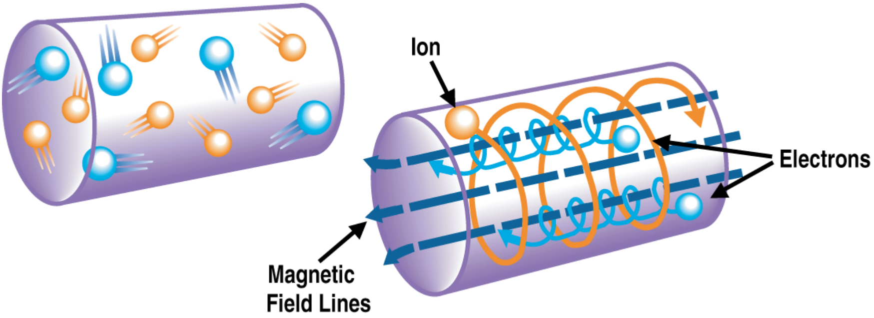
- The motion of the charged particle can be resolved into motion $\perp$ to B and motion $\parallel$ to B.

Magnetic Confinement

- Associated with $v_\perp$ is $E_\perp$ and with $v_\parallel$ is $E_\parallel$.
  - $E_\parallel$ represents one degree of motion (i.e., $E_\parallel = kT/2$)
  - $E_\perp$ represents two degrees of motion (i.e., $E_\perp = kT$)
- The component $E_\perp$ ($v_\perp$) leads to circular orbits in the plane containing $E_\perp$ and perpendicular to B (where $mv_\perp = \sqrt{2mE_\perp} = \sqrt{2mkT}$).
- Motion in the direction of $v_\parallel$ (parallel to B) is unaffected by B ($v_\parallel \times B \equiv 0$).
- The total motion is the superposition of the circular orbits and the motion along field lines, leading to helical trajectories.
- Thus, both ions and electrons are "held" confined by the magnetic force and move along the field lines. This situation is in contrast to that with electric fields which "hold" confined only one charge species.

Magnetic Confinement

- We exploit the fact that moving charged particles in a **plasma** respond to the presence of an imposed magnetic field due to the **“Lorentz force”**.



- However, for practical devices end losses must be eliminated.

Magnetic Confinement

- A plasma tends to exclude externally applied electric fields due to rearrangement of charges.
- How does the plasma react to an applied magnetic field? Consider the following:

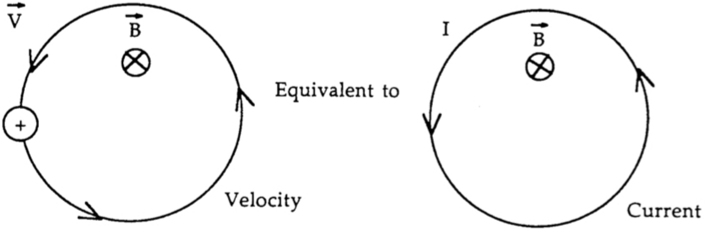


Figure 10: Current generated by moving particle.

Magnetic Confinement

- By convention the current direction is that of positive charge flow. A current has associated with it a magnetic field (Ampere's Law). Schematically,

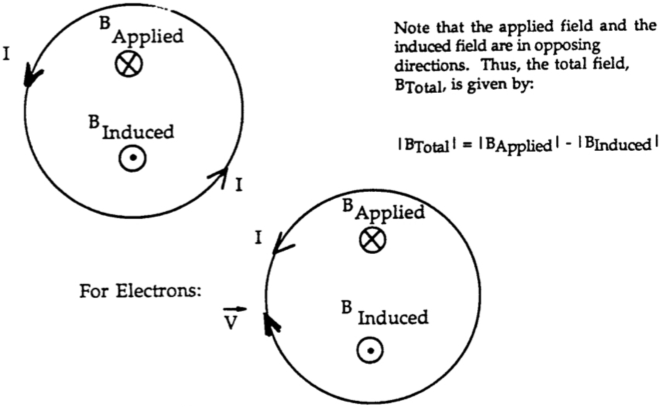


Figure 11: Plasma diamagnetism.

