Nuclear Fusion and Radiation

Lecture 5 (Meetings 9, 10, 11 & 12) - Problem 3

Eugenio Schuster

schuster@lehigh.edu
Mechanical Engineering and Mechanics
Lehigh University

Power Balance in Fusion Plasmas

Problem: (80 points) Consider a DT plasma with $n_e = 10^{20}/m^3$, $kT_e = kT_i \triangleq kT = 10keV$, α's build-up concentration $r_\alpha \triangleq n_\alpha/n_e = 0.2$, and impurity concentration $r_I \triangleq n_I/n_e = 0.01$ with atomic number $Z_I = 6$. Radiation losses by Bremsstrahlung must be considered. Give both analytical expressions and numerical values for:

(a) (10+5 points) The fuel fraction $r_{DT} \triangleq n_{DT}/n_e$, assuming $n_D = n_T \triangleq n_{DT}$.

(b) (10+10 points) The necessary auxiliary power to achieve $Q = 2$.

(c) (25+20 points) The necessary energy confinement time τ_E to achieve steady-state operation.

NOTE: No credit will be given to numerical values if analytical expressions are wrong. Remember that $E_\alpha = 3.5MeV$ and $1eV = 1.602 \times 10^{-19}J$. Use $C = 5 \times 10^{-37} \frac{Wm^3}{keV^{1/2}}$ as Bremsstrahlung power constant.

Power Balance in Fusion Plasmas

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Quasi-neutrality condition:

$$
\begin{aligned}
n_e &= n_p \\
n_e &= n_D Z_D + n_T Z_T + n_\alpha Z_\alpha + n_I Z_I \\
n_e &= n_D + n_T + 2n_\alpha + Z_I n_I \\
n_e &= n_D + n_T + 2r_\alpha n_e + Z_I r_I n_e
\end{aligned}
$$

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$$\begin{aligned} n_e &= n_p \\ n_e &= n_D Z_D + n_T Z_T + n_\alpha Z_\alpha + n_I Z_I \\ n_e &= n_D + n_T + 2n_\alpha + Z_I n_I \\ n_e &= n_D + n_T + 2r_\alpha n_e + Z_I r_I n_e \end{aligned}$$

Assuming $n_D = n_T \triangleq n_{DT}$,

$$\begin{aligned} n_e &= 2n_{DT} + 2r_\alpha n_e + Z_I r_I n_e \\ n_{DT} &= \frac{1}{2}\left(1 - 2r_\alpha - Z_I r_I\right) n_e \end{aligned}$$

Therefore,

$$r_{DT} = \frac{n_{DT}}{n_e} = \frac{1}{2}\left(1 - 2r_\alpha - Z_I r_I\right) = 0.27 \tag{1}$$

Power Balance in Fusion Plasmas

(b) The necessary auxiliary power to achieve $Q = 2$.

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Reactor gain:

$$Q = \frac{P_f}{P_{aux}} = \frac{\overbrace{5P_\alpha}^{DT}}{P_{aux}} \Rightarrow P_{aux} = \frac{5P_\alpha}{Q} = \frac{5}{2}P_\alpha$$

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Alpha power:

$$\begin{aligned} P_\alpha &= n_D n_T \langle\sigma v\rangle_{DT} E_\alpha \\ &= n_{DT}^2 \langle\sigma v\rangle_{DT} E_\alpha \\ &= \frac{n_e^2}{4}\left(1 - 2r_\alpha - Z_I r_I\right)^2 \langle\sigma v\rangle_{DT} E_\alpha \end{aligned} \quad (2)$$

Therefore,

$$P_\alpha = 4.4554 \times 10^4 \frac{W}{m^3} \quad (3)$$

$$P_{aux} = \frac{5}{2}P_\alpha = 1.1138 \times 10^5 \frac{W}{m^3} \quad (4)$$

Power Balance in Fusion Plasmas

(c) The necessary energy confinement time τ_E to achieve steady-state operation.

