Nuclear Fusion and Radiation

Lecture 5 (Meetings 9, 10, 11 & 12)

Eugenio Schuster

schuster@lehigh.edu

Mechanical Engineering and Mechanics

Lehigh University

Power Balance in Fusion Plasmas

Now that we have established methods for calculating reaction rates in a plasma, we are in a position to consider energy/power balances in a fusioning plasma. Consider the following schematic representation of a fusioning plasma:

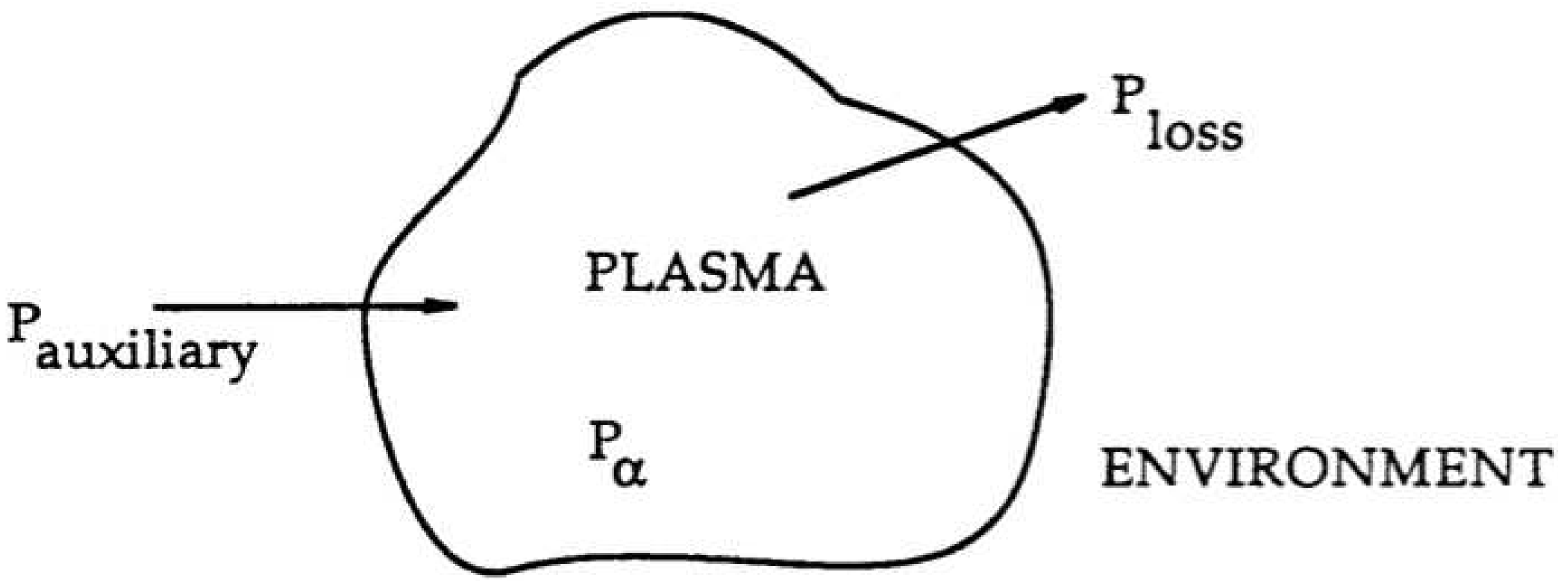

Figure 1: Power balance.

Power Balance in Fusion Plasmas

- P_{aux} denotes the external power input required to heat fuel to fusion conditions and to sustain temperature at fusion conditions.
- P_α denotes the power associated with DT produced α's assumed to be deposited within the plasma.
- P_{loss} denotes the power loss from the plasma.

What mechanisms might result in energy loss from the plasma?

Power Balance in Fusion Plasmas

- The temperature of the environment is much less than that of the plasma. Therefore, very steep gradients in temperature will exist between the plasma and its environment, driving heat out of the plasma (heat conduction).
- The density of ions and electrons in the environment is much less than in the plasma. Therefore, very steep gradients in density will exist between the plasma and its environment, driving particles out of the plasma (particle transport/diffusion). Note also that escaping particles carry energy out with them.
- Processes in the plasma can generate electromagnetic (EM) radiation and much of this radiation will not be absorbed in the plasma, but will escape from the plasma leading to radiative power loss from the plasma.