Magnetic Confinement

- This effect on the applied field is called diamagnetism and plasmas are diamagnetic, i.e., they tend to reduce the externally applied field and $|B_{plasma}| < |B_{vacuum}|$.

- Let us consider the strength of the magnetic field required to hold the plasma. Here we take a "fluid" approach and invoke the concept of magnetic pressure:

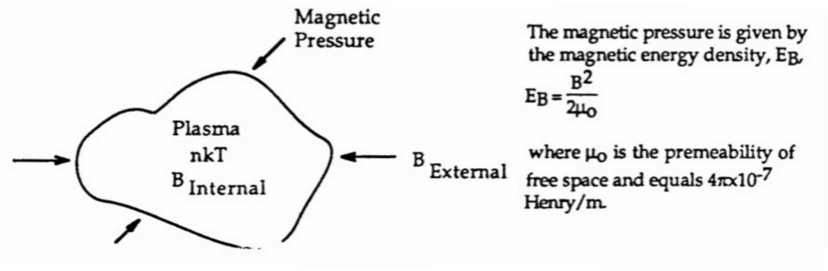


Figure 12: Pressure balance.

Magnetic Confinement

- Assuming the internal pressure of the plasma (kinetic plus magnetic) is balanced by the external magnetic pressure:

$$nkT + \frac{B_{int}^2}{2\mu_0} = \frac{B_{ext}^2}{2\mu_0}$$

  Recall that the kinetic pressure is defined as $p = nkT$ (See Lecture 4).

- We now define the parameter beta, β, such that

$$\beta = \frac{nkT}{\frac{B_{ext}^2}{2\mu_0}} = \frac{\frac{B_{ext}^2}{2\mu_0} - \frac{B_{int}^2}{2\mu_0}}{\frac{B_{ext}^2}{2\mu_0}} = 1 - \frac{B_{int}^2}{B_{ext}^2}$$

- If a plasma is "perfectly diamagnetic", $B_{int} \to 0$ and $\beta \to 1$. This would be the case of the most efficient use of the applied external field – i.e., the external field is totally used to "hold" the plasma kinetic pressure (nkT).

Magnetic Confinement

- Consider again the standard plasma ($n_e = 10^{20}/m^3$ and $kT_i = kT_e \triangleq kT = 10keV$). By definition, $n_i \triangleq n_D + n_T$ (no impurities, negligible α-particle population). Moreover, $n_D = n_T$ is assumed.
- Then, the quasi-neutrality condition yields $n_e = n_D + n_T = n_i$, which implies $n_D = n_T = n_e/2$ and $n \triangleq n_e + n_i = 2n_e$.
- Thus, $p = 2n_e kT = 0.32 \times 10^6 J/m^3$.
- If $\beta = 1$,

$$\frac{B_{ext}^2}{2\mu_0} = 0.32\times10^6 \frac{J}{m^3} \Rightarrow B_{ext}^2 = (0.32\times10^6)\frac{J}{m^3}(2)(4\pi\times10^{-7}\frac{H}{m}) = 0.803T^2$$

  and

$$B_{ext} \sim 0.9T$$

  Note that $H \triangleq \frac{J}{A^2}$. Therefore, $\sqrt{\frac{J}{m^3}\frac{H}{m}} = \sqrt{\frac{J}{m^3}\frac{J}{A^2 m}} = \frac{J}{Am^2} \triangleq T$.

- Magnet technology now yields fields in the range of $8 - 12T$ for steady state fields. Therefore, $0.9T$ is a "low" field.

Magnetic Confinement

- Now we consider situations for which $\beta < 1$, less than 100% efficient use of B_{ext}. In general,

$$\frac{B_{ext}^2}{2\mu_0} = \frac{0.32 \times 10^6 \frac{J}{m^3}}{\beta} \Rightarrow B_{ext}^2 = \frac{(0.32 \times 10^6)\frac{J}{m^3}(2)(4\pi \times 10^{-7})\frac{H}{m}}{\beta} = \frac{0.803T^2}{\beta}$$

- Consider a low β case for which $\beta = 0.05$. For this case

$$B_{ext} \sim 4T$$

This is still well within technology limits.