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Since

$$P_L = \frac{\frac{3}{2}nkT}{\tau_E}$$

we can write

$$P_L = P_{aux} + P_\alpha - P_b$$

and then

$$\tau_E = \frac{\frac{3}{2}nkT}{P_{aux} + P_\alpha - P_b}$$

$$\tau_E = \frac{\frac{3}{2}nkT}{\frac{5}{2}P_\alpha + P_\alpha - P_b}$$

$$\tau_E = \frac{\frac{3}{2}nkT}{\frac{7}{2}P_\alpha - P_b}$$

Power Balance in Fusion Plasmas

Bremsstrahlung power:

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Effective atomic number:

$$\begin{aligned}
Z_{eff} &= \sum_j \frac{n_j^{ion} \langle Z_j^{ion} \rangle^2}{n_e} \\
&= \frac{n_D Z_D^2 + n_T Z_T^2 + n_\alpha Z_\alpha^2 + n_I Z_I^2}{n_e} \\
&= \frac{n_D + n_T}{n_e} + \frac{4n_\alpha + Z_I^2 n_I}{n_e} \\
&= 2r_{DT} + 4r_\alpha + Z_I^2 r_I
\end{aligned}$$

By using (1),

$$\begin{aligned}
Z_{eff} &= (1 - 2r_\alpha - Z_I r_I) + 4r_\alpha + Z_I^2 r_I \\
&= 1 + 2r_\alpha + Z_I (Z_I - 1) r_I \qquad (5)
\end{aligned}$$

Therefore,

$$Z_{eff} = 1.7 \qquad (6)$$

Power Balance in Fusion Plasmas

Bremsstrahlung power (we use (5)):

$$P_b = C n_e^2 \sqrt{T} Z_{eff} = C n_e^2 \sqrt{T} \left[1 + 2 r_\alpha + Z_I \left(Z_I - 1\right) r_I\right] \tag{7}$$

Therefore,

$$P_b = 2.6879 \times 10^4 \frac{W}{m^3} \tag{8}$$

Power Balance in Fusion Plasmas

The ion density is given by

$$
\begin{aligned}
n_i &= \sum_j n_j^{ion} \\
&= n_D + n_T + n_\alpha + n_I \\
&= 2n_{DT} + r_\alpha n_e + r_I n_e \\
&= (1 - 2r_\alpha - Z_I r_I)n_e + r_\alpha n_e + r_I n_e \\
&= (1 - r_\alpha - (Z_I - 1)r_I)n_e \qquad (9)
\end{aligned}
$$

Therefore, the total density is given by

$$
\begin{aligned}
n = n_e + n_i &= n_e + (1 - r_\alpha - (Z_I - 1)r_I)n_e \\
&= (2 - r_\alpha - (Z_I - 1)r_I)n_e \qquad (10) \\
&= 1.75 n_e \\
&= 1.75 \times 10^{20}/m^3
\end{aligned}
$$

Power Balance in Fusion Plasmas

By using (2), (7), and (10), finally we can write

$$\begin{aligned}
\tau_E &= \frac{\frac{3}{2}nkT}{P_{aux} + P_\alpha - P_b} \\
&= \frac{\frac{3}{2}nkT}{\frac{5}{2}P_\alpha + P_\alpha - P_b} \\
&= \frac{\frac{3}{2}nkT}{\frac{7}{2}P_\alpha - P_b} \\
&= \frac{\frac{3}{2}((2 - r_\alpha - (Z_I - 1)r_I)n_e)kT}{\frac{7}{2}\frac{n_e^2}{4}\left(1 - 2r_\alpha - Z_I r_I\right)^2 \langle\sigma v\rangle_{DT} E_\alpha - C n_e^2 \sqrt{T}\left[1 + 2r_\alpha + Z_I\left(Z_I - 1\right) r_I\right]}
\end{aligned}$$

Therefore,

$$\tau_E = 3.2584s \qquad (11)$$