Power Balance in Fusion Plasmas

The plasma energy is given by

$$E_{content} = \int_{volume} \left(\frac{3}{2} n_i k T_i + \frac{3}{2} n_e k T_e \right) dV$$

$n_e/T_e, n_i/T_i$: electron, ion density/temperature.

The quasineutrality condition is written as

$$n_p = n_e$$

n_p, n_e : number (i.e., density) of protons, electrons.

Consider energy content of **purely hydrogenic** DT plasma:

- There are no ions present other than fuel ions (no impurities and no α particles)), i.e. $n_i = n_D + n_T$.
- All species have same temperature, i.e. $T_i = T_e \triangleq T$.
- D and T have optimal 50-50 mixture, i.e. $n_D = n_T$.

Power Balance in Fusion Plasmas

For the assumed **purely hydrogenic** DT plasma, we have

$$n_p = n_D Z_D + n_T Z_T = n_D + n_T = n_i$$

In this case, the quasineutrality conditions reduces to

$$n_i = n_e$$

The overall plasma density is always defined as

$$n = n_i + n_e$$

Therefore, for the assumed **purely hydrogenic** DT plasma,

$$n = n_i + n_e = 2n_e = 2n_i$$

Recalling that $T_i = T_e \triangleq T$, we can then rewrite

$$E_{content} = \int_{volume} 3n_e kT dV = \int_{volume} \frac{3}{2} nkT dV$$

Power Balance in Fusion Plasmas

In general, densities and temperatures will be functions of position and time. After integrating over the volume, we can write

$$E_{content} = \frac{3}{2}\bar{n}k\bar{T}V$$

where $\bar{n}$ and $\bar{T}$ represent the appropriate volume average of n and T. For convenience, we drop the upper-bar notation from now on. All variables are time-dependent and volume-averaged. With the appropriate volume averages, we can write the following power balance for our plasma:

$$\frac{d(\frac{3}{2}nkTV)}{dt} = P_{aux}V + P_\alpha V - P_L V$$

At <u>steady state</u>, $d()/dt = 0$, and

$$P_{aux}V + P_\alpha V = P_L V \iff P_{aux} + P_\alpha = P_L$$

Power Balance in Fusion Plasmas

For macroscopic calculations, the term P_L is generally defined in terms of the energy content of the plasma and a global/ overall energy confinement time, τ_E. Thus,

$$P_L \triangleq \frac{\frac{3}{2}nkT}{\tau_E}$$

The above equation says nothing about the physics or mechanisms for energy loss, it simply defines a global energy confinement time

$$\tau_E \triangleq \frac{\frac{3}{2}nkT}{P_L}$$

For the **purely hydrogenic** DT plasma ($n = 2n_e$),

$$P_L = \frac{3n_e kT}{\tau_E}, \quad \tau_E = \frac{3n_e kT}{P_L}$$

Power Balance in Fusion Plasmas

The α-power density, P_α, can be written as

$$P_\alpha = n_D n_T < \sigma V > E_\alpha$$

where it is understood that the densities and the reaction rate parameter are volume averages. $E_\alpha = 3.5$ MeV denotes the fraction of energy released by the DT fusion reactor that is carried by the α particle.

For the **purely hydrogenic** DT plasma ($n_D = n_T = n_e/2$),

$$P_\alpha = \frac{n_e^2}{4} < \sigma V > E_\alpha$$

Power Balance in Fusion Plasmas

We can also define a plasma condition in which the plasma is energy/ power self sufficient, a situation in which the losses, P_L, are exactly balanced by the α-power, P_α. Thus,

$$P_L = P_\alpha \iff P_{aux} = 0$$

This condition is called “ignition”. In terms of plasma parameters the ignition condition can be written as

$$n_D n_T < \sigma V > E_\alpha = \frac{\frac{3}{2} nkT}{\tau_E}$$

Power Balance in Fusion Plasmas

For the **purely hydrogenic** DT plasma ($n_e = n_i = n_D + n_T$, $n = n_i + ne = 2n_e = 2n_i$, $n_D = n_T = \frac{n_i}{2} = \frac{n_e}{2}$),

$$\frac{n_e^2}{4} < \sigma V > E_\alpha = \frac{3n_e kT}{\tau_E}$$

where the volume average symbol has been dropped for convenience. Rearranging the above yields

$$n_e \tau_E|_{IGN}^{DT} = \frac{12kT}{< \sigma V >_{DT} E_\alpha}$$

Since

$$< \sigma V >_{DT} = f(kT)$$

the ignition parameter is a function of kT ($n_e \tau_E|_{IGN}^{DT} = g(kT)$).

