Nuclear Fusion and Radiation

Lecture 1 (Meetings 1 & 2)

Eugenio Schuster

schuster@lehigh.edu
Mechanical Engineering and Mechanics
Lehigh University

Nuclear Energy Release

Consider a nuclear reaction of the form:

$$a + b \Rightarrow d + e \tag{1}$$

where a and b are the reactants, and d and e are the products. In such a reaction it is observed that the relativistic total energy of the system, E, is conserved, where E is given by

$$E = T + M_o c^2 \tag{2}$$

$T =$ kinetic energy
$M_o =$ rest mass
$c =$ speed of light in vacuum (299,792,458 m/s)
$M_o c^2 =$ rest mass energy

Nuclear Energy Release

Conservation of the relativistic total energy for the reaction can be written as:

$$(T + M_o c^2)_a + (T + M_o c^2)_b = (T + M_o c^2)_d + (T + M_o c^2)_e \tag{3}$$

Rearranging this equation yields:

$$[(M_{o_a} + M_{o_b}) - (M_{o_d} + M_{o_e})]c^2 = (T_d + T_e) - (T_a + T_b) \tag{4}$$

or:

$$\left(\sum M_r - \sum M_p\right) c^2 = \sum T_p - \sum T_r \tag{5}$$

where the subscripts r and p refer to reactants and products, respectively.

Nuclear Energy Release

The Q value of a nuclear reaction is defined as:

$$Q \triangleq \underbrace{\left(\sum M_r - \sum M_p\right)}_{\Delta M} c^2 = \sum T_p - \sum T_r \tag{6}$$

If $\sum M_r > \sum M_p$, $Q > 0$ and $\sum T_p > \sum T_r$, and the reaction is designated "exothermic", that is, there is energy release and this energy release is manifested in the excess kinetic energy of the products.

If $\sum M_r < \sum M_p$, $Q < 0$ and $\sum T_p < \sum T_r$, and the reaction is called "endothermic".

In summary

$$Q \triangleq (\Delta M)c^2 = \Delta T. \tag{7}$$

Nuclear Energy Release

The characteristics of a nucleus are defined in terms of three parameters

$$ {}^{A}_{Z}M \tag{8} $$

where,

A is the mass number (number protons + neutrons)
M is the nuclear rest mass
Z is the atomic number (number of protons)

The nuclear forces holding the nucleus together are often expressed as "binding" energy, B, denoted by

$$ B = [Zm_p + (A - Z)m_n - M]c^2 \tag{9} $$

where m_p is the proton mass and m_n is the neutron mass.

Note that $B > 0$ implies that the sum of the masses of the individual components (protons and neutrons) of the nucleus is larger than the mass of the nucleus. The "lost" mass is converted into the energy that holds the nucleus together.

Nuclear Energy Release

Thus, the nuclear rest mass, M, can be written as

$$M = Zm_p + (A-Z)m_n - \frac{B}{c^2} \tag{10}$$

Again consider the reaction

$$a + b \Rightarrow d + e \tag{11}$$

$$Q = [(M_a + M_b) - (M_d + M_e)]c^2 \tag{12}$$

Rewriting this equation in terms of the binding energy yields

$$\begin{aligned}\frac{Q}{c^2} \quad = \quad & Z_a m_p + (A-Z)_a m_n - \frac{B_a}{c^2} + Z_b m_p + (A-Z)_b m_n - \frac{B_b}{c^2} \\ & - Z_d m_p - (A-Z)_d m_n + \frac{B_d}{c^2} - Z_e m_p - (A-Z)_e m_n + \frac{B_e}{c^2}\end{aligned}$$

Nuclear Energy Release

It is observed in nuclear reactions that charge and number of nucleons are conserved. Thus,

$\sum Z$ = constant $\Rightarrow Z_a + Z_b = Z_d + Z_e$
Then: $(Z_a + Z_b)m_p - (Z_d + Z_e)m_p \equiv 0$

$\sum A$ = constant $\Rightarrow A_a + A_b = A_d + A_e \Rightarrow (A-Z)_a + (A-Z)_b = (A-Z)_d + (A-Z)_e$
Then: $[(A-Z)_a + (A-Z)_b)]m_n - [(A-Z)_d + (A-Z)_e)]m_n \equiv 0$

Therefore

$$\frac{Q}{c^2} = \frac{(\sum B_p - \sum B_r)}{c^2} \tag{13}$$

$$Q = \sum Bp - \sum Br \tag{14}$$

Thus, Q is positive if $\sum B_p > \sum B_r$, that is, if nucleons in the product nuclei are more tightly bound than nucleons in the reactant nuclei.