Magnetic Confinement

- We also note that for a given β and $B_{ext} = B$,

$$n \propto \beta B^2/T$$

and for a fixed temperature T,

$$n \propto \beta B^2$$

- Recall that the fusion power density, P_f, is given by $P_f \propto n^2 < \sigma V > E_f$ and, thus, at fixed temperature

$$P_f \propto n^2 \propto \beta^2 B^4$$

- This emphasizes the importance of β (fixed by physics) and B (fixed by technology) in determining the fusion power density.

Magnetic Confinement

- In summary, we can say that confinement by magnetic fields appears feasible and we will pursue this approach in greater detail in subsequent lectures.

- We will study both "open" and "closed" configurations:

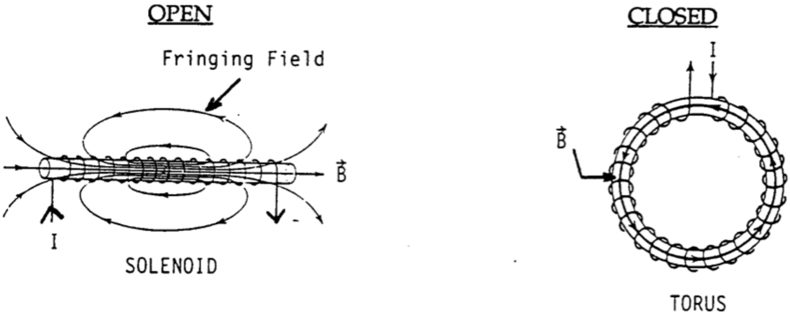


Figure 13: Open & closed magnetic configurations.

Open Configurations

- To examine confinement in open systems, consider the following schematic:

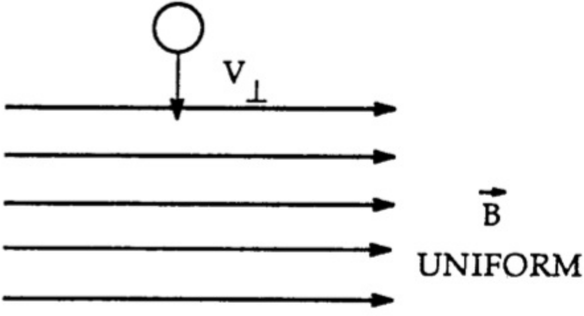


Figure 14: Perpendicular travel in uniform magnetic field.

Open Configurations

- We inject particles with only $V_\perp$ and they are confined in circular orbits. However, collisions will give these particles significant $V_\|$ and these particles will eventually escape the confinement region along field lines.
- Consider a solenoid $\sim 1km$ in length and $V_\| \sim 10^6 m/s$ (estimated velocity for Deuteron at 10keV (see Lecture 4)).
- In this case, the energy confinement time, τ_E, is approximated by

$$\tau_E \sim \frac{10^3 m}{10^6 m/s} \sim 10^{-3} s$$

- For ignition with $D-T$, we require $n_e \tau_E \sim 3 \times 10^{20} m^{-3} s$ (Lawson Criterion (see Lecture 5)) and, therefore, the electron density, n_e, must be

$$n_e \sim 3 \times 10^{20} m^{-3} s / 10^{-3} s \sim 3 \times 10^{23} / m^3.$$

Open Configurations

- At these densities, and at $T = 10keV$, the pressure, p, is $p = 2n_e kT \sim 3000 \times p_{sp}$ (purely hydrogenic plasma: $n = 2n_e$) where p_{sp} is the pressure of the standard plasma ($n_e \sim 10^{20}/m^3$ and $T \sim 10keV$).

- Thus,

$$p = 3 \times 10^3 (0.32MJ/m^3) \sim 10^9 J/m^3 \sim 10^4 \text{ atmospheres.}$$

- Let us estimate the field strength required to hold a plasma at such a pressure. Even if $\beta = 1$,

$$B_{ext} = [(2n_e kT)(2\mu_o)]^{1/2} = [(.96 \times 10^9)(2)(4\pi \times 10^{-7})]^{1/2} \sim 49T.$$

Such a steady-state field strength is well beyond technological limits.

