# Nuclear Fusion and Radiation

## Lecture 3 (Meetings 5 & 6)

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# Fusion Reactions

- It was previously noted (lecture 1) that for the $D - T$ fusion reaction to take place a relative kinetic energy of $\sim 0.36MeV$ would be required to overcome the Coulomb barrier.
- However, experimental observations indicate that substantial fusion of $D - T$ takes place at energies well below the barrier height.
- Classical mechanics cannot explain these observations and we must turn to quantum mechanics to understand what is happening. Consider particles approaching a rectangular potential barrier:

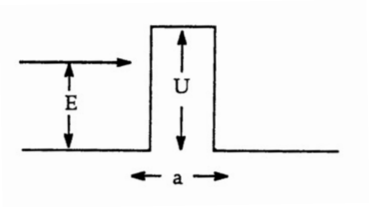



Figure 1: Potential barrier

# Fusion Reactions

- According to classical mechanics the incident particles would be totally reflected.

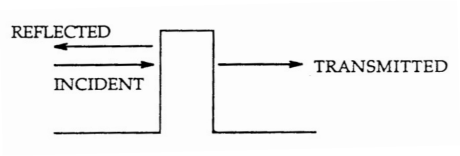



Figure 2: Potential barrier

- However, according to quantum mechanics there can be partial reflection and partial transmission of the incident beam. Using a graphical <u>wave</u> representation:

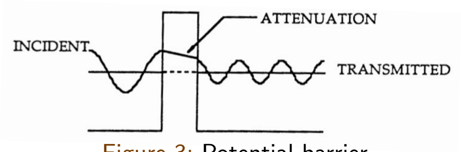



Figure 3: Potential barrier

# Fusion Reactions

- Calculation of the transmission probability is complicated and will not be considered here.

- It can be shown that the probability for transmission, $P_T$, has the dependencies,

$$P_T = f(E, U, a)$$

- Theory predicts significant transmission at energies well below barrier height.

- The probability for a given fusion reaction to take place depends on:
  - The probability for transmission (with formation of a compound nucleus)
  - The probability for the given exit channel to occur.

# Fusion Reactions

$$a + b \to d + e$$

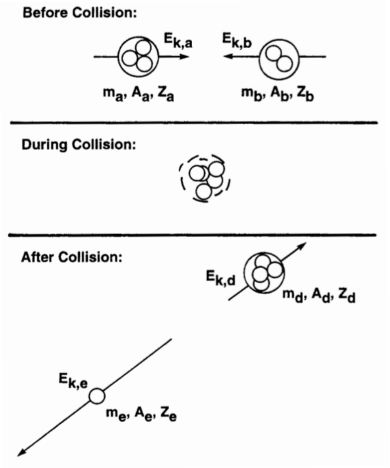



Figure 4: Head-on nuclear fusion reaction (HSMK)

# Fusion Reactions

For example,

$${}_{1}^{2}D +{}_{1}^{3} T \rightarrow ({}_{2}^{5}He)^{*}$$

exhibits two exit channels:

- Nuclear Elastic Scattering:

$$({}_{2}^{5}He)^{*} \rightarrow{}_{1}^{2} D +{}_{1}^{3} T$$

- Fusion:

$$({}_{2}^{5}He)^{*} \rightarrow{}_{2}^{4} He +{}_{0}^{1} n$$

*The combined probabilities for compound nucleus formation and de-excitation by a given exit channel represents the reaction probability*.

# Cross Sections

- Historically *reaction probabilities* have been designated as reaction "cross sections."

- We first give a functional definition for the cross section $\sigma$. Consider a test particle of species 1 (e.g., $D$) moving in a medium of target particles of species 2 (e.g., $T$). For this case we make the following definition:

$$\text{Reaction Probability per Test Particle per Unit Length Traveled in Medium} = \frac{P}{l} \triangleq \sigma n_2$$

  where $\sigma$ is the cross section for reaction, $n_2$ is the number density of the medium, $P$ is the probability per test particle, and $l$ is the traveled length.

- Note that $\sigma$ has dimensions of $l^2$ (e.g., $cm^2$, $m^2$) and $n_2$ has dimensions of $\#/l^3$ (e.g., $\#/cm^3$, $\#/m^3$), and $\sigma n_2$ has units of $(\#)/l$ or $l^{-1}$, that is, probability/unit length traveled by the test particle in the medium.

# Cross Sections

- We now give a geometrical interpretation for the cross section.

