Nuclear Fusion and Radiation

Lecture 4 (Meetings 7 & 8)

Eugenio Schuster

schuster@lehigh.edu
Mechanical Engineering and Mechanics
Lehigh University

Energy Loss in Collisions

- We examined Coulomb scattering collisions between two charged particles and developed an estimate for the Coulomb scattering cross section using simple physical arguments and a geometrical interpretation for the cross section. This approach resulted in the following estimate for σ_{CS}:

$$\sigma_{CS} \sim \frac{1.6 \times 10^4}{T^2(keV)} barns \tag{1}$$

- We noted that the above expression was a reasonable estimate when

$$T = \frac{1}{2} M_o V^2, \qquad M_o = \frac{M_1 M_2}{M_1 + M_2} \tag{2}$$

is the center of mass kinetic energy ($V \equiv$ relative velocity).

Energy Loss in Collisions

- We then examined the effects of beam slowing down in the target to estimate the impact of Coulomb scattering on the fusion energy gain parameter, G.
- By assuming that a series of s successive collisions would degrade the beam to the point where fusion reactions could be neglected, we obtained the following estimate for G including Coulomb scattering effects:

$$G = \frac{\sigma_f}{\left(\sigma_f + \frac{\sigma_{CS}}{s}\right)} \frac{Q}{E_o} \tag{3}$$

where $Q/E_o \equiv G_o$ is the energy gain in absence of Coulomb scattering.

- We considered the $D - T^{cold}$ scattering case where

$$E_o = 100keV, \quad Q = 17.6MeV, \quad s = 1 \tag{4}$$

We obtained $\sigma_{CS} = 4.4b$. Since $\sigma_f \approx 5b$, we obtained

$$G = \frac{5}{\left(5 + \frac{4.4}{1}\right)} \frac{17.6}{0.1} = 0.53G_o \approx 93 \tag{5}$$

This gain is still unrealistically high ... Let us estimate a more realistic s value.

Energy Loss in Collisions

- Using conservation of momentum/energy in a head-on collision, we obtain:

$$T_1 = \frac{(M_1 - M2)^2}{(M_1 + M_2)^2} T_o, \qquad T_2 = \frac{4 M_1 M2}{(M_1 + M_2)^2} T_o \tag{6}$$

  NOTE: You will need to prove these relationships in Homework 2.

- When $M_1 = M_2$ we obtain

$$T_2 = T_o \text{ and } T_1 = 0 \tag{7}$$

- When $M_1 = D$ and $M_2 = T$ we obtain

$$T_2 \approx \frac{24}{25} T_o \text{ and } T_1 \approx \frac{1}{25} T_o \tag{8}$$

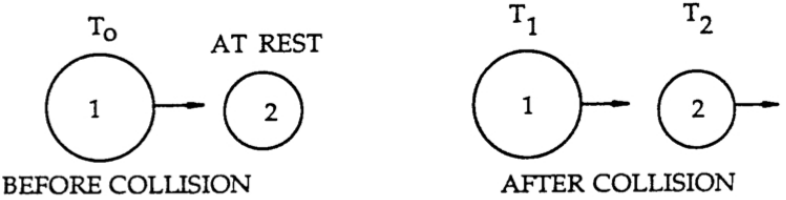

Figure 1: $D - T$ collision.

Energy Loss in Collisions

- Thus, in a head-on collision between D and T nearly all the incident energy is lost to the triton. In other words, for this case, the energy loss in a head-on collision, ϵ_l^{ho}, is

$$\epsilon_l^{ho} = 0.96T_o \tag{9}$$

- Of course not every collision is head-on. We will define an "average" energy loss per collision,

$$\epsilon_l^{av} = \frac{1}{2}\epsilon_l^{ho} \tag{10}$$

which for D on T^{cold} is

$$\epsilon_l^{av} = 0.48T_o \approx 0.5T_o \tag{11}$$

Energy Loss in Collisions

- Thus, if the deuteron energy is $100keV$ initially, after one collision, on average, its energy will be $50keV$. During a second collision the deuteron will, on average, again lose approximately one-half of its energy, resulting in an average energy of about $25keV$. Similarly, after three collisions, the average energy of the deuteron will have been degraded to about $12.5keV$.

- At $12.5keV$, $\sigma_f^{D-T} \sim 2 \times 10^{-3} b$ and we can reasonably neglect fusion contributions after about 3 collisions. Using $s = 3$, we obtain the following estimate for G:

$$G^{D-T^{cold}} \sim \frac{5}{\left(5 + \frac{4.4}{3}\right)} \frac{17.6}{0.1} \sim 136 \tag{12}$$

- Thus, with $s = 3$, the estimate of G is about 45% greater than with $s = 1$.

