ME 343 – Control Systems

Lecture 40

November 30, 2009

Observability

Problem Definition: "An unforced system is said to be observable if and only if it is possible to determine any (arbitrary initial) state $x(0)$ by using only a finite record, $y(\tau)$ for $0 \le \tau \le T$, of the output"

Theorem: "A system is controllable if and only if the matrix

$$\overline{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

Observability Matrix

is full-rank."

Observer Design

We consider the linear, time-invariant system

$$\dot{x} = Ax + Bu,$$
$$y = Cx + Du.$$

and we look for an "observer" of the state of the form

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}),$$
$$\hat{y} = C\hat{x} + Du.$$

In this case we have the error $\tilde{x} = x - \hat{x}$ dynamics

$$\dot{\tilde{x}} = (A - LC)\tilde{x}$$

We should note that we can modify the dynamics (eigenvalues) of the error system by proper selection of the gain *L*. If the system is observable, it is always possible to find an observer gain *L* to set the eigenvalues of the error dynamics at arbitrary values.

Observer Design

By noting that

$$eig\{A - LC\} = eig\{(A - LC)^T\} = eig\{A^T - C^T L^T\}$$

we can conclude that the observer eigenvalue placement problem is similar to the controller eigenvalue placement problem

$$eig\{A - BK\}$$

By making

$$A = A^T, B = C^T$$

we can use the same eigenvalue placement formulas developed by state feedback control design. After obtaining *K*, we obtain *L* as

$$L = K^T$$

Output Feedback

We consider the linear, time-invariant system

$$\dot{x} = Ax + Bu,$$
$$y = Cx + Du.$$

with state feedback $u = -Kx$

such that the closed-loop control system has the desired dynamics

$$\dot{x} = (A - BK)x$$

We design, in addition, an "observer" of the state of the form

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}),$$
$$\hat{y} = C\hat{x} + Du.$$

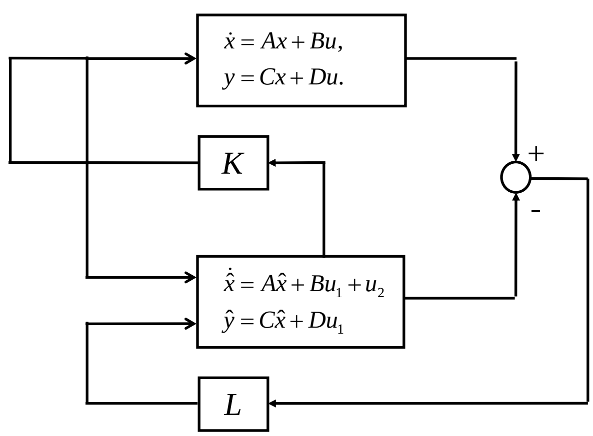
such that the error $\tilde{x} = x - \hat{x}$ has the desired dynamics

$$\dot{\tilde{x}} = (A - LC)\tilde{x}$$

Output Feedback

We consider now the following feedback law

$$u = -K\hat{x}$$



Output Feedback

System

$$\dot{x} = Ax - BK\hat{x}$$
$$y = Cx - DK\hat{x}$$

Observer

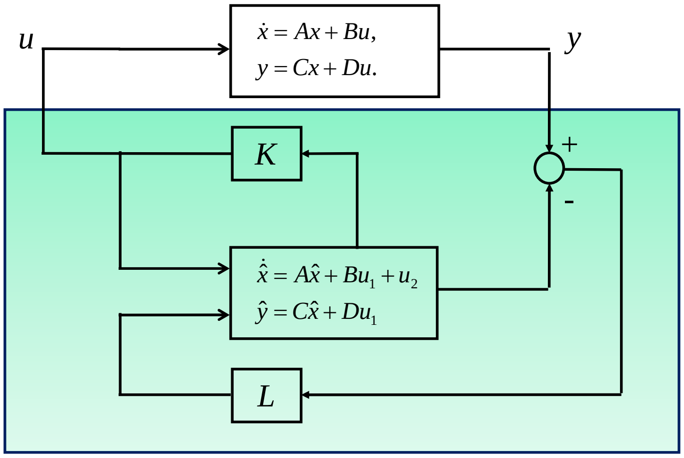
$$\dot{\hat{x}} = (A - LC - BK)\hat{x} + LCx$$
$$\hat{y} = (C - DK)\hat{x}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & (A - LC - BK) \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$
$$\begin{bmatrix} y \\ \hat{y} \end{bmatrix} = \begin{bmatrix} C & -DK \\ 0 & C - DK \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

Output Feedback

We consider now the following feedback law

$$u = -K\hat{x}$$



Output Feedback

State Feedback

$$u = -K\hat{x}$$

Observer

$$\dot{\hat{x}} = (A - LC - BK)\hat{x} + Ly$$
$$\hat{y} = (C - DK)\hat{x}$$

Output Feedback

$$\dot{\hat{x}} = (A - LC - BK)\hat{x} + Ly$$
$$u = -K\hat{x}$$

$$A_{cont} = (A - LC - BK), B_{cont} = L$$
$$C_{cont} = -K, D_{cont} = 0$$

$$\boxed{G_{cont}(s) = \frac{U(s)}{Y(s)} = -K\left[sI - (A - LC - BK)\right]^{-1} L}$$