Power Balance in Fusion Plasmas

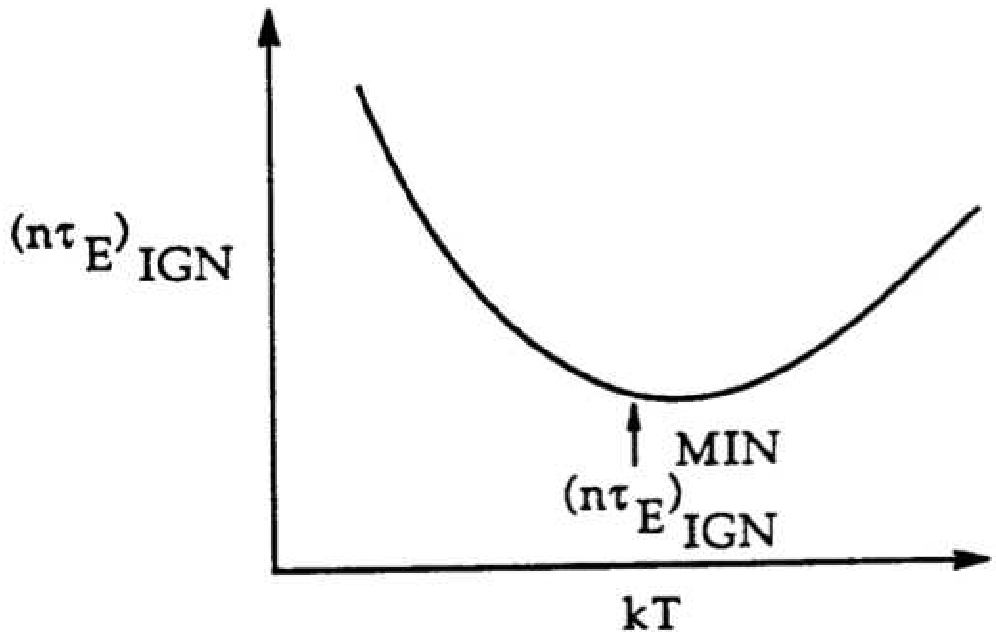

Figure 2: Ignition parameter. In this figure n denotes indeed the electron density n_e.

Note that $n_e\tau_E|_{IGN}$ as defined here is a requirement on the product $n_e\tau_E$. As we shall see later the value of $n_e\tau_E$ actually achieved for a given plasma must be compared with the required $n_e\tau_E$ to judge if the plasma can ignite.

Energy Gain in a Fusion Reaction

We now turn to fusion power/ energy gain in a fusioning plasma. In discussing energy gain in our beam-target reactor system, we defined the gain in terms of the ratio of fusion energy out to injected energy in the beam. For our fusioning plasma we have a similar definition:

$$\text{Power Gain} \equiv \frac{P_f}{P_{aux}} \equiv Q, \quad P_f = n_D n_T < \sigma V > E_f$$

where $E_f = 17.6$ MeV, which makes $P_f = 5P_\alpha$.

Historically the symbol Q has been used for the power gain (Note that this symbol is also used to denote energy release in fusion reactions!!!). For our steady-state plasma,

$$P_{aux} = P_L - P_\alpha \iff Q = \frac{P_f}{P_L - P_\alpha}$$

Energy Gain in a Fusion Reaction

Note that at ignition, $Q \rightarrow \infty$ ($P_L = P_\alpha$). We can divide numerator and denominator oft the above equation by P_α, yielding

$$Q = \frac{P_f/P_\alpha}{P_L/P_\alpha - 1}$$

What is the value of Pf/P_α for the DT reaction?

$$\frac{P_f}{P_\alpha} = \frac{17.6}{3.5} \sim 5$$

What is the value of P_L/P_α for the DT reaction in the case of a **purely hydrogenic** plasma?

$$\frac{P_L}{P_\alpha} = \frac{\frac{3n_e kT}{\tau_E}}{\frac{n_e^2}{4} < \sigma V >_{DT} E_\alpha} = \frac{12kT}{< \sigma V >_{DT} E_\alpha} \frac{1}{n_e \tau_E}$$

Energy Gain in a Fusion Reaction

But

$$\frac{12kT}{< \sigma V >_{DT}\ E_\alpha} \equiv n_e \tau_E|_{IGN}^{DT} = g(kT)$$

Thus, at a given kT value

$$\frac{P_L}{P_\alpha} = \frac{n_e \tau_E|_{IGN}^{DT}}{n_e \tau_E|_{ACTUAL}^{DT}}$$

Then,

$$Q_{DT} = \frac{5}{\frac{n_e \tau_E|_{IGN}^{DT}}{n_e \tau_E|_{ACT}^{DT}} - 1}$$

Various goals of the fusion program are discussed in terms of values of Q.