Deuteron Scattering on Cold Electrons

- As a result of our estimates, it appears that the inclusion of $D - T^{cold}$ Coulomb scattering does not degrade our energy gain to the point where our idealized fusion reactor becomes unattractive (i.e., too low gain).
- However, in addition to $D - T^{cold}$ scattering, we must consider $D - e^{cold}$ scattering.
- We first calculate σ_{CS}^{D-e} for collisions between beam deuterons and target electrons (assumed to be at rest, i.e., "COLD" electrons). For this case, $M_D/M_e \sim 2.01/0.00055 \sim 3655$. Thus,

$$M_o \sim \frac{M_D M_e}{M_D + M_e} \sim M_e \tag{13}$$

  In addition, $V \sim V_D$. Thus,

$$T = \frac{1}{2} M_e V_D^2 = \frac{M_e}{M_D} T_D \tag{14}$$

Deuteron Scattering on Cold Electrons

- Therefore, the Coulomb scattering cross section is given by

$$\sigma_{CS}^{D-e} \sim \frac{1.6 \times 10^4}{T_D^2} \left(\frac{M_D}{M_e} \right)^2 \tag{15}$$

- Assuming an incident beam energy of $100 keV$,

$$\sigma_{CS}^{D-e} \sim \frac{1.6 \times 10^4}{100^2} (3655)^2 \sim 2 \times 10^7 b \tag{16}$$

- The associated G value at $s = 1$ would be

$$G \sim \frac{5}{2 \times 10^7} \frac{17.6}{0.1} \sim 4.4 \times 10^{-5} \tag{17}$$

- This exceeding low G value would certainly render our reactor unattractive.

Deuteron Scattering on Cold Electrons

- Before we draw this conclusion, let us examine the validity of the assumption, $s = 1$, for the $D - e$ case. For $M_1 = D$ and $M_2 = e$, the energy loss in a head-on collision is

$$\epsilon_l^{ho} = \frac{4M_D M_e}{(M_D + M_e)^2} T_D \overset{M_D >> M_e}{\longrightarrow} \epsilon_l^{ho} \approx \frac{4M_e}{M_D} T_D \sim 10^{-3} T_D \qquad (18)$$

and

$$\epsilon_l^{av} \sim 5 \times 10^{-4} T_D \qquad (19)$$

- With such a small value of ϵ_l^{av} we can view the slowing down process of the deuterons on the electrons as being continuous. Thus, we can define a differential energy loss per collision, dT_D/ds, by

$$\frac{dT_D}{ds} = -\epsilon_l^{av} \sim -5 \times 10^{-4} T_D \Rightarrow \frac{dT_D}{T_d} = -5 \times 10^{-4} ds \qquad (20)$$

- Solving this equation yields

$$\ln(T_D/T_o) = -5 \times 10^{-4} s \qquad (21)$$

where T_o is the initial energy and s represents the number of collisions to reach the energy T_D.

Deuteron Scattering on Cold Electrons

- Rearranging gives

$$s = \frac{\ln(T_o/T_D)}{5 \times 10^{-4}} \tag{22}$$

- For $T_o = 100keV$ and $T_D = 10keV$ (this is the temperature at which we can assume that the probability for fusion is negligible), $T_o/T_D = 10$, and

$$s = \frac{2.3}{5 \times 10^{-4}} = 0.46 \times 10^4 \sim 4600 \text{ collisions} \tag{23}$$

- Using this value for s, we reevaluate G to be

$$G^{D-e^{cold}} = \frac{5}{5 + \frac{2\times 10^7}{4.6\times 10^3}} \frac{17.6}{0.1} \sim 0.2 \tag{24}$$

- This value of G is still too low for a reactor. Thus, we have shown that Coulomb scattering on the cold target electrons is an energy loss mechanism which prevents useful fusion energy gain. Would this situation be different if we could somehow create a population of energetic, "HOT", electrons?

Deuteron Scattering on Hot Electrons

- Again, we first consider the Coulomb scattering cross section and note that the reduced mass is still given by

$$M_o = \frac{M_D M_e}{M_D + M_e} \sim M_e \tag{25}$$

- However, is V still given by V_D? No! The assumption of a hot electron population implies that

$$T_e \sim T_D \iff \frac{1}{2} M_e V_e^2 \sim \frac{1}{2} M_D V_D^2 \tag{26}$$

Thus,

$$\left(\frac{V_e}{V_D}\right)^2 \sim \frac{M_D}{M_e} \sim 3655 \Rightarrow \frac{V_e}{V_D} = \left(\frac{M_D}{Me}\right)^{1/2} \sim 60 \tag{27}$$

- Thus, $V_e >> V_D$, and

$$V = V_e - V_D \sim V_e = \left(\frac{M_D}{Me}\right)^{1/2} V_D \tag{28}$$

Therefore,

$$T = \frac{1}{2} M_o V^2 = T_e = \frac{1}{2} M_e V_D^2 \left(\frac{M_D}{Me}\right) = T_D \tag{29}$$