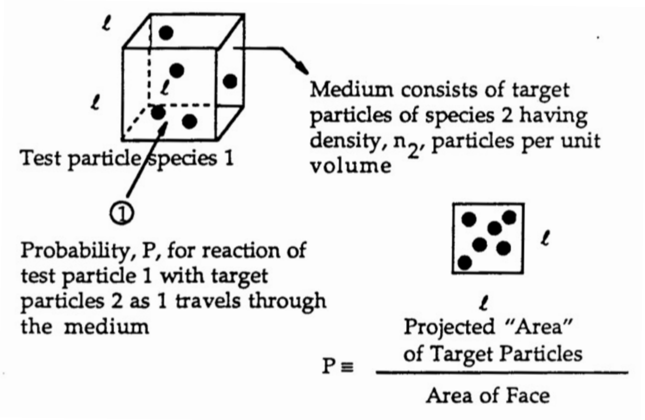



Figure 5: Cross section geometrical interpretation

# Cross Sections

- If each target particle has "cross sectional area," $\sigma$, then

$$P = \sigma \frac{n_2 l^3}{l^2} = \sigma n_2 l$$

- The probability per unit length traveled by the test particle in the medium is

$$\frac{P}{l} = \sigma n_2,$$

just as we had before!

- If we have $n_1$ test particles per unit volume of the medium (in addition to $n_2$ target particles per unit volume) and if the velocity of the test particles (with respect to the target particles) is $v$, then the total path length traveled by the $n_1$ particles per sec per unit volume is $n_1 v$ $[\frac{\#}{l^3} \times \frac{l}{t} = \frac{l}{l^3 t} = \frac{l}{Vt}]$.

# Cross Sections

- Thus, the reaction rate "density," the number of reactions between particles 1 and 2 per unit volume of medium per sec is

$$\text{Reaction Rate Density} = \frac{P}{Vt} = \frac{P}{l} \times \frac{l}{Vt} = (\sigma n_2)(n_1 v)$$

  - $\sigma n_2 =$ reaction probability per unit length traveled by one particle 1.
  - $n_1 v =$ total path length traveled by <u>all</u> particles 1 per unit volume per sec.

- In the CGS system the reaction rate density, $R$, has the following units:

$$\begin{aligned} R = (\sigma n_2)(n_1 v) &= \left((cm^2)\left(\frac{\#}{cm^3}\right)\right)\left(\left(\frac{\#}{cm^3}\right)\left(\frac{cm}{sec}\right)\right) \\ &= \left(\frac{\#}{cm}\right)\left(\frac{cm}{cm^3 sec}\right) \\ &= \frac{\#}{cm^3 sec} \end{aligned}$$

- Note that the quantity $n_1 v$ is usually known as the "flux" $\phi$ $[\frac{\#}{l^3} \times \frac{l}{t} = \frac{\#}{l^2 t}]$.

# Cross Sections

- The determination of $\sigma$ as a function of $E$ is the work of nuclear physicists. Experimental values of cross sections are needed. Theory only provides a guide.

- Sometimes we can get an order of magnitude estimate for a cross section by use of the geometric argument $\sigma \sim \pi R^2$ (area of projected circle).

- For example, the $D-T$ reaction has $R_{DT} \sim 4 \times 10^{-15} m = 4 \times 10^{-13} cm$. Therefore,

$$\sigma \sim \pi R^2 \approx 0.5 \times 10^{-24} cm^2 \tag{1}$$

- This, of course, is NOT an accurate cross section value but indicates the order of magnitude of nuclear cross sections, $10^{-24} cm^2$. For this reason this dimension has been given a name as follows

$$10^{-24} cm^2 \equiv 1 barn$$

# Cross Sections

- Consider first the fusion reaction

$$ {}^1_1H + {}^1_1H \rightarrow {}^2_1D + \beta^+ + \nu \qquad (Q = 1.44MeV) $$

The cross section for this reaction is too small to be observed in the laboratory. However, the cross section can be inferred from stellar processes and can be estimated based on theory ($\sim 10^{-9}b$).

- The two reactions

$$ \begin{array}{rcll} {}^1_1H + {}^2_1D & \rightarrow & {}^3_2He + \gamma & \qquad (Q = 5.5MeV) \\ {}^1_1H + {}^3_1T & \rightarrow & {}^4_2He + \gamma & \qquad (Q = 19.8MeV) \end{array} $$

have small, but observed cross sections ($\sim 10^{-6}b$). These cross sections, however, are too small to be of interest for fusion power.