Energy Gain in a Fusion Reaction

The goal of "breakeven" defined as $Q = 1$, for which

$$Q_{DT} = \frac{5}{\frac{n_e\tau_E|_{IGN}^{DT}}{n_e\tau_E|_{ACT}^{DT}} - 1} = 1,$$

yields

$$\frac{n_e\tau_E|_{IGN}^{DT}}{n_e\tau_E|_{ACT(Q=1)}^{DT}} = 6 \Rightarrow n_e\tau_E|_{ACT(Q=1)}^{DT} = \frac{1}{6} n_e\tau_E|_{IGN}^{DT}$$

For $Q_{DT} = 1$, the achieved $n_e\tau$ must be $\sim 1/6$ of that required for ignition. At $kT = 10keV$,

$$n_e\tau_E|_{IGN}^{DT} \sim 3 \times 10^{20} \frac{s}{m^3} \Rightarrow n_e\tau_E|_{ACT(Q=1)}^{DT} \sim 5 \times 10^{19} \frac{s}{m^3}$$

Energy Gain in a Fusion Reaction

The goal of breakeven, achieving as much fusion power release as auxiliary power into the plasma, has been the major goal of the fusion program for over forty years. However, it should be emphasized that $Q = 1$ will not yield net electricity, $Q > 1$ is required for a useful reactor.

Another goal of the fusion program is to achieve plasma conditions such that the α-power deposited in the plasma is of the same order of magnitude as the auxiliary power to the plasma–this is an interesting physics regime since now the α-power plays a major role in heating the plasma. For this condition,

$$P_{aux} = P_\alpha.$$

Energy Gain in a Fusion Reaction

Therefore,

$$Q = \frac{P_f}{P_{aux}} = \frac{P_f}{P_\alpha} = 5 \Rightarrow Q_{DT} = \frac{5}{\frac{n_e\tau_E|_{IGN}^{DT}}{n_e\tau_E|_{ACT}^{DT}} - 1} = 5,$$

which implies

$$\frac{n_e\tau_E|_{IGN}^{DT}}{n_e\tau_E|_{ACT(Q=5)}^{DT}} = 2.$$

For $Q_{DT} = 5$, the achieved $n_e\tau$ must be $\sim 1/2$ of that required for ignition. At $kT = 10keV$, this results in the requirement

$$n_e\tau_E|_{ACT(Q=5)}^{DT} = \frac{1}{2} n_e\tau_E|_{IGN}^{DT} \sim 1.5 \times 10^{20}\frac{s}{m^3}$$

Particle Balances

Up to now we have been considering power balances. It is also important to consider particle balances in a fusioning plasma. Schematically,