Open Configurations

- Thus, even an extremely long solenoid is not a practical means for achieving confinement in an open configuration because of the high loss rate of particles out of the ends of the system.
- In general, the basic problem with open configurations as confinement schemes is the inherently large rate of end-losses.
- Research in open configurations has focused on the identification of methods by which the end-losses could be reduced.
- Historically, the first approach to emerge and the approach which has been the most widely used is called the magnetic “mirror” and makes use of electromagnetic “plugging” to reduce end losses. The essential features of the magnetic mirror are shown in Figures 15 and 16.

Magnetic Mirror

- Consider a particle injected at point “0” under a particularly shaped magnetic field line:

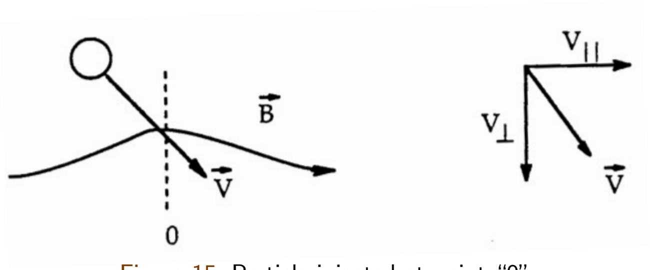


Figure 15: Particle injected at point “0”.

Magnetic Mirror

- As the particle moves along the field line (helical motion), $F_M = q(V \times B)$, and the force is always $\perp B$.

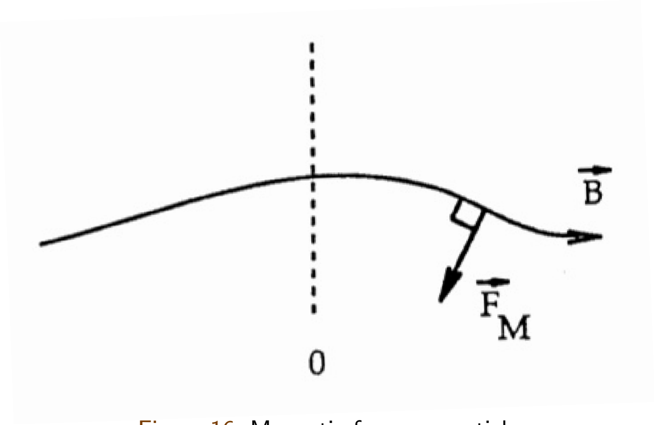


Figure 16: Magnetic force on particle.

Magnetic Mirror

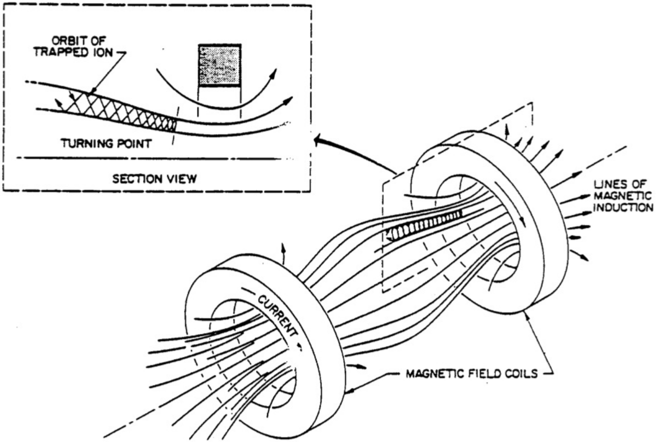


Figure 17: Coil arrangement and particle orbit in a magnetic mirror.

Magnetic Mirror

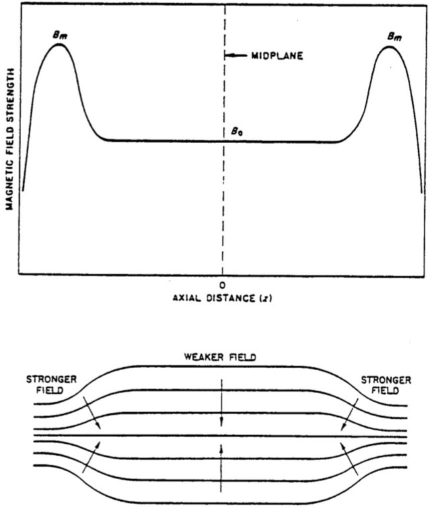


Figure 18: Schematic of field distribution in a magnetic mirror.