# Cross Sections

- Next consider the reactions

$$
\begin{aligned}
{}^2_1D + {}^2_1D &\rightarrow {}^3_2He + {}^1_0n \qquad (Q = 3.3MeV) \\
{}^2_1D + {}^2_1D &\rightarrow {}^3_1T + {}^1_1H \qquad (Q = 4.03MeV) \\
{}^2_1D + {}^3_1T &\rightarrow {}^4_2He + {}^1_0n \qquad (Q = 17.6MeV) \\
{}^2_1D + {}^3_2He &\rightarrow {}^4_2He + {}^1_1H \qquad (Q = 18.3MeV)
\end{aligned}
$$

- The cross section versus deuteron energy for these reactions is given in Fig. 6. The following points are noted:
  - $\sigma_{DT}$ is by far the largest cross section
  - Most of the $\sigma_{DT}$ curve is significant well below $0.36MeV$ $(360keV)$
    - Remember that $0.36MeV$ $(360keV)$ is the approximate Coulomb barrier height
  - $\sigma_{DT}$ peaks at $\sim 110keV$, where $\sigma_{DT} \sim 5b$
    - Note that this is approximately an order of magnitude larger than the estimate based on $\pi R^2$ (See equation (1)).

# Cross Sections

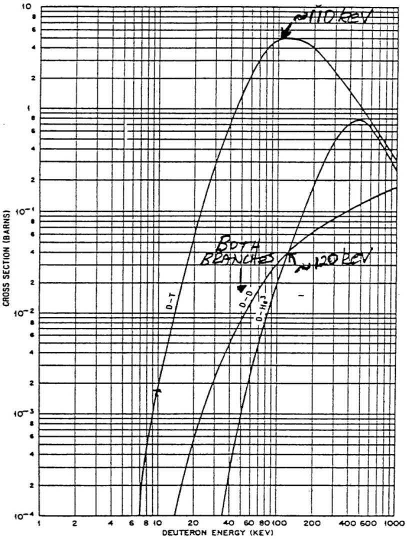



Figure 6: $D-D$, $D-T$ and $D-He^3$ cross sections.

# Fueling Strategy

- As we shall see later in the course, under the proper conditions some of the fusion reactions listed above have large enough cross sections to yield reaction rates which are interesting for fusion power production.
- We must first address a more fundamental question:

**"Are there sufficient reserves of fuels for a fusion energy economy?"**

- Let us examine this question for a $D-T$ based fusion power economy. What are the "fuel" requirements?
  - Deuterium is a stable isotope and can be recovered from water relatively easily
    - The Deuterium isotope has a 0.015 % abundance
  - Tritium, on the other hand, is radioactive decaying by beta decay to $^3He$, i.e.,

$$^3_1T \xrightarrow{\beta} {^3_2He}, T_{1/2} \sim 12.4 \text{ years}$$

Note: We will learn more about the $\beta$ radioactive decay later in the term.

# Fueling Strategy

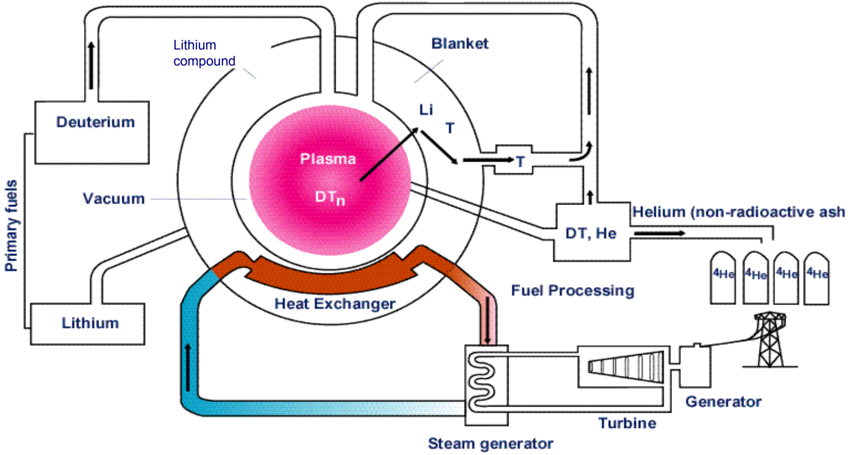
- Because of the short half-life (time taken for the population of a radioactive isotope to fall to half its original value) of $T$, which is just 12.4 years, there are not sufficient amounts of $T$ in nature to sustain a $D-T$ fusion power economy. Therefore, $T$ for $D-T$ fusion power must be manmade by the following strategy:

$$\begin{aligned} {}_1^2D + {}_1^3T &\rightarrow {}_2^4He + {}_0^1n \\ {}_0^1n + {}_3^6Li(7.5\%) &\rightarrow {}_2^4He + {}_1^3T \\ {}_0^1n + {}_3^7Li(92.5\%) &\rightarrow {}_0^1n + {}_2^4He + {}_1^3T \end{aligned}$$

- As we shall show later, we can design a fusion reactor so that it will breed the tritium it consumes by using the $D-T$ generated neutrons and lithium.
- Thus, the fuel requirement for $T$ becomes a resource requirement for lithium. Lithium and deuterium (water) resources are sufficient for $1000's$ of years of energy production via $D-T$ fusion reactors.
- A reactor based on the $D-D$ fusion reaction would require only water!