Deuteron Scattering on Hot Electrons

- At a deuteron energy $T_D(= T_e) = 100keV$,

$$\sigma_{CS}^{D-e^h} \sim \frac{1.6 \times 10^4}{100^2} \sim 1.6b \tag{30}$$

Using $s = 1$, we reevaluate G to be

$$G^{D-e^h} = \frac{5}{5 + 1.6} \frac{17.6}{0.1} \sim 0.76G_o = 133 \tag{31}$$

- At a deuteron energy $T_D(= T_e) = 10keV$,

$$\sigma_{CS}^{D-e^h} \sim \frac{1.6 \times 10^4}{10^2} \sim 160b \tag{32}$$

Using $s = 1$, we reevaluate G to be

$$G^{D-e^h} = \frac{10^{-3}}{10^{-3} + 160} \frac{17.6}{0.01} \sim 6.25 \times 10^{-6}G_o = 0.001 \tag{33}$$

Effect of Coloumb Scattering

Scattering	Energy	s	Gain	Reference
None	100 keV		176	Lecture 3 - Slide 20
$D-T^{cold}$	100 keV	1	93	Lecture 3 - Slide 40
	100 keV	3	136	Lecture 4 - Slide 6
$D-e^{cold}$	100 keV	1	4×10^{-5}	Lecture 4 - Slide 8
	100 keV	4600	0.2	Lecture 4 - Slide 10
$D-e^{hot}$	100 keV	1	133	Lecture 4 - Slide 12
	10 keV	1	0.001	Lecture 4 - Slide 12

- Results depend greatly on T and s! Results will also depend greatly on the efficiency to heat and confine the gas.
- However, it seems that if we can heat electrons and ions, and keep them hot we should have the basis for a fusion power producer.

Hot Electron Population Consequences

- In order to obtain the "hot" electron population we desire for attractive fusion energy gain, we must heat our target. Heating causes collisions and collisions eventually cause ionization.
- In doing so, we will create a system consisting of ions and electrons, and some neutral atoms. This mixture of ions, electrons and neutrals is called a "plasma".
- In many ways the plasma behaves like ordinary gases/fluids except that it can carry electric currents and can generate magnetic and electric fields.
- For our purposes we will consider only "fully" ionized plasmas, i.e., plasmas in which the neutral content is negligible. We will neglect any effects of neutrals in our discussions.

Hot Electron Population Consequences

- The ions and electrons making up our plasma are NOT monoenergetic.
- At each point in space of the plasma, the particles exhibit a distribution of energies/ velocities (magnitude/ direction).
- An essential task of plasma physics is to determine the appropriate particle distributions since they will be required to calculate various reaction rates in the plasma.
- Consider a container of particles. We can view these particles as constituting a gas and having macroscopic characteristics such as pressure and temperature ($pV = NRT$) – measures of average properties of the system of particles. These properties may be constant with respect to time.

Hot Electron Population Consequences

- On a microscopic level there is much activity. The particles are continuously colliding with one another and the velocity (direction and magnitude) and energy of an individual particle are continuously changing.

- There is a distribution of velocities (energies) which characterizes the details of the particles state at any time.

- We would expect that most of the particles will have a velocity (energy) which is near the average value for this system,

$$\sum_{i}^{N} \frac{E_i}{N} \tag{34}$$

- However, some particles will acquire energies above the average value and some will acquire energies below the average value.

Hot Electron Population Consequences

- For example, a given particle may acquire an exceptionally high energy as a result of several consecutive favorable collisions.
- We will here define the probability, p, for one favorable collision as

$$p = \frac{1}{c} \tag{35}$$

  where c is a constant.

- Then the probability for n consecutive favorable collisions is:

$$p(n) = \left(\frac{1}{c}\right)^n \tag{36}$$

- Taking the logarithm of both sides

$$\ln(p) = n \ln\left(\frac{1}{c}\right) = -n \ln(c) \tag{37}$$

Hot Electron Population Consequences

- We now define the constant a by $a \equiv \ln(c)$. Thus,

$$\ln(p) = -an \tag{38}$$

- Solving for $p(n)$ yields

$$p(n) = e^{-an} \tag{39}$$

- We can associate the n consecutive favorable collisions with an energy, E, above the average energy, and write

$$p(E) = e^{-bE} \tag{40}$$

  where b is a constant.

- The above equation is the general form of the probability (or distribution) function for large numbers of particles based on statistics.