Magnetic Mirror

- This force $F = F_M$ can be divided into $F_\perp$ and $F_\|$ with respect to the field direction at point "0".

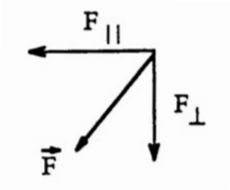


Figure 19: Force components.

- Because of the spatial variation of B, there will be an $F_\|$ component acting on the particles and this $F_\|$ will oppose $V_\|$, the initial velocity of the particle. Thus, the particle is slowed down in the $V_\|$ direction.

Magnetic Mirror

- We noted earlier that F_M does no work on the particle. Thus, the total energy of the particle must remain constant as it moves along B. Thus,

$$E_T = E_\perp + E_\parallel = \text{ constant of motion}$$

and in terms of positions "0" (center) and "M" (magnets)

$$E_T = E_\perp(0) + E_\parallel(0) = E_\perp(M) + E_\parallel(M)$$

- As the particles move from regions of lower B into regions of higher B, $E_\parallel$ is converted into $E_\perp$ by virtue of F.

- In principle, if $B(M)$ is large enough with respect to $B(0)$, $E_\parallel(M)$ can go to zero and the particle will be mirrored/reflected/confined at the ends of this configuration.

Magnetic Mirror

- It can also be shown that, if the field does not vary “too” abruptly, then the magnetic flux contained in the circular orbit of the particle is also a constant of the particle motion:

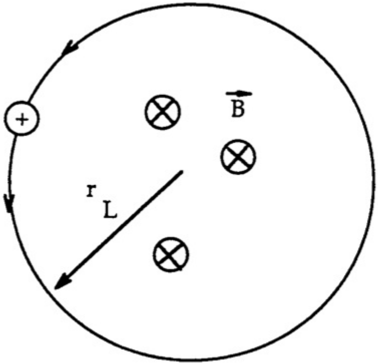


Figure 20: Projected orbit of the particle.

Magnetic Mirror

- Since

$$\Phi = \int B \cdot dA \sim B\pi r_L^2$$

- Thus,

$$\pi r_{L_0}^2 B(0) = \pi r_{L_M}^2 B(M) = \text{ constant of motion}$$

- Recall that

$$r_L = \frac{\sqrt{2ME_\perp}}{qB} \Rightarrow \Phi = B\pi r_L^2 \propto B\left(\frac{E_\perp}{B^2}\right) \propto \frac{E_\perp}{B}$$

- Thus, $\Phi =$ constant of motion implies

$$\frac{E_\perp}{B} = \text{ constant of motion}$$

where

$$\frac{E_\perp}{B} \equiv \mu = IA$$

is the magnetic moment of the orbiting particle and I is the current associated with the orbiting particle and A is the area of the orbit.

Magnetic Mirror

- The $E_\perp/B$ condition yields:

$$\frac{E_\perp(0)}{B(0)} = \frac{E_\perp(M)}{B(M)}$$

- We now combine conservation of energy and constancy of magnetic moment to obtain:

$$E_\parallel(M) = E_\parallel(0) + E_\perp(0) - E_\perp(M)$$

and

$$E_\perp(M) = E_\perp(0) \times \frac{B(M)}{B(0)}$$

Magnetic Mirror

- Defining the mirror ratio, R, by $R = \frac{B(M)}{B(0)}$, we write

$$E_{||}(M) = E_{||}(0) + E_{\perp}(0) - RE_{\perp}(0) = E_{||}(0) - E_{\perp}(0)(R-1)$$

- The particle will be mirrored/reflected at M when $E_{||}(M) \leq 0$. This condition requires

$$E_{||}(0) \leq E_{\perp}(0)(R-1) \iff \frac{E_{||}(0)}{E_{\perp}(0)} \leq R-1 \iff \frac{V_{||}^2(0)}{V_{\perp}^2(0)} \leq R-1$$

or adding $E_{\perp}(0)/E_{\perp}(0) = 1$ to both sides $\left(E_T = E_{\perp} + E_{||}\right)$,

$$\frac{E_T(0)}{E_{\perp}(0)} \leq R \iff \frac{E_{\perp}(0)}{E_T(0)} \geq \frac{1}{R}$$