Nuclear Energy Release

Since A (mass number) is conserved in nuclear reactions it is useful to talk about binding energy per nucleon, B/A, for a nucleus. In this context consider the equation

$$(Q/A) = \sum (B/A)_p - \sum (B/A)_r \tag{15}$$

Plots of B/A are helpful in identifying energetically favorable reactions. Consider Fig. 1. The following points are noted with regard to this figure:

- B/A is a maximum at $A \sim 60$
- B/A decreases as $A \to 200$
- B/A decreases for light nuclei.

Nuclear Energy Release

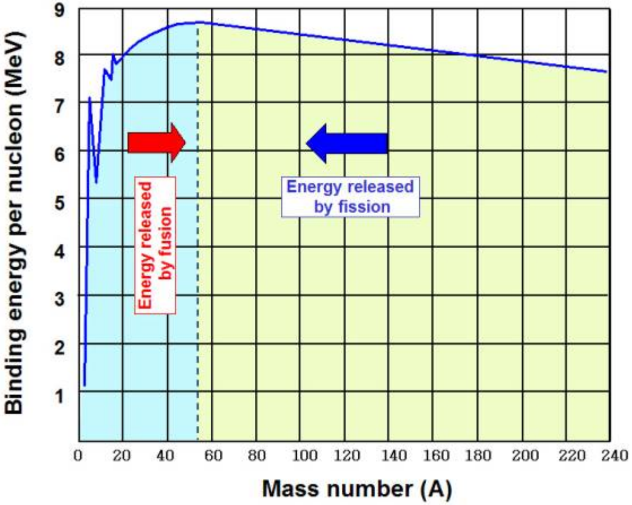


Figure: Average binding energy B/A in Mev per nucleon as a function of mass number A

Nuclear Energy Release

- In discussing nuclear reactions it is convenient to define the amu (atomic mass unit) which is based on ${}^{12}_{6}C$ (the atom) as having an atomic mass of exactly 12 amu. On this basis the mass equivalence of the amu is

$$1\ amu = 1.66 \times 10^{-27}\ kg \tag{16}$$

- What is the energy equivalence of the amu? If we could convert the mass associated with 1 amu to kinetic energy release, then:

$$\Delta T = 1.66 \times 10^{-27}(kg)(3 \times 10^{8} m/s)^2 = 1.49 \times 10^{-10} J \tag{17}$$

- Note that this is not much energy in conventional terms: 1 jelly donut $\sim 10^6 J \sim 1 MJ$.

Nuclear Energy Release

- It is not very convenient to characterize the energy release of nuclear reactions in Joules (because the release is so small in this unit) and generally we use the electron volt (eV) which is defined as follows.
- When a singly charged particle (e.g. an electron) is accelerated through a potential difference of one volt it is said that the particle kinetic energy has increased by one electron volt ($1\ eV$). Since,

$$\Delta T = qV \tag{18}$$

Then,

$$\Delta T(1eV) = (1.602 \times 10^{-19} C)(1V) = 1.602 \times 10^{-19} J \tag{19}$$

Nuclear Energy Release

- Recasting the amu in terms of the electron volt yields

$$\Delta T(1amu) = \frac{1.49 \times 10^{-10} J}{1.602 \times 10^{-19} J/eV} = 9.31 \times 10^{8} eV = 931 MeV \qquad (20)$$

- In nuclear reactions the energy release is generally well below 931 MeV. Thus, mass conversion to energy is generally less than 1 amu.
- Note that in chemical reactions the Q values are $\sim 2 - 5\ eV$. Thus, the energy associated with bond formation and breakage (rearrangement of electronic structure) is $\sim 10^6$ less than the energy involved in nuclear rearrangements.

Fusion Reactions

- Fusion involves the coalescing of light nuclei to form heavier (more stable) nuclei.
- Whereas only a few elements can undergo fission, there are many possible fusion reactions.
- Consider first fusion of the lightest elements - hydrogen and its isotopes. Reactions of possible interest include:

$$\begin{aligned}
{}^1_1H + {}^1_1H &\rightarrow {}^2_1D + {}^0_1e\,(\beta^+) + \nu \\
{}^1_1H + {}^2_1D &\rightarrow {}^3_2He + {}^0_0\gamma \\
{}^1_1H + {}^3_1T &\rightarrow {}^4_2He + {}^0_0\gamma \\
{}^2_1D + {}^2_1D &\rightarrow {}^3_2He + {}^1_0n \\
{}^2_1D + {}^3_1T &\rightarrow {}^4_2He + {}^1_0n
\end{aligned}$$

Exercise: Calculate the Q value for the DT fusion reaction.