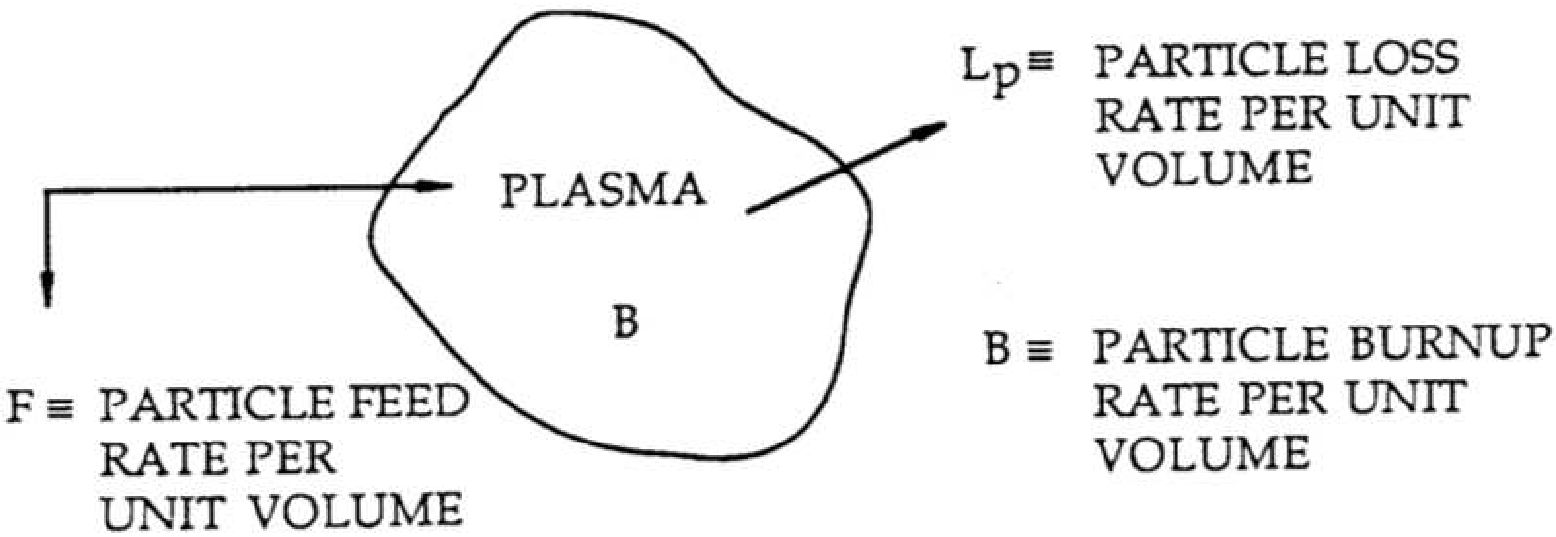

Figure 3: Particle balance.

Particle Balances

Thus, for the assumed **purely hydrogenic** DT plasma,

$$\frac{d(n_D + n_T)}{dt} = F - L_p - B$$

where $n_D + n_T = n_i$ represents the appropriate volume average of the fuel density. As with the P_L term in the power balance we define a global/macroscopic particle confinement time, τ_p, such that

$$L_p \equiv \frac{(n_D + n_T)}{\tau_p}$$

The burnup rate is given by

$$B \equiv \frac{n_e^2}{4} <\sigma V> \times 2 = \frac{n_e^2}{2} <\sigma V> = \frac{n_i^2}{2} <\sigma V> = \frac{(n_D + n_T)^2}{2} <\sigma V>$$

Again, average symbol has been dropped for convenience.

Particle Balances

At steady state, $d(n_D + n_T)/dt = 0$ and

$$F = L_p + B = \frac{(n_D + n_T)}{\tau_p} + \frac{(n_D + n_T)^2}{2} < \sigma V >$$

Defining $n_{DT} \triangleq n_D = n_T$, we can rewrite

$$F = \frac{(2n_{DT})}{\tau_p} + \frac{(2n_{DT})^2}{2} < \sigma V >$$

It is also useful to define a quantity called the fractional burnup, f_b, where

$$f_b \equiv \frac{B}{F} = \frac{\frac{(2n_{DT})^2}{2} < \sigma V >}{\frac{2n_{DT}}{\tau_p} + \frac{(2n_{DT})^2}{2} < \sigma V >} = \frac{n_{DT} < \sigma V >}{\frac{1}{\tau_p} + n_{DT} < \sigma V >}$$

Particle Balances

What is this quantity?

Remember that the mean life of a particle between fusion events is given by

$$\tau_{DT} = \frac{1}{n_{DT} < \sigma V >}$$

Thus,

$$f_b = \frac{\frac{1}{\tau_{DT}}}{\frac{1}{\tau_p} + \frac{1}{\tau_{DT}}} \frac{\tau_p}{\tau_p} = \frac{\frac{\tau_p}{\tau_{DT}}}{1 + \frac{\tau_p}{\tau_{DT}}}$$

For ignition at $10keV$ with standard plasma, $n_e = 10^{20}/m^3$:

$$n_e \tau_E|_{IGN}^{DT} \sim 3 \times 10^{20} \frac{s}{m^3}$$

Particle Balances

and

$$\tau_E \sim 3s$$

Assume $\tau_p = 2\tau_E \sim 6s$. From previous calculations $\tau_{DT} \sim 180s$. Thus,

$$f_b = \frac{\frac{6}{180}}{1 + \frac{6}{180}} \sim 0.032$$

About 3.2% of the feed is burned up and the rest appears as particle loss!!