Hot Electron Population Consequences

- From gas kinetic theory it is derived that the distribution of particles in energy when equilibrium has been reached (distribution does not change with time) is given by the Maxwell-Boltzmann Distribution (called a "Maxwellian"):

$$n(E) = K \frac{E^{1/2}}{(kT)^{3/2}} e^{-\frac{E}{kT}} \tag{41}$$

  where $n(E)$ is the number of particles (per unit volume) per unit energy whose energy is in the range E and $E + dE$ and

  - K is a constant
  - E is the particle energy
  - T is the system temperature
  - k is the Boltzmann constant

Hot Electron Population Consequences

- The graphical representation of a “Maxwellian” distribution takes the form shown in Fig 2:

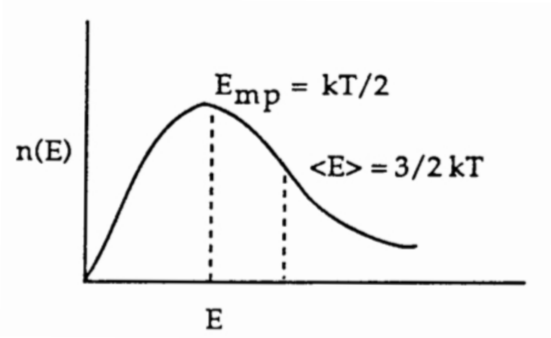

Figure 2: Boltzman distribution.

where E_{mp} denotes the most probable energy and $\langle E \rangle$ denotes the average energy.

Hot Electron Population Consequences

- Boltzmann's constant is obtained from the gas constant

$$pV = NRT \qquad R(\text{Gas constant}) = 8.31 J/mole - K, N : \# \text{ of moles} \quad (42)$$

$$\begin{aligned} k \equiv R/A_N &= \frac{8.31 J/mole - K}{6.02 \times 10^{23}(\#/mole)} \frac{1}{1.6 \times 10^{-16}(J/keV)} \\ &= 8.63 \times 10^{-8} \frac{keV}{K} \end{aligned} \quad (43)$$

Note that $p = \frac{NA_N}{V} kT = nkT$ (We will use this later!), where $n = \frac{NA_N}{V}$.

- We now define kinetic temperature, kT. If $kT = 1 keV$

$$T = \frac{1 keV}{8.63 \times 10^{-8} keV/K} \sim 1.16 \times 10^7 K \sim 10 \text{ million } K \quad (44)$$

Averages over Distributions

- Given the distribution function, it is useful to define certain average quantities.
- Consider the quantity f characterizing a particle. The average of f is defined as

$$< f > \equiv \frac{\int f(x)n(x)dx}{\int n(x)dx} \tag{45}$$

where $n(x)$ is the distribution function.

- For example for a M-B distribution, the average particle energy, $< E >$, is

$$< E > \equiv \frac{\int_0^\infty En(E)dE}{\int_0^\infty n(E)dE} = \frac{3}{2}kT \tag{46}$$

This corresponds to $1/2kT$ for each degree of freedom (x, y, z).

- Note that in a plasma at a kinetic temperature, kT, of $1keV$, the average particle energy is

$$< E > = \frac{3}{2}kT = 1.5keV \tag{47}$$

Averages over Distributions

- There is also a most probable energy, E_{mp}, defined as the energy value satisfying the condition, $dn(E)/dE = 0$. For a M-B distribution

$$E_{mp} = \frac{kT}{2} \tag{48}$$

- Schematically, $< E >$ and E_{mp} are shown in Fig 2.

Averages over Distributions

- We can also define a distribution in speed:

$$n(E)dE = n(V)dV \tag{49}$$

- Note that V is the speed, not velocity. Velocity is a vector, $\bar{V}$, and can assume negative values as well as positive values.

- Solving for the distribution in speed

$$n(V) = n(E)dE/dV \tag{50}$$

Averages over Distributions

- Noting that

$$\begin{aligned} E &= \frac{1}{2}mV^2 && (51) \\ dE &= mV\,dV && (52) \\ dE/dV &= mV && (53) \end{aligned}$$

- The condition

$$n(V)dV = n(E)dE \iff n(V) = n(E)\frac{dE}{dV}$$

allows us to write the M-B distribution in speed as

$$\begin{aligned} n(V) &= n(E)mV \\ &= K\frac{\left(\frac{1}{2}mV^2\right)^{1/2}}{(kT)^{3/2}}e^{-\frac{1}{2}\frac{mV^2}{kT}}mV \\ &= K\left(\frac{1}{2}\right)^{1/2}\frac{m^{3/2}V^2}{(kT)^{3/2}}e^{-\frac{1}{2}\frac{mV^2}{kT}} && (54) \end{aligned}$$

Averages over Distributions

- The average speed, $< V >$, for a M-B distribution is given by

$$< V > = \frac{\int_0^\infty V n(V) dV}{\int_0^\infty n(V) dV} = \left(\frac{8kT}{\pi m} \right)^{1/2} \tag{55}$$

- The most probable speed, V_{mp}, is defined as the speed value satisfying the condition, $dn(V)/dV = 0$. For a M-B distribution is given by

$$V_{mp} = \left(\frac{2kT}{m} \right)^{1/2} \tag{56}$$

- Also, the energy at the most probable speed, $E(V_{mp})$, is

$$E|_{V_{mp}} = \frac{1}{2} m \left[\left(\frac{2kT}{m} \right)^{1/2} \right]^2 = kT \tag{57}$$

NOTE: You will have to prove the expressions for $< E >$, E_{mp}, $< V >$, V_{mp} in Homework 2.