# Nuclear Fusion Power Plant Based on DT Fueling Strategy

- Neutron escapes to the walls.
- Energy of the neutron can be captured to create electricity.
- Neutron is also used to produce Tritium needed for fueling.
- Energetic **alpha particle** remains in the plasma $\rightarrow$ **'self-heating' source**.



# Other Energy Units

- In examining energy resources it is useful to introduce a quantity of energy called the terawatt–yr ($TW - YR$) defined as

$$1TW - YR \equiv 10^{12}(J/sec)(365)(3600)(24)(sec/yr) \sim 3.15 \times 10^{19} J$$

- Consider the following electricity consumption points:

$$\begin{aligned} 1975: &\quad \text{US consumed} \sim 2.6TW - YR \\ &\quad \text{WORLD consumed} \sim 8.5TW - YR \\ 2025: &\quad \text{US consumption} \sim 4.5TW - YR \\ &\quad \text{WORLD consumption} \sim 27TW - YR \end{aligned}$$

# Other Energy Units

- List of countries by electricity consumption

  https://en.wikipedia.org/wiki/List_of_countries_by_electricity_consumption

- List of countries by total primary energy consumption

  https://en.wikipedia.org/wiki/List_of_countries_by_total_primary_energy_consumption_and_production

- Energy in the United States

  https://en.wikipedia.org/wiki/Energy_in_the_United_States

  https://www.eia.gov/energyexplained/us-energy-facts/
  https://www.eia.gov/energyexplained/us-energy-facts/data-and-statistics.php
  https://www.eia.gov/energyexplained/us-energy-facts/images/consumption-by-source-and-sector.png

# Fusion Gain in Ideal Reactor

- So far everything looks promising for a $D - T$ fusion reactor, so let us try to construct an "idealized" fusion reactor (we will learn that this is rather different from a "practical" reactor but it will teach us important lessons).
- Consider the following system:

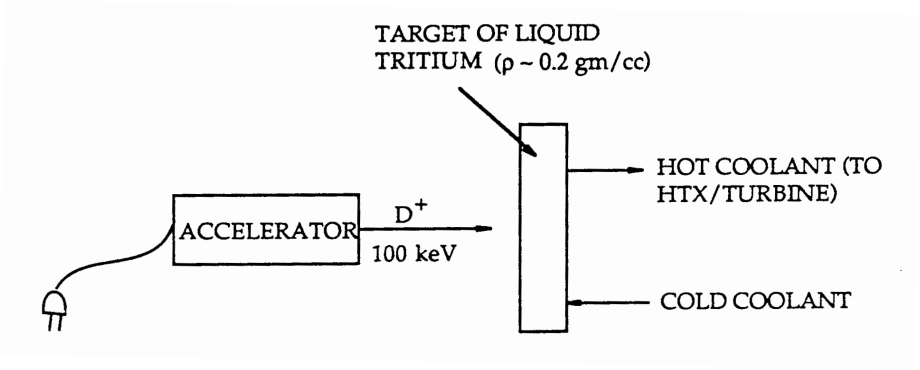



Figure 7: Idealized fusion reactor.

# Fusion Gain in Ideal Reactor

- The probability for a $D-T$ fusion reaction per cm of $D^+$ path length in the target, $P/l$, is given by

$$P/l = \sigma n_2 = \underbrace{(5\times 10^{-24} cm^2)}_{\sigma \text{ (from table)}} \underbrace{\frac{0.2 g/cm^3}{3 g/mol}(6.02\times 10^{23} \#/mol)}_{n_2 = n_T} = 0.2 \frac{\#}{cm}$$

- NOTE:
  - The mass of a proton is $1.00727647 amu \approx 1 amu$. The mass of a neutron is $1.008665 amu \approx 1 amu$. Since the electron mass is 1,836 times smaller than the proton mass, the atom mass is approximately equal to the nucleus mass.
  - The atom mass is approximately equal to $A \times amu = A \times 1.66053907 \times 10^{-27} kg$.
  - Since we have $N_A \triangleq 6.02214076 \times 10^{23} atoms/mol$, the mass of a mol is

$$\begin{aligned} M_{mol}(^AX) &\approx A \times 1.66053907 \times 10^{-27} kg/atom \times 6.02214076 \times 10^{23} atoms/mol \\ &\approx A \times 1.66053907 \times 10^{-24} g/atom \times 0.602214076 \times 10^{24} atoms/mol \\ &\approx A \times g/mol \end{aligned}$$

    Note that $1/1.66053907 \approx 0.602214076 \Rightarrow 1.66053907 \times 0.602214076 \approx 1!$
  - *The molar mass of any atom is approximately equal to its mass number $A$ (grams).*
  - Particle density $[\frac{\#Particles}{V}]$: $n = \underbrace{\rho/M_{mol}(^AX)}_{\#moles/V} \times N_A$, where $\rho =$ mass density.

