ME 343 – Control Systems

Lecture 38

November 20, 2009

Controllability

Problem Definition: "A system is said to be controllable if and only if it is possible, by means of the input, to transfer the system from any initial state $x(0)$ to any other state $x(t)$ in a finite time $t \geq 0$."

Theorem: "A system is controllable if and only if the matrix

$$\overline{C} = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$$ Controllability Matrix

is full-rank."

State Feedback

We consider the linear, time-invariant system

$$\dot{x} = Ax + Bu,$$
$$y = Cx + Du.$$

and we look for a state gain *K* such that

$$u = -Kx$$

In this case we have the closed-loop system

$$\dot{x} = Ax - BKx = (A - BK)x$$

We should note that we can modify the dynamics (eigenvalues) of the system by state feedback. If the system is controllable, it is always possible to find a state gain *K* to set the eigenvalues of the closed-loop system at arbitrary values.

State Feedback

Given the desired characteristic equation $\alpha(s)$, we can compute the closed-loop characteristic equation.

$$a_k(s) = \det(sI - A + BK)$$

By equating coefficients of identical power of $a_k(s)$ and $\alpha(s)$, we can obtain n algebraic equations for the coefficients of K.

Example: Desired eigenvalues: *-1*, *-3*.

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Note: This method becomes rather cumbersome when n is large.