Plasmas for Fusion

- We have so far concluded that substantial energy gain from fusion would require a hot electron population.

- We further observed that the formation of a hot electron population would establish a gas-like mixture of hot electrons and hot ions. This gas-like mixture is called a "plasma" and has the interesting property that it can carry electric current.

- Exactly how "hot" the plasma must be to obtain acceptable energy gain is a question we will address later.

- We also noted that the particles which comprise the plasma are NOT monoenergetic but are characterized by distributions in energy/speed/velocity.

Plasmas for Fusion

- A knowledge of these distributions would be important in calculating reaction rates for various processes (fusion/ scattering) in the plasma.

- Using arguments based on statistics and scattering collisions, we established that the energy distribution should have an exponential factor e^{-bE}.

- Then we introduced the concept of the Maxwell-Boltzmann (M-B) distribution which is derived from statistical mechanics and applies to systems of colliding particles in thermodynamic equilibrium.

- Assuming we can represent the distribution of ions and electrons in the plasma as function of their energy E_i and E_e by a M-B distribution, we have:

$$n(E_{i,e}) = K \frac{E_{i,e}^{1/2}}{(kT_{i,e})^{3/2}} e^{-\frac{E_{i,e}}{kT_{i,e}}} \tag{58}$$

Plasmas for Fusion

- Remember that $kT_{i,e}$ represents the kinetic temperature of the plasma species (ions or electrons) and that a kinetic temperature of $1keV$ is equivalent to $\sim 10^7 K$.

- Using the M-B distribution we defined the average particle energy and speed for the plasma:

$$< E_{i,e} > \equiv \frac{\int_0^\infty E_{i,e} n(E_{i,e}) dE_{i,e}}{\int_0^\infty n(E_{i,e}) dE_{i,e}} = \frac{3}{2} kT_{i,e} \tag{59}$$

$$< V_{i,e} > \equiv \frac{\int_0^\infty V_{i,e} n(V_{i,e}) dV_{i,e}}{\int_0^\infty n(V_{i,e}) dV_{i,e}} = \left(\frac{8kT_{i,e}}{\pi m_{i,e}} \right)^{1/2} \tag{60}$$

- NOTE: Most of the plasmas considered in this course will assume $E_i = E_e$.

Plasmas for Fusion

- The question naturally arises as to the validity of assuming that a M-B distribution can be applied to a plasma in which fusion is taking place.
- The M-B distribution strictly applies only to a system of particles in thermodynamic equilibrium, that is, a system in which there are NO strong sinks / sources of particles or energy.
- In fusion plasmas we generally have strong sources (fueling/ auxiliary heating/ heating by α particles) and sinks (losses of particles and energy from the plasma).
- Nevertheless, it is observed that many phenomena in plasmas are well represented by assuming a M-B distribution.
- When the M-B distribution is not an acceptable representation, we must go back to statistical mechanics/ kinetic theory and solve (as best we can) for the actual distribution.

Fusioning Plasmas and Reaction Rates

- Consider a plasma formed from a mixture of deuterium (D) and tritium (T) gas, which we have heated up to some temperature T. Assume that the D and T distributions are characterized by identical temperature.

- Also note that in this system we are not considering firing a beam into our plasma, i.e., we do not consider a D^+ beam on a T plasma. The reason for this is that once we have formed a plasma, it is much more energy efficient to obtain fusion from D-T reactions between plasma particles than from a D^+ beam on a T plasma.

- In our plasma we can talk about the relative speed of any given Deuteron to any given Triton and we can define this relative speed as

$$V = |\bar{V}| = |\bar{V}_D - \bar{V}_T| \tag{61}$$

Fusioning Plasmas and Reaction Rates

- And we can perform calculations in the center of mass system such that

$$E = \frac{1}{2} M_o V^2, \qquad M_o = \frac{M_1 M_2}{M_1 + M_2} \tag{62}$$

- We can define the reaction rate between D and T in our plasma at a relative speed V as

$$R_{DT}(V) = n_D n_T \sigma_{DT}(V) V \tag{63}$$

- We recall that the distribution in speed follows the following Maxwell-Boltzmann law:

$$f(V) \propto V^2 e^{-\frac{M_o V^2}{2kT_{D,T}}} \tag{64}$$

Fusioning Plasmas and Reaction Rates

- The total reaction rate over all relative speeds is

$$\begin{aligned} R_{DT} &= \frac{\int_0^\infty R_{DT}(V) f(V) dV}{\int_0^\infty f(V) dV} \\ &= \frac{\int_0^\infty n_D n_T f(V) \sigma_{DT}(V) V dV}{\int_0^\infty f(V) dV} \\ &= \frac{n_D n_T \int_0^\infty f(V) \sigma_{DT}(V) V dV}{\int_0^\infty f(V) dV} \end{aligned} \tag{65}$$

- We make the following definition

$$< \sigma V > \equiv \frac{\int_0^\infty f(V) \sigma(V) V dV}{\int_0^\infty f(V) dV} \tag{66}$$

- Then,

$$R_{DT} = n_D n_T < \sigma V >_{DT} \tag{67}$$

or in general,

$$R_{1,2} = n_1 n_2 < \sigma V >_{1,2} \tag{68}$$

- Note that $< \sigma V >_{1,2}$ will depend on the species 1 and 2 and on the kinetic temperature, kT, characterizing their M-B distributions. The dimensions of $< \sigma V >$ are $length^3/time$ (m^3/s or cm^3/s).