Radiative Processes in Plasmas

We previously examined power/energy balances in plasmas on a macroscopic scale and we lumped all energy loss phenomena into a single term:

$$P_L \equiv \frac{\frac{3}{2}nkT}{\tau_E} = \frac{3n_e kT}{\tau_E}$$

We noted P_L might consist of at least three contributions:

- diffusion losses $\propto dn/dx$ (and whatever energy the "leaking" particle carries)
- conduction losses $\propto dT/dx$
- radiative losses

We will now turn to a discussion of the radiative losses.

Radiative Processes in Plasmas

- There are a number of processes by which EM radiation can be produced in, and eventually lost from, a plasma.
- For fusioning plasmas the most important process is in general Bremsstrahlung (braking radiation). Whenever a charge experiences an acceleration/ deceleration (i.e., change in velocity), it will radiate electromagnetic energy.
- In a plasma, coulomb scattering collisions will cause acceleration/ deceleration of charged particles and will therefore result in the emission of Bremsstrahlung radiation.

Radiative Processes in Plasmas

Consider a scattering collision between target ion Z_1 and projectile (ion or electron) Z_2. The acceleration, a_2, of Z_2 produced by the collision is:

$$a_2 = \frac{F_{12}}{M_2}$$

where

$$F_{12} = \text{ Force between } Z_1 \text{ and } Z_2 \propto Z_1 Z_2$$

Therefore

$$a_2 \propto \frac{Z_1 Z_2}{M_2}$$

It can be shown that the intensity of the emitted radiation is proportional to $\|a_2\|^2$ and thus the Bremsstrahlung radiation intensity is proportional to $(Z_1 Z_2 / M_2)^2$.

Radiative Processes in Plasmas

In a plasma, the primary source of Bremsstrahlung radiation is that resulting from electron acceleration / deflections (note the mass dependence of the intensity) and essentially all from electron-ion collisions. It can be shown that the Bremsstrahlung radiation power density, P_b, is given by:

$$P_b^Z = CT_e^{1/2} n_e \sum_{\text{ions } i} n_i < \bar{Z}_i >^2$$

where C is a constant and $< \bar{Z}_i >$ represents the average charge state of species i, which need not be fully ionized.

In discussing plasma radiation it is useful to define the <u>effective atomic number</u> of the plasma as

$$Z_{eff} = \frac{\sum_i n_i < \bar{Z}_i >^2}{n_e}$$

Radiative Processes in Plasmas

The Bremsstrahlung radiation power density is rewritten as

$$P_b^Z = C n_e^2 T_e^{1/2} Z_e f f$$

Note that for a **purely hydrogenic** DT plasma

$$< \bar{Z}_i >= 1 \text{ and } \sum_i n_i < \bar{Z}_i >^2 = n_D + n_T$$

and by charge neutrality

$$n_e = n_D + n_T = n_i.$$

Then, $Z_{eff} \equiv 1$, and

$$P_b = C T_e^{1/2} n_e^2$$

Radiative Processes in Plasmas

$$P_b^Z = \underbrace{C n_e^2 T_e^{1/2}}_{\text{Hydrogenic Term}} \quad Z_{eff}$$

where $[T_e] = keV$. The Bremsstrahlung constant is given by

$$C = 5 \times 10^{-37} \frac{W m^3}{\sqrt{keV}} = 5 \times 10^{-43} \frac{MW m^3}{\sqrt{keV}}$$

Consider the standard purely hydrogenic plasma, $n_e = n_i = 10^{20}/m^3$, $T_e = T_i = 10 keV$, where $Z_{eff} = 1$. Then,

$$P_b^{Z=1} = 5 \times 10^{-43} (10^{20})^2 (10)^{1/2} \sim 0.016 MW/m^3$$

In addition, recall that for the standard plasma $P_\alpha \sim 0.15 MW/m^3$. Thus, for this case $P_b \sim P_\alpha/10$.