# Fusion Gain in Ideal Reactor

- The probability for a $D-T$ fusion reaction per cm of $D^+$ path length in the target, $P/l$, is given by

$$P/l = \sigma n_2 = \underbrace{(5 \times 10^{-24} cm^2)}_{\sigma \text{ (from table)}} \underbrace{\frac{0.2 g/cm^3}{3 g/mol}(6.02 \times 10^{23} \#/mol)}_{n_2 = n_T} = 0.2 \frac{\#}{cm}$$

- On this basis we can easily make the target thick enough ($> 5$ cm) to achieve a reaction probability approaching unity. Let us define an energy gain, $G_o$, for our system:

$$G_o = \frac{\text{Fusion Energy/Incident D}}{\text{Energy of Incident D}} = \frac{17.6 MeV/\text{D-T Fusion}}{0.1 MeV/\text{Incident D}} = 176$$

THIS IS A VERY HIGH GAIN. THE "REACTOR" LOOKS VERY ATTRACTIVE. IS THERE ANYTHING WRONG WITH THIS ANALYSIS?

# Competing Processes

- We neglected the effect of "Coulomb scattering" (one possible *exit channel* (see slide 6)) in our calculation of $G_o$.

- Coulomb or Rutherford scattering (elastic scattering between charged particles) is the primary source of competition for fusion.

- In Coulomb scattering the incident $D+$ looses energy and because the fusion cross section decreases as the incident particle energy decreases, the probability for fusion decreases.

- If the Coulomb scattering cross section is large with respect to the fusion cross section, the $D+$ will be "down-scattered" in energy before it can engage in significant fusion reaction.

# Coulomb Scattering Cross Sections

- The detailed derivation of the Coulomb scattering cross section can be found in the textbook (Principles of Fusion Energy, Chapter 3). We shall use some simpler arguments here to estimate $\sigma_{CS}$.
- We would expect significant Coulomb scattering when two nuclei "feel" the presence of the Coulomb potential relative to their mutual kinetic energy. Consider the following schematic representation.

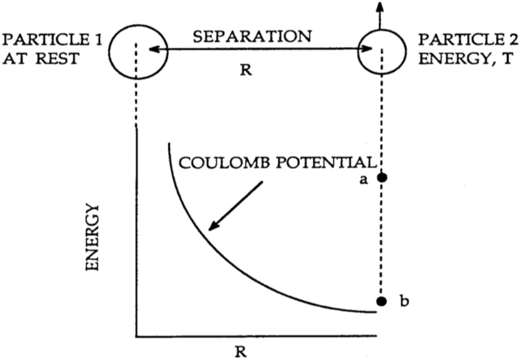



Figure 8: Coulomb potential.

# Coulomb Scattering Cross Sections

- If the kinetic energy of particle $2$ corresponds to point $a$, particle $2$ will not "feel" the Coulomb potential, that is, since $T_a >> E_c(R)$ the Coulomb potential is not strong enough to deflect (scatter) particle $2$ significantly.
- On the other hand, if the kinetic energy of particle $2$ corresponds to point $b$, $T_b \approx E_C(R)$ and particle $2$ will "feel" the Coulomb potential. In this case, particle $2$ will be deflected significantly.
- Based on these arguments we can define the effective "range," $R_{eff}$, of the Coulomb potential of particle $1$ with respect to particle $2$ by the following condition:

$$E_{1,2}(R_{eff}) = T_{1,2} \iff \frac{Z_1 Z_2}{4\pi\epsilon_o R_{eff}} = T_{1,2}$$

- Taking $Z_1 = Z_2 = 1.6 \times 10^{-19} C$ and $\epsilon_o = 8.85 \times 10^{-12} F/m$,

$$R_{eff} = \frac{Z_1 Z_2}{4\pi\epsilon_o T} = \frac{2.3 \times 10^{-28}}{T(J)} m = \frac{1.44 \times 10^{-12}}{T(keV)} m \quad (T = T_{1,2})$$

# Coulomb Scattering Cross Sections

- If we employ a geometric interpretation for $\sigma_{CS}$:

$$\sigma_{CS} \sim \pi \left( \frac{R_{eff}}{2} \right)^2 = \frac{\pi}{4} \frac{(1.44 \times 10^{-12})^2}{T^2(keV)} m^2 = \frac{1.6 \times 10^4}{T^2(keV)} barns$$

- The above expression gives a very good estimate for large angle (significant deflection) Coulomb scattering events when $T$ is the "center of mass energy" of the two particles:

$$T \triangleq \frac{1}{2} M_o V^2; \; M_o \triangleq \frac{M_1 M_2}{M_1 + M_2} \quad (V \equiv \text{relative velocity} \Rightarrow V^2 = (V_1 - V_2)^2)$$

- It must be emphasized that this expression is only an estimate.
- It does, however, give reasonable values for large angle scattering.
- Moreover, it gives the correct energy dependence, $\sigma_{CS} \sim 1/T^2$, for all Coulomb scattering events (large and small angle).