- The Maxwellian reaction rate parameter $< \sigma V >$ (averaged over a M-B distribution) has been evaluated for many fusion reactions and over a wide range of temperatures. Table 3 and Figure 4 give results of such evaluations.

Fusioning Plasmas and Reaction Rates

T,keV	T(d,n)^{4}He	D(d,n)^{3}He	D(d,p)T	^{3}He(d,p)^{4}He	T(t,2n)^{4}He
1	.548 E-26	.692 E-28	.830 E-28	.302 E-31	.328 E-27
1.5	.589 E-25	.647 E-27	.729 E-27	.132 E-29	.218 E-26
2	.263 E-24	.260 E-26	.282 E-26	.142 E-28	.709 E-26
3	.171 E-23	.145 E-25	.150 E-25	.275 E-27	.303 E-25
4	.558 E-23	.423 E-25	.424 E-25	.177 E-26	.746 E-25
5	.129 E-22	.894 E-25	.877 E-25	.666 E-26	.140 E-24
6	.242 E-22	.157 E-24	.152 E-24	.183 E-25	.226 E-24
7	.398 E-22	.246 E-24	.235 E-24	.409 E-25	.329 E-24
8	.594 E-22	.355 E-24	.335 E-24	.796 E-25	.447 E-24
9	.826 E-22	.482 E-24	.451 E-24	.140 E-24	.579 E-24
10	.109 E-21	.626 E-24	.582 E-24	.227 E-24	.722 E-24
15	.265 E-21	.156 E-23	.141 E-23	.127 E-23	.156 E-23
20	.424 E-21	.273 E-23	.243 E-23	.379 E-23	.251 E-23
25	.559 E-21	.403 E-23	.357 E-23	.818 E-23	.351 E-23
30	.665 E-21	.541 E-23	.476 E-23	.145 E-22	.454 E-23
35	.745 E-21	.683 E-23	.598 E-23	.227 E-22	.557 E-23
40	.803 E-21	.826 E-23	.721 E-23	.323 E-22	.660 E-23
45	.843 E-21	.969 E-23	.844 E-23	.430 E-22	.763 E-23
50	.871 E-21	.111 E-22	.966 E-23	.544 E-22	.865 E-23
60	.897 E-21	.139 E-22	.121 E-22	.782 E-22	.107 E-22
70	.900 E-21	.166 E-22	.144 E-22	.102 E-21	.128 E-22
80	.890 E-21	.193 E-22	.167 E-22	.124 E-21	.148 E-22
90	.871 E-21	.218 E-22	.190 E-22	.144 E-21	.169 E-22
100	.849 E-21	.243 E-22	.212 E-22	.161 E-21	.191 E-22
150	.728 E-21	.358 E-22	.317 E-22	.220 E-21	.305 E-22
200	.628 E-21	.462 E-22	.414 E-22	.244 E-21	.424 E-22
250	.552 E-21	.559 E-22	.503 E-22	.251 E-21	.536 E-22
300	.495 E-21	.650 E-22	.585 E-22	.250 E-21	.631 E-22
energy yield W(J)	2.818E-12	5.24E-13	6.46E-13	2.93E-12	1.81E-12

Figure 3: Maxwellian reaction rate parameters (m^3/s) (G.H. Miley, H.Towner, and N. Ivich, 1974).

Fusioning Plasmas and Reaction Rates

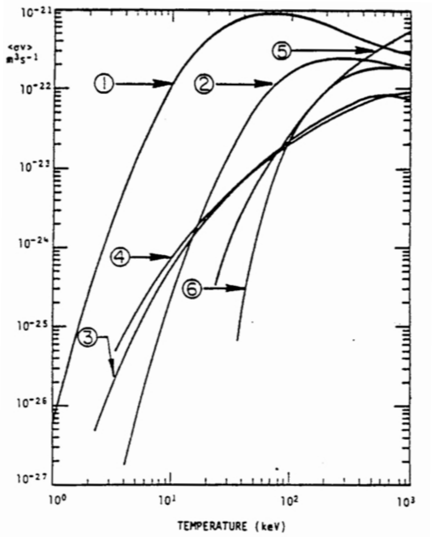

Figure 4: Maxwellian reaction rate parameters (m^3/s) (G.H. Miley, H.Towner, and N. Ivich, 1974).