Power Balance with P_b

The power balance equation in Slide 7 needs to be modified to include the effect of the radiation loss. In this case, we can write the following power balance for our plasma:

$$\frac{d(\frac{3}{2}nkTV)}{dt} = P_{aux}V + P_{\alpha}V - P_{L}V - P_{b}V$$

At steady state, $d()/dt = 0$, and

$$P_{aux}V + P_{\alpha}V = P_{L}V + P_{b}V \iff P_{aux} + P_{\alpha} = P_{L} + P_{b}$$

Power Balance with P_b

We will now examine the power balance equation for a purely hydrogenic plasma ($Z_{eff} = 1$) with P_b included explicitly for the ignition case ($P_{aux} = 0$). For this case,

$$\begin{aligned} P_\alpha &= P_L + P_b \\ \frac{n_e^2}{4} < \sigma V >_{DT} E_\alpha &= \frac{3 n_e k T}{\tau_E} + C T_e^{1/2} n_e^2 Z_{eff} \end{aligned}$$

This yields,

$$n_e \tau_E |_{IGN, P_b}^{DT} = \frac{12 kT}{< \sigma V >_{DT} E_\alpha - 4 C T^{1/2}}$$

Comparing the above with the result in the absence of radiation loss,

$$n_e \tau_E |_{IGN}^{DT} = \frac{12 kT}{< \sigma V >_{DT} E_\alpha}$$

Power Balance with P_b

we note that $n_e\tau_E|_{IGN,P_b}^{DT} > n_e\tau_E|_{IGN}^{DT}$. Consider the ratio

$$\frac{n_e\tau_E|_{IGN,Pb}^{DT}}{n_e\tau_E|_{IGN}^{DT}} = \frac{<\sigma V>_{DT} E_\alpha}{<\sigma V>_{DT} E_\alpha - 4CT^{1/2}}$$

Multiply numerator and denominator by $n_e^2/4$

$$\frac{n_e\tau_E|_{IGN,Pb}^{DT}}{n_e\tau_E|_{IGN}^{DT}} = \frac{\frac{n_e^2}{4} <\sigma V>_{DT} E_\alpha}{\frac{n_e^2}{4} <\sigma V>_{DT} E_\alpha - Cn_e^2T^{1/2}}$$

For the standard case this yields

$$\frac{n_e\tau_E|_{IGN,Pb}^{DT}}{n_e\tau_E|_{IGN}^{DT}} = \frac{0.15}{0.15 - 0.016} = 1.12$$

The $n_e\tau_E|_{IGN}$ required increases by 12% compared to the case where Bremsstraahlung radiation loss is neglected!

Power Balance with P_b

We will now examine the power balance equation for a non-purely hydrogenic (plasma with impurities ($Z_{eff} \neq 1$)) withP_b included explicitly for the ignition case ($P_{aux} = 0$). We consider the case with iron impurity (which could be introduced into the plasma from surrounding walls). Assume $n_{Fe}/n_e = 0.01(1\%)$ and $< \bar{Z}_{Fe} >= 14$. Thus, iron ($Z = 26$) is not fully ionized. We assume that this value of the charge state is obtained from spectroscopic measurements. Thus,

$$Z_{eff} = \frac{\sum_i n_i < \bar{Z}_i >^2}{n_e} = \frac{n_D + n_T + n_{Fe} < \bar{Z}_{Fe} >^2}{n_e}$$

$$= \frac{n_D + n_T}{n_e} + \frac{n_{Fe} < \bar{Z}_{Fe} >^2}{n_e}$$

Power Balance with P_b

We are given the second term in Z_{eff}, how do we obtain $(n_D + n_T)/n_e$?. We apply charge neutrality!! Charge neutrality requires that the number of electrons per unit volume equals the number of positive charges per unit volume. Thus,

$$
\begin{aligned}
n_e &= n_p \\
n_e &= n_D Z_D + n_T Z_T + n_{Fe} < \bar{Z}_{Fe} > \qquad (Z_D = Z_T = 1)
\end{aligned}
$$

Dividing both sides of this equation by n_e yields

$$1 = \frac{n_D + n_T}{n_e} + \frac{n_{Fe} < \bar{Z}_{Fe} >}{n_e}$$

and

$$\frac{n_D + n_T}{n_e} = 1 - \frac{n_{Fe}}{n_e} < \bar{Z}_{Fe} >$$

Power Balance with P_b

Thus,

$$Z_{eff} = 1 - \frac{n_{Fe}}{n_e} < \bar{Z}_{Fe} > + \frac{n_{Fe}}{n_e} < \bar{Z}_{Fe} >^2$$

For our case

$$Z_{eff} = 1 - (0.01)(14) + (0.01)(14)^2 = 2.82$$

With the standard plasma

$$P_b^{Fe} = P_b^{Z=1} 2.82 = (0.016)(2.82) = 0.045 MW$$

Now that we have already analyzed the impact of the iron impurities on the radiation losses, it is also of interest to consider the impact of the iron impurities on both the fuel density and the plasma power density.