# Energetic Deuterons on “Cold” Targets

- Consider the following situation:

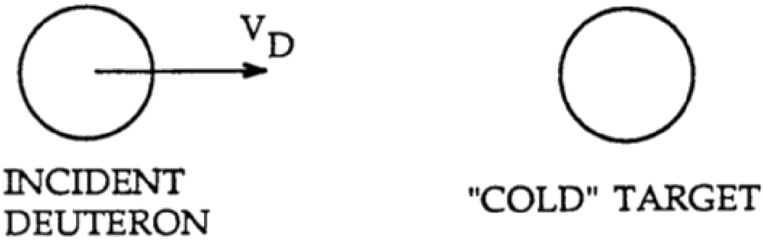



Figure 9: Energetic vs. cold deuteron/triton collision

# Energetic Deuterons on "Cold" Targets

- For the deuteron on deuteron collision case

$$M_o \triangleq \frac{M_D M_D}{M_D + M_D} = \frac{M_D^2}{2M_D} = \frac{M_D}{2}$$

$$T \triangleq \frac{1}{2} M_o V^2 = \frac{1}{2} \frac{M_D}{2} V_D^2 = \frac{1}{2} T_D \quad \text{(target at rest: } V = V_D\text{)}$$

Thus,

$$\sigma_{CS}^{D-D(\text{cold})} = \frac{1.6 \times 10^4}{(\frac{T_D}{2})^2} b = \frac{6.4 \times 10^4}{T_D^2} b$$

- For our idealized reactor, $T_D = 100keV$,

$$\sigma_{CS}^{D-D(\text{cold})} = \frac{6.4 \times 10^4}{100^2} b = 6.4b$$

Note that $\sigma_f^{D-D} \approx 0.03b$ at $100keV$. Therefore, $\sigma_{CS}/\sigma_f \approx 200$.

# Energetic Deuterons on "Cold" Targets

- For the deuteron on triton collision case

$$M_o = \frac{M_D M_T}{M_D + M_T} = \frac{M_D(3M_D/2)}{M_D + (3M_D/2)} = \frac{3M_D^2/2}{5M_D/2} = \frac{3M_D}{5}$$

$$T \equiv \frac{1}{2}M_o V^2 = \frac{1}{2}\frac{3M_D}{5}V_D^2 = \frac{3}{5}T_D \quad \text{(target at rest: } V = V_D\text{)}$$

Thus,

$$\sigma_{CS}^{D-T(\text{cold})} = \frac{1.6 \times 10^4}{(\frac{3T_D}{5})^2}b = \frac{4.4 \times 10^4}{T_D^2}b$$

- For our idealized reactor, $T_D = 100keV$,

$$\sigma_{CS}^{D-T(\text{cold})} = \frac{4.4 \times 10^4}{100^2}b = 4.4b$$

Note that $\sigma_f^{D-T} \approx 5b$ at $100keV$. Therefore, $\sigma_{CS}/\sigma_f \approx 1$.

# Energetic Deuterons on “Cold” Targets

- We will now estimate the energy gain of our idealized fusion reactor when $D-T$ Coulomb scattering is taken into account. Consider the following:

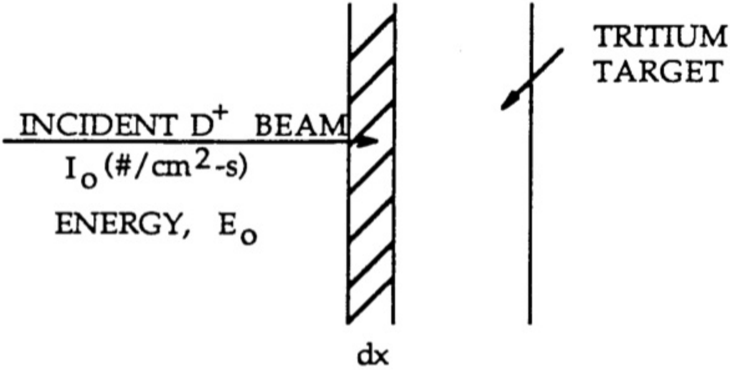



Figure 10: Incident $D$ beam on $T$ target

- As the beam penetrates into the target, beam particles can engage in two types of interactions with target particles:
  (1) $D - T$ fusion
  (2) $D - T$ Coulomb scattering

- Note that if fusion takes place, a deuteron is removed from incident beam.