Fusioning Plasmas and Reaction Rates

(1) $D + T \rightarrow n + {}^4He$

(2) $D + {}^3He \rightarrow H + {}^4He$

(3) $D + D \rightarrow H + T$

(4) $T + T \rightarrow {}^4He + 2n$

(5) $T + {}^3He \rightarrow$ (various products)

(6) $H + {}^{11}B \rightarrow 3({}^4He)$.

NOTE: The curve for $D + D \rightarrow {}^3He + n$ is about the same as curve (3)

Fusioning Plasmas and Reaction Rates

- It is useful to compare the reaction rate parameter $< \sigma V >$ in a DT plasma with the σV value for a D$^+$ beam on T (cold) target.
- From Table 3, $< \sigma V >_{DT}$ at $10keV$ is equal to $1.09 \times 10^{-22} m^3/s$.
- For the D$^+$ beam on T (cold) target case, σ_{DT} at $10keV$ is around $2 \times 10^{-3} b$.
- Next we calculate V for the beam case,

$$V = \left(\frac{2E}{m}\right)^{1/2} = \left[\frac{2 \times 10(keV) \times 1.6 \times 10^{-16}(J/keV)}{\underbrace{(3.34 \times 10^{-27}(kg)}_{2.0141amu \times 1.6605 \times 10^{-27}\frac{kg}{amu}}}\right]^{1/2} = 0.98 \times 10^6 \frac{m}{s} \quad (69)$$

Thus,

$$\sigma V_{DT}^{10keV} \approx 2 \times 10^{-3}(b) \times 10^{-28}(m^2/b) \times 0.98 \times 10^6 (m/s) \approx 2 \times 10^{-25} m^3/s \quad (70)$$

- This means that $< \sigma V >_{DT}^{10keV} / \sigma V_{DT}^{10keV} \sim 500!!!$

Fusioning Plasmas and Reaction Rates

- The results of the calculation can be illustrated as follows:

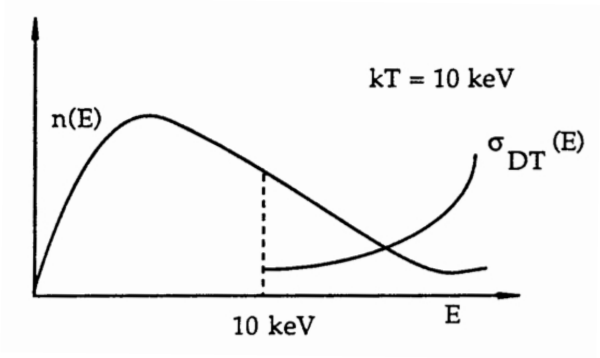

Figure 5: Effect of high energy particles on $<\sigma V>$.

- The high energetic particles within the distribution are indeed the ones producing most of the fusion reactions because the DT cross section σ_{DT} increases in this energy range and peaks at 100 keV.

Fusioning Plasmas and Reaction Rates

- Consider the reaction rate:

$$R_{1,2} = n_1 n_2 < \sigma V >_{1,2} \quad (71)$$

- If n_1 consists of particles 1, 2 and 3, and n_2 consists of particles 4, 5 and 6, then the number of different ways in which n_1 and n_2 can interact in binary collisions is $n_1 \times n_2$. Therefore, the product $n_1 n_2$ appears in $R_{1,2}$

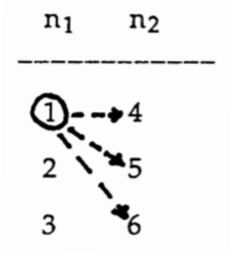

Figure 6: Possible binary collisions.

- However, if the reacting particles are all the same species, the situation is different. In this case, all six particles are of the same species and the number of different ways in which these particles can interact in binary collisions is given by the following table:

n_1	Interactions
1	5
2	4
3	3
4	2
5	1
[illegible]	15

Figure 7: Possible binary collisions.

Fusioning Plasmas and Reaction Rates

- The number of interactions is given by the series

$$\frac{n_1(n_1-1)}{2} \rightarrow \frac{6 \times 5}{2} = 15 \tag{72}$$

- When $n_1 >> 1$,

$$\frac{n_1(n_1-1)}{2} \rightarrow \frac{n_1^2}{2} \tag{73}$$

- Therefore,

$$R_{1,1} = \frac{n_1^2}{2} < \sigma V >_{1,1} \tag{74}$$

Additional Fusion Plasma Parameters

Power Density in Plasmas. The power density, $P_{1,2}$, of a given reaction is:

$$P_{1,2} = n_1 n_2 < \sigma V >_{1,2} Q_{1,2} \tag{75}$$

- In many reactions, energy appears both in the form of charged particles and in the form of neutrons. Most of the charged particle energy is deposited in the plasma. The plasma is transparent to the neutron (no charge).
- For a "standard" plasma ($n_D = n_T = 5 \times 10^{19} \#/m^3$, $kT_D = kT_T = 10keV$):

$$\begin{aligned} P_{DT}^{10keV} &= (5 \times 10^{19} \frac{1}{m^3})^2 (1.1 \times 10^{-22} \frac{m^3}{s})(17.6MeV)(1.6 \times 10^{-13} \frac{J}{MeV}) \\ &= 774 \times 10^3 \frac{J}{m^3 s} \\ &= 0.774 \frac{MW}{m^3} \end{aligned}$$

