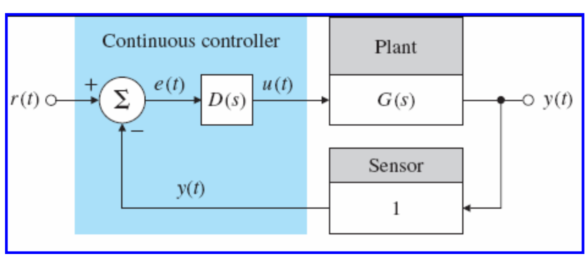
ME 343 – Control Systems

Lecture 41

December 2, 2009

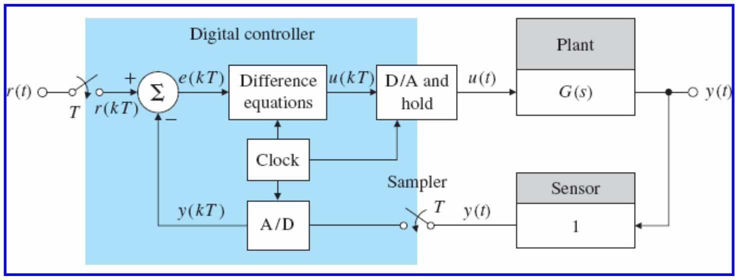
Digitization

Continuous-time Feedback System



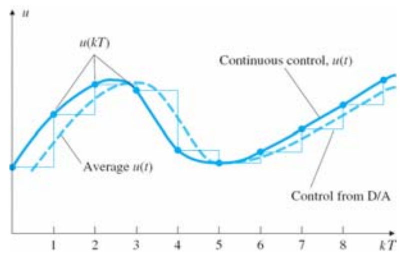
Digitization

Discrete-time Feedback System



$r(kT)$, $e(kT)$, $u(kT)$, $y(kT)$: sampled signals

T:	sample period
$1/T$:	sample rate (Hz)
A/D:	analog-to-digital converter
D/A:	digital-to-analog converter

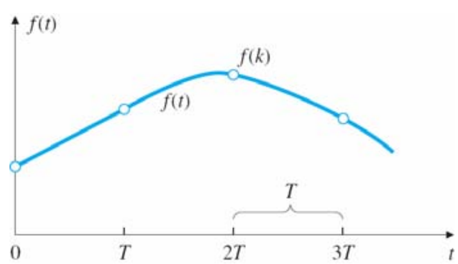


Z-Transform

Function *f(kT)* of time:

$f(k)=f(kT)$ is the sampled version of $f(t)$, and $k=0, 1, 2, 3, \ldots$ refer to the discrete sample times $t_0=0$, $t_1=T$, $t_2=2T$, $t_3=3T$, …

$$Z\{f(k)\} = F(z) = \sum_{k=0}^{\infty} f(k) z^{-k}$$



"Differentiation" property:

$$Z\{f(k-1)\} = z^{-1} F(z)$$

Z-Transform Table

L-Transform	Waveform	Z-Transform
$\frac{1}{s}$	$1(kT)$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s+\alpha}$	$e^{-\alpha kT}$	$\frac{z}{z-e^{-\alpha T}}$
$\frac{1}{(s+\alpha)^2}$	$kTe^{-\alpha kT}$	$\frac{Tze^{-\alpha T}}{(z-e^{-\alpha T})^2}$
$\frac{\beta}{s^2+\beta^2}$	$\sin(\beta kT)$	$\frac{z\sin(\beta T)}{z^2-2\cos(\beta T)z+1}$
$\frac{s}{s^2+\beta^2}$	$\cos(\beta kT)$	$\frac{z(z-\cos(\beta T))}{z^2-2\cos(\beta T)z+1}$

Z-Transform

Discrete-time System

$$y[k]+a_{k-1}y[k-1]+\cdots+a_n y[k-n]=b_o u[k]+\cdots+b_m u[k-m]$$

$$\Big\updownarrow \textbf{ Z Transform}$$

$$\left(1+a_1 z+\cdots+a_{n-1}z^{n-1}+a_n z^{-n}\right)Y(s)=\left(b_o+\cdots+b_m z^{-m}\right)U(s)$$

$$\boxed{T(z)=\frac{Y(z)}{U(z)}=\frac{b_o+\cdots+b_m z^{-m}}{1+a_1 z+\cdots+a_{n-1}z^{n-1}+a_n z^{-n}}}$$

Relationship between *s* and *z*

Discrete-time System:

$$f(t) = e^{-\alpha t} \longrightarrow \frac{1}{s+\alpha}$$

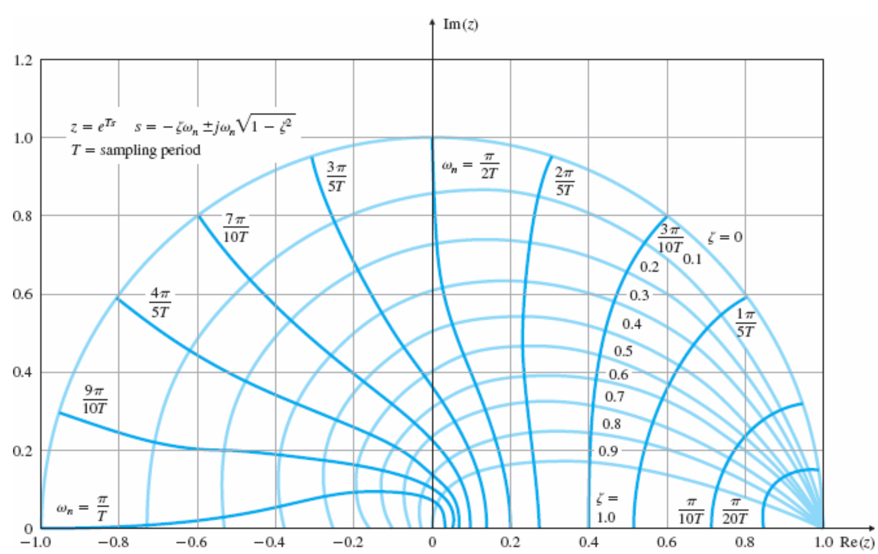
Continuous-time System:

$$f(kT) = e^{-\alpha kT} \longrightarrow \frac{z}{z - e^{-\alpha T}}$$

The pole at $s=-\alpha$ in the s-domain corresponds to a pole at $z=e^{-\alpha T}$ in the z-domain:

$$\boxed{z = e^{sT}}$$

Relationship between *s* and *z*



Relationship between s and z