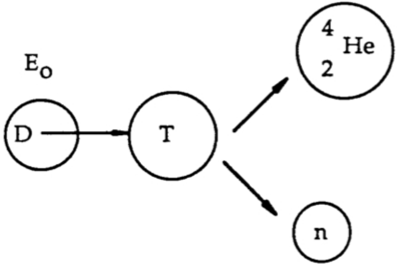



Figure 11: $D - T$ fusion reaction

# Energetic Deuterons on "Cold" Targets

- If scattering takes place, the scattered deuteron (with less energy!!!) is still available for fusion or additional scattering.

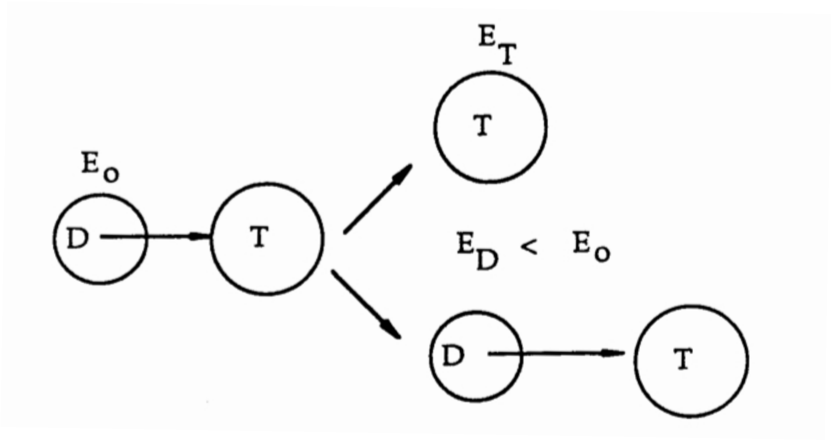



Figure 12: $D - T$ scattering

# Energetic Deuterons on "Cold" Targets

- After Coulomb scattering, a beam particle has been degraded in energy and:
  (1) the probability for fusion has decreased
  (2) the probability for Coulomb scattering has increased

From Table (Experiments) From Formula (Estimation): $\sigma_{CS} \propto \frac{1}{E^2}$

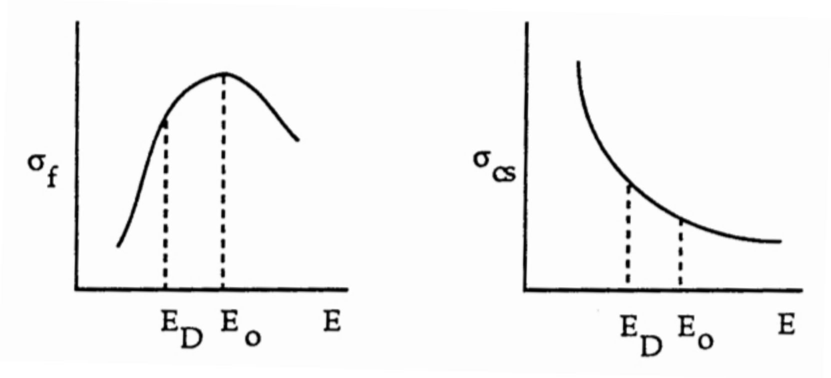



Figure 13: Fusion vs. scattering probabilities

# Energetic Deuterons on "Cold" Targets

- The following table gives specific values for the DT reaction:

| $E(keV)$ | $\sigma_f(b)$ | $\sigma_{CS}(b)$ | $\sigma_f/\sigma_{CS}$ |
|---|---|---|---|
| 100 | 5 | 4.4 | 1.13 |
| 50 | 1.5 | 17.6 | 0.085 |
| 25 | 0.15 | 70.4 | 0.002 |

- Thus, as the beam penetrates into the target, the beam energy is degraded and the probability for fusion decreases dramatically.

- Let us try to estimate the effect of beam degradation on the fusion energy gain, $G$.