Magnetic Mirror

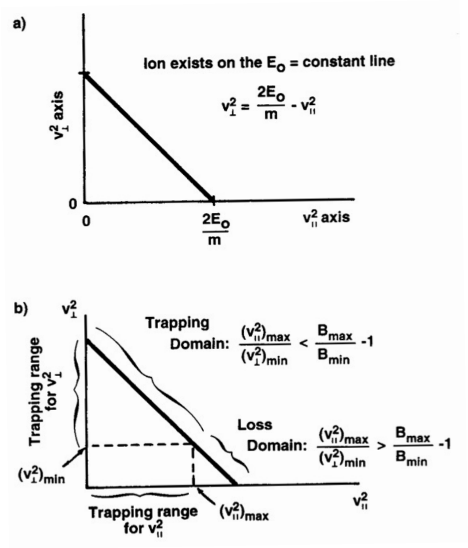


Figure 21: Magnetic force on particle.

Magnetic Mirror

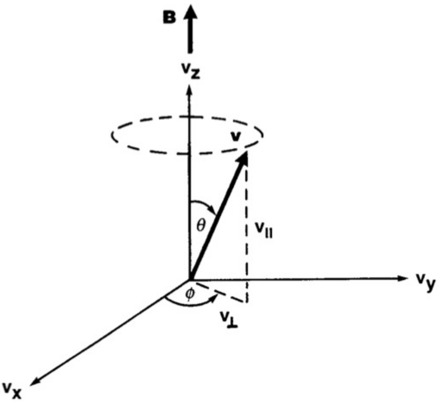


Figure 22: Condition on angle of injection.

$$\sin^2(\theta) = \frac{V_\perp^2(0)}{V_T^2(0)} = \frac{E_\perp(0)}{E_T(0)} \geq \frac{1}{R}$$

Magnetic Mirror

- Thus perpendicular injection and operation with as high a mirror ratio as possible, favors reflection/ end-loss plugging/ confinement.

- To determine the required injection angle, we must examine $B(0)$, $B(M)$ and R. Recall that the fusion power density, P_f, is given by

$$P_f \propto \beta^2 B^4$$

where $B \equiv B(0)$.

- Thus, $B(0)$ will be set by the desired P_f and by the allowable β. Let us assume $B(0) = 4T$. Technology limits will set $B(M)$. Assume $B(M) = 12T$. Thus, $R = 12T/4T = B(M)/B(0) = 3$. Thus, for reflection

$$\frac{E_\perp(0)}{E_T(0)} \geq \frac{1}{3}$$

Magnetic Mirror

- The ratio of $\perp$ energy to total energy at the injection point must be greater than $1/3$ for reflection.

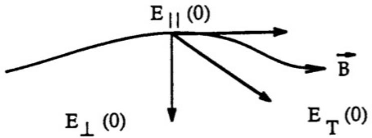


Figure 23: Injection angle.

If $E_{||}(0)$ is too large, the particle will escape.

- Note that although $E_{\perp}(0)/E_T(0)$ may satisfy the reflection condition at injection, coulomb scattering collisions will alter the orientation of the particle with respect to E.
- Thus, we must average the assembly of particles over all orientations to obtain the actual losses. We need the particle distribution in velocity and space to calculate the losses accurately.

Magnetic Mirror

- Experiments and theory began to accumulate which showed that this "classical mirror" would not work as the basis for a fusion reactor for the following two reasons:
  - Electromagnetic end plugs were found to be too leaky to give adequate confinement for reactor-level Q values ($Q = P_f / P_{inj}$).
  - The reason for this is that the mirror ratio, R, is limited by technology to ~ 3, and $R \sim 10$ would be required for adequate Q values. Thus, $Q < 1$.
  - Convex curvature of the field lines leads to a plasma (fluid) instability.
- Alternative concepts such as the "minimum B mirror" (concave (in all directions) field lines) and the "tandem mirror" (electrostatic end plugs) have been proposed to overcome these deficits.
- A breakthrough in magnet technology could be a real enabler.