# ME 343 – Control Systems

Lecture 27

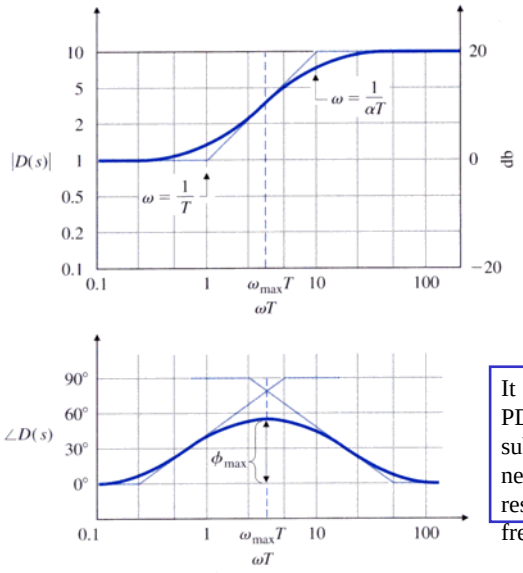
October 26, 2009



## Specifications in the Frequency Domain

1. The crossover frequency $\omega_c$, which determines bandwith $\omega_{BW}$, rise time $t_r$ and settling time $t_s$.
2. The phase margin PM, which determines the damping coefficient $\zeta$ and the overshoot $M_p$.
3. The low-frequency gain, which determines the steady-state error characteristics.

# Frequency Response – Phase Lead Compensators




$$D(s) = \frac{Ts+1}{\alpha Ts+1}, \quad \alpha < 1$$

$$\alpha = \frac{1-\sin\phi_{MAX}}{1+\sin\phi_{MAX}}$$

$$\log\omega_{MAX} = \frac{1}{2}\left[\log\left(\frac{1}{T}\right) + \log\left(\frac{1}{\alpha T}\right)\right]$$

It is a high-pass filter and approximates PD control. It is used whenever substantial improvement in damping is needed. It tends to increase the speed of response of a system for a fixed low-frequency gain.



# Frequency Response – Phase Lead Compensators

1. Determine the open-loop gain $K$ to satisfy error or bandwidth requirements:
   - To meet error requirement, pick $K$ to satisfy error constants ($K_p$, $K_v$, $K_a$) so that $e_{ss}$ specification is met.
   - To meet bandwidth requirement, pick $K$ so that the open-loop crossover frequency is a factor of two below the desired closed-loop bandwidth.

2. Determine the needed phase lead → α based on the PM specification. $\alpha = \frac{1-\sin\phi_{MAX}}{1+\sin\phi_{MAX}}$

3. Pick $\omega_{MAX}$ to be at the crossover frequency.

4. Determine the zero and pole of the compensator.
   $z = 1/T = \omega_{MAX}\,\alpha^{1/2}$ $\quad p = 1/\alpha T = \omega_{MAX}\,\alpha^{1/2}$

5. Draw the compensated frequency response and check PM.

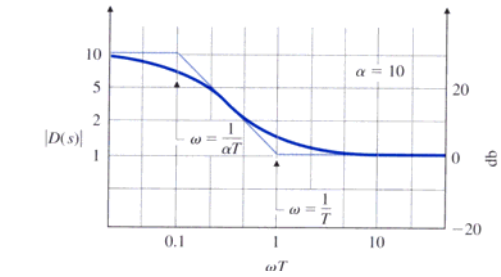
6. Iterate on the design. Add additional compensator if needed.

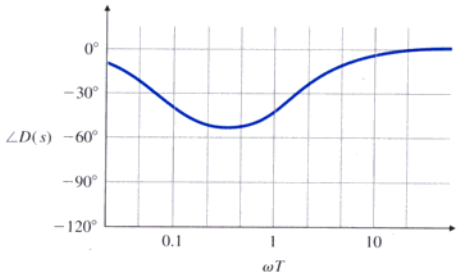
# Frequency Response – Phase Lead Compensators

**Example: FPE 6.45**



# Frequency Response – Phase Lag Compensators




$$D(s) = \alpha \frac{Ts+1}{\alpha Ts+1}, \qquad \alpha > 1$$

It is a low-pass filter and approximates PI control. It is used to increase the low frequency gain of the system and improve steady state response for fixed bandwidth. For a fixed low-frequency gain, it will decrease the speed of response of the system.

## Frequency Response – Phase Lag Compensators

1. Determine the open-loop gain $K$ that will meet the PM requirement without compensation.
2. Draw the Bode plot of the uncompensated system with crossover frequency from step 1 and evaluate the low-frequency gain.
3. Determine $\alpha$ to meet the low frequency gain error requirement.
4. Choose the corner frequency $\omega=1/T$ (the zero of the compensator) to be one decade below the new crossover frequency $\omega_c$.
5. The other corner frequency (the pole of the compensator) is then $\omega=1/\alpha T$.
6. Iterate on the design



## Frequency Response – Phase Lag Compensators

**Example: FPE 6.55**