# Energetic Deuterons on "Cold" Targets

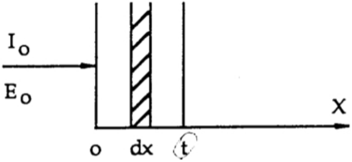



Figure 14: Beam degradation

- Remember that in slide 10 we defined the reaction rate density as

$$R = (\sigma n_2) \underbrace{(n_1 v)}_{\phi}$$

  where $\phi$ denotes the particle flux (this often is referred to as intensity $I$).
- Therefore, the number of fusion reactions at $E_o$ in $dx$ about $x$ is given by:

$$R(x)dx = \underbrace{I(x)}_{\phi(x)} \underbrace{\sigma_f}_{\sigma} \underbrace{n}_{n_2} dx \qquad [\phi] = [I] = [nv] = \frac{\#}{cm^2 s}$$

- We neglect fusion reactions at $E_D < E_O$

# Energetic Deuterons on “Cold” Targets

- The total number of fusion reactions in the target per $cm^2 s$ is

$$\int_0^t R(x)dx$$

Note that $t$ denotes “thickness” (not “time”). See Fig. (14).

- The fusion energy gain, $G$, is thus,

$$G = \frac{\int_0^t QR(x)dx}{I_o E_o} = \frac{Q\int_0^t R(x)dx}{I_o E_o}$$

- Now,

$$\int_0^t R(x)dx = \int_0^t I(x)\sigma_f n dx = \sigma_f n \int_0^t I(x)dx$$

- How does $I(x)$ change as function of $x$? What is the expression for $I(x)$?

# Energetic Deuterons on “Cold” Targets

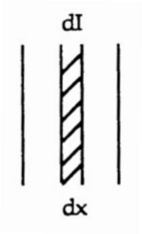



Figure 15: Beam degradation

- To calculate $I(x)$ consider the reduction in beam intensity, $dI$, in $dx$ about $x$,

$$\begin{aligned} \frac{dI}{dx} &= -I(x)\sigma_f n - I(x)\sigma_{CS} n \\ dI &= -I(x)\sigma_f n dx - I(x)\sigma_{CS} n dx \end{aligned}$$

We neglect fusion reactions at $E_D < E_O \Rightarrow$ Particles scattered just one time are also “removed” from the beam (similarly to fused particles).

# Energetic Deuterons on "Cold" Targets

- The Coulomb scattering term in this equation assumes that each scattering event degrades the incident deuteron energy below the point at which it could engage in a fusion reaction. A "better" assumption would be:

$$\begin{aligned} \frac{dI}{dx} &= -I(x)\sigma_f n - I(x)\frac{\sigma_{CS}}{s}n \\ dI &= -I(x)\sigma_f n dx - I(x)\frac{\sigma_{CS}}{s}n dx \end{aligned}$$

  where $s$ is the number of Coulomb scattering collisions required for "substantial" degradation of $E_D$.

  Here we neglect fusion reactions at $E_D < E_{threshold} < E_O$ with the assumption that it takes $s$ scatterings for the particle to see its energy $E_D$ go below $E_{threshold}$. Therefore, only particles scattered a minimum of $s$ times are "removed" from the beam (similarly to fused particles).

# Energetic Deuterons on “Cold” Targets

- Thus,

$$\frac{dI}{dx} = -I(x)n\left(\sigma_f + \frac{\sigma_{CS}}{s}\right) \iff I(x) = -\frac{dI}{dx}\frac{1}{n\left(\sigma_f + \frac{\sigma_{CS}}{s}\right)}$$

- Now,

$$\begin{aligned} \int_0^t R(x)dx = \sigma_f n \int_0^t I(x)dx &= \frac{\sigma_f n}{n\left(\sigma_f + \frac{\sigma_{CS}}{s}\right)} \int_0^t \left(-\frac{dI}{dx}\right) dx \\ &= \frac{\sigma_f}{\left(\sigma_f + \frac{\sigma_{CS}}{s}\right)} \int_{I_o}^{I_t} (-dI) \end{aligned}$$

- If we take the thickness $t$ large enough to totally attenuate $I_o$ (i.e., $I_t \equiv 0$),

$$\int_0^t R(x)dx = \frac{\sigma_f I_o}{\left(\sigma_f + \frac{\sigma_{CS}}{s}\right)}$$

# Energetic Deuterons on "Cold" Targets

- Thus,

$$G = \frac{Q \frac{\sigma_f I_o}{\left(\sigma_f + \frac{\sigma_{CS}}{s}\right)}}{I_o E_o} = \frac{Q}{E_o} \frac{\sigma_f}{\left(\sigma_f + \frac{\sigma_{CS}}{s}\right)}$$

- Now we return to the $D$ on $T$ (cold) case and make the most pessimistic assumption about $s$, i.e., $s = 1$. Then,

$$G = \frac{5}{\left(5 + \frac{4.4}{1}\right)} \frac{17.6}{0.1} = 0.53 G_o \approx 93$$

- Thus, $D - T$ scattering degrades $G$ by only about a factor of $2$ and the resulting gain is still substantial.

WHAT IS STILL MISSING? WHAT MUST BE CONSIDERED IN ADDITION TO $D - T$ (COLD) COULOMB SCATTERING?