Fusion Reactions

The Q value for the DT reaction is $17.6MeV$. Consider the amount of energy release per gram of tritium consumed (1 tritium mol $\approx 3gr$):

$$E(T) = \frac{6.02 \times 10^{23}(\#/mole)}{3gr/mole} \times 17.6MeV = 2.01 \times 10^{23} \times 17.6\frac{MeV}{gr}$$

Converting to Joules ($1MeV = 1.6 \times 10^{-13}J$), we obtain

$$E(T) = 2.01 \times 10^{23} \times 17.6 \times 1.6 \times 10^{-13}\frac{J}{gr} = 5.65 \times 10^{11}\frac{J}{gr}$$

We now define an energy unit called the Megawatt-day (MWd) as follows:

$$1MWd = 10^6(J/sec) \times 3600 \times 24(sec/d) = 8.64 \times 10^{10}J \tag{21}$$

Fusion Reactions

Thus, the energy yield from a DT reaction is

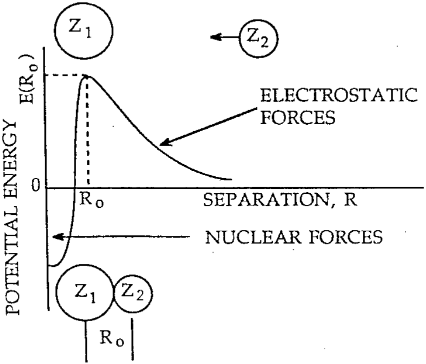
$$E = \frac{5.65 \times 10^{10} (J/gm)}{8.64 \times 10^{10} (J/MWd)} = 6.54 \frac{MWd}{gr}$$

Note that one fission event yields $\sim 200 MeV$ or about ten times as much energy as a DT fusion event. However, the result is different when compared on a per gram basis (1 U^{235} mol $\approx 235 gr$).

$$\begin{aligned} E(U^{235}) &= \frac{6.02 \times 10^{23} (\#/mole)}{235 gr/mole} \times 200 MeV \times 1.6 \times 10^{-13} \frac{J}{MeV} \\ &= 8.2 \times 10^{10} \frac{J}{gr} \\ E(U^{235}) &= \frac{8.2 \times 10^{10} (J/gr)}{8.64 \times 10^{10} (J/MWd)} = 0.95 \frac{MWd}{gr} \end{aligned}$$

Fusion Reactions

- Based on the previous calculations, it is obvious that we can get substantial energy release from a few grams of D and T. We must now turn to the probabilities for fusion.
- Consider schematically the mechanics of a two particle nuclear reaction.



Fusion Reactions

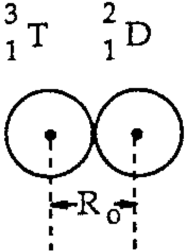
The electrostatic potential energy between two point charges, $E(R)$, is given by

$$E(R) = \frac{Z_1 Z_2}{4\pi\epsilon_o R} \tag{22}$$

where Z is the charge in coulombs (C), R is the separation in meters (m), and $\epsilon_o = 8.85 \times 10^{-12} F/m$ is the permittivity of free space. NOTE: Farad, $F = C^2/J$.

Let us estimate the barrier height, $E(Ro)$ for D on T. The nuclear separation at contact is approximately $R_o = 4 \times 10^{-15} m$.

Fusion Reactions



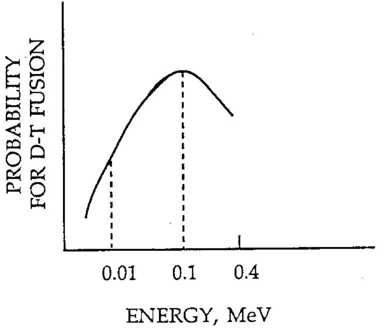
$$E(R_o) = \frac{(1.6 \times 10^{-19})^2 (C^2)}{4 \times \pi \times 8.85 \times 10^{-12} (C^2/J/m) \times 4 \times 10^{-15} (m)}$$
$$= 5.76 \times 10^{-14} J$$

Noting that $1 MeV = 1.6 \times 10^{-13} J$,

$$E(Ro) = \frac{5.76 \times 10^{-14} J}{1.6 \times 10^{-13} J/MeV} \sim 0.36 MeV$$

Fusion Reactions

- Thus, if the mechanics of fusion relied purely on classical effects, the mutual kinetic, energy of the D on T would have to be $\sim 0.36 MeV$ before any fusion could take place, i.e., the barrier height would have to be overcome before any reaction would take place.
- However, if we examine the experimentally observed probability for reaction as a function of energy, the following results emerge.



Thus, the experimental evidence is that there is significant probability for fusion at energies well below the coulomb barrier height.

What is happening?