- This power density is divided between neutrons and alpha particles as follows:

$$\begin{aligned} P_n &\sim 0.774 \times \frac{14.1}{17.6} \sim 0.62 MW/m^3 \text{ (escapes plasma)} \\ P_\alpha &\sim 0.774 \times \frac{3.5}{17.6} \sim 0.15 MW/m^3 \text{ (stays in plasma to heat it)} \end{aligned}$$

Additional Fusion Plasma Parameters

Reaction Mean Life of Particle in Plasma. The reaction rate

$$R_{1,2} = n_1 n_2 < \sigma V >_{1,2} \tag{76}$$

gives the number of reactions per cubic meter per second.

- The number of reactions per sec per particle of type 1 is

$$R_{1,2}/n_1 = n_2 < \sigma V >_{1,2} \tag{77}$$

- The reaction mean life for species 1 is defined as

$$\tau_1 \equiv \frac{1}{n_2 < \sigma V >_{1,2}} = \text{average time between reactions} \tag{78}$$

- For the standard plasma, $n_D = n_T = n_{DT}$ and

$$\tau_D = \tau_T = \tau_{DT} = \frac{1}{n_{DT} < \sigma V >_{DT}} = \frac{1}{(5 \times 10^{19} \frac{1}{m^3})(1.1 \times 10^{-22} \frac{m^3}{s})} \sim 180s$$

Additional Fusion Plasma Parameters

Reaction Mean Free Path. The reaction mean free path, λ, is defined as

$$\lambda \equiv \text{average distance travelled by a particle between reactions} \tag{79}$$

- Using the definition of mean life, we can write

$$\lambda_{D(T)} = \tau_{DT} < V >_{D(T)} \tag{80}$$

- For intance,

$$< V >_D = \left(\frac{8kT}{\pi M_D} \right)^{1/2}$$

- For the standard plasma,

$$< V >_D \sim 1.1 \times 10^6 m/s \qquad \lambda_D \sim 2 \times 10^8 m$$

Additional Fusion Plasma Parameters

Plasma Pressure. Using the ideal gas law $PV = NRT$, we obtain

$$P = \frac{N}{V} RT = nkT \text{ (see slide 21)} \quad (81)$$

- For plasmas it is more convenient to talk about number of particles/unit volume, i.e., number density, and kinetic temperature (kT).
- Assume $n_i = n_D + n_T, n_D = n_T = n_i/2$. This implies that D and T are the only ions in the plasma. This is a purely hydrogenic plasma.
- Thus,

$$P_{DT} = n_D kT_D + n_T kT_T + n_e kT_e$$

- Charge neutrality in a plasma requires $n_e = n_p$ (#electrons = #protons). In a purely hydrogenic plasma, $n_p = n_i = n_D + n_T$. Therefore, the quasineutrality condition reduces to $n_e = n_i$. Thus,

$$P_{DT} = \frac{n_i}{2} kT_D + \frac{n_i}{2} kT_T + n_i kT_e$$

Additional Fusion Plasma Parameters

- Further assume $T_D = T_T = T_e$. Thus,

$$P_{DT} = 2n_i kT_i = 2n_e kT_e$$

- For the standard plasma,

$$P_{DT} = 0.32\frac{MJ}{m^3} \sim 3.1atm$$

- Note that for an ideal gas at $273K$, $1gm - mole = 22.4l$ at $1atm$. Then,

$$\frac{\#}{V} = \frac{6.023 \times 10^{23}}{22.4 \times 10^3} \sim 2.7 \times 10^{25}\frac{\#}{m^3} \qquad (82)$$

- The plasma has a density of $n = n_e + n_i = 2n_i = 2(n_D + n_T) \sim 2 \times 10^{20}\frac{\#}{m^3}$ but the temperature is $\sim 10^8 K$ at $10keV$!!!

Additional Fusion Plasma Parameters

Energy Density. The energy density, D_E, is given by

$$D_E = \frac{3}{2} n_D k T_D + \frac{3}{2} n_T k T_T + \frac{3}{2} n_e k T_e \tag{83}$$

- Again assuming $n_D = n_T = n_i/2$ $(n_D + n_T = n_i)$ and $T_D = T_T = T_e$, and invoking charge neutrality $(n_i = n_e)$,

$$D_E = 3 n_i k T_i = 3 n_e k T_e = \frac{3}{2} P = \frac{3}{2} 0.32 \frac{MJ}{m^3} \sim 0.5 \frac{MJ}{m^3}$$

- Note that although D_E and P have the same units they differ greatly in physical interpretation $(P = F/A)$. The total energy content of the plasma is simply D_E times the volume of the plasma.