Power Balance with P_b

We still assume $n_D = n_T$. The neutrality condition

$$\frac{n_D + n_T}{n_e} = 1 - \frac{n_{Fe}}{n_e} < \bar{Z}_{Fe} >$$

yields

$$n_D^{Fe} = \frac{1}{2}\left[1 - \frac{n_{Fe}}{n_e} < \bar{Z}_{Fe} >\right] n_e \iff n_D^{Fe} = 0.43 n_e$$

In the case with no impurities (**purely hydrogenic** plasma), the neutrality condition $n_e = n_D + n_T$ yields $n_D^{Z=1} = 0.5 n_e$. Then,

$$\frac{P_\alpha^{Fe}}{P_\alpha^{Z=1}} = \frac{(0.43 n_e)^2}{(0.5 n_e)^2} \sim 0.74 \Rightarrow P_\alpha^{Fe} = 0.74 \times 0.15 \sim 0.11 MW/m^3$$

Power Balance with P_b

In this case, the power balance equation is written as

$$\begin{aligned} P_\alpha &= P_L + P_b \\ n_D n_T < \sigma V >_{DT} E_\alpha &= \frac{\frac{3}{2} nkT}{\tau_E} + C T_e^{1/2} n_e^2 Z_{eff} \\ (0.43 n_e)^2 < \sigma V >_{DT} E_\alpha &= \frac{\frac{3}{2}(1.87 n_e) kT}{\tau_E} + 2.82 C T_e^{1/2} n_e^2 \end{aligned}$$

where we have used

$$\begin{aligned} n = n_e + n_i &= n_e + n_D + n_T + n_{Fe} \\ &= n_e + 2 n_D^{Fe} + 0.01 n_e \\ &= n_e + 2(0.43) ne + 0.01 n_e \\ &= 1.87 n_e \end{aligned}$$

Power Balance with P_b

This yields,

$$n_e \tau_E|^{Fe}_{IGN,P_b} = \frac{\frac{3}{2}\frac{1.87}{0.43^2}kT}{< \sigma V >_{DT} E_\alpha - \frac{2.82}{0.43^2}CT^{1/2}}$$

Comparing the above with the result in the absence of impurities,

$$n_e \tau_E|^{DT}_{IGN,P_b} = \frac{12kT}{< \sigma V >_{DT} E_\alpha - 4CT^{1/2}}$$

we compute the ratio

$$\frac{n_e \tau_E|^{Fe}_{IGN,P_b}}{n_e \tau_E|^{DT}_{IGN,Pb}} = \frac{\frac{3}{2}\frac{1.87}{0.43^2}}{12} \frac{< \sigma V >_{DT} E_\alpha - 4CT^{1/2}}{< \sigma V >_{DT} E_\alpha - \frac{2.82}{0.43^2}CT^{1/2}}$$

Multiply numerator and denominator by $n_e^2/4$, we obtain

Power Balance with P_b

$$\frac{n_e\tau_E|^{Fe}_{IGN,P_b}}{n_e\tau_E|^{DT}_{IGN,Pb}} = \frac{\frac{3}{2}\frac{1.87}{0.43^2}}{12}\frac{\frac{n_e^2}{4} < \sigma V >_{DT} E_\alpha - n_e^2 C T^{1/2}}{\frac{n_e^2}{4} < \sigma V >_{DT} E_\alpha - \frac{2.82}{4\times 0.43^2} n_e^2 C T^{1/2}}$$

For the standard case this yields

$$\frac{n_e\tau_E|^{Fe}_{IGN,P_b}}{n_e\tau_E|^{DT}_{IGN,Pb}} = \frac{\frac{3}{2}\frac{1.87}{0.43^2}}{12}\frac{0.15 - 0.016}{0.15 - \frac{2.82}{4\times 0.43^2}0.016} \approx 1.9$$

The $n_e\tau_E|_{IGN}$ required increases by 90% compared to the case where there are no impurities!

Impurities (high Z materials) are deleterious to the plasma power balance in two ways:

1. Enhancement of P_b by Z_{eff} factor
2. Reduction of P_α by fuel depletion effect

This turns into an increase in the $n_e\tau_E|_{IGN}$ requirement!