ME 343 – Control Systems

Lecture 26

October 23, 2009

Specifications in the Frequency Domain

1. The crossover frequency ω_c, which determines bandwith ω_{BW}, rise time t_r and settling time t_s.
2. The phase margin PM, which determines the damping coefficient ζ and the overshoot M_p.
3. The low-frequency gain, which determines the steady-state error characteristics.

Specifications in the Frequency Domain

The phase and the magnitude are NOT independent!

Bode's Gain-Phase relationship:

$$\angle G(j\omega_o) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{dM}{du}W(u)du$$

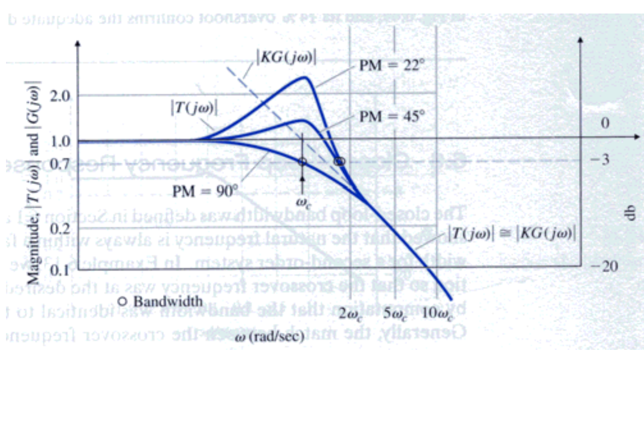
$$M = \ln\left|G(j\omega)\right|$$

$$u = \ln(\omega/\omega_o)$$

$$W(u) = \ln\left(\coth\left|u\right|/2\right)$$

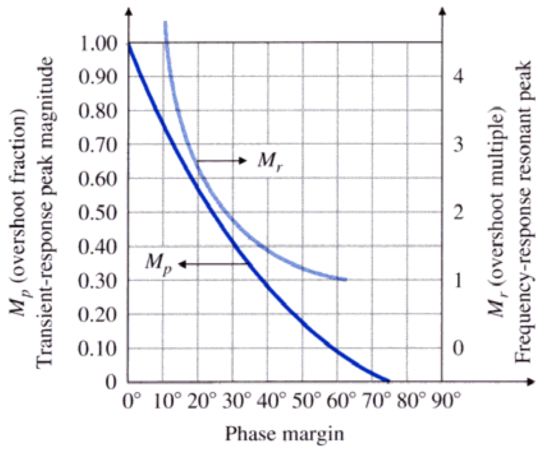
Specifications in the Frequency Domain

The crossover frequency: $\omega_c \le \omega_{BW} \le 2\omega_c$



Specifications in the Frequency Domain

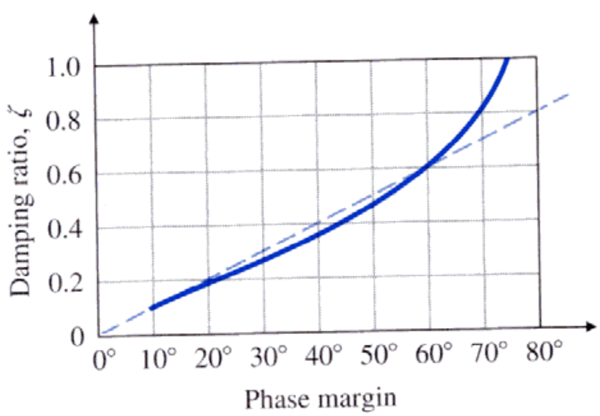
The Phase Margin: PM vs. M_p



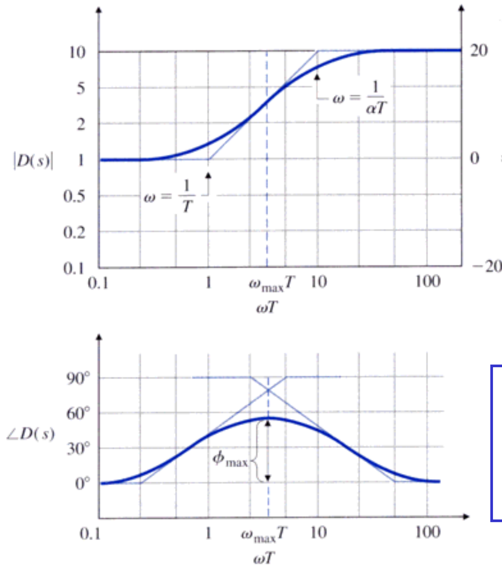
Specifications in the Frequency Domain

The Phase Margin: PM vs. ζ

$$\zeta \cong \frac{PM}{100}$$



Frequency Response – Phase Lead Compensators



$$D(s)=\frac{Ts+1}{\alpha Ts+1}, \qquad \alpha<1$$

$$\alpha=\frac{1-\sin\phi_{MAX}}{1+\sin\phi_{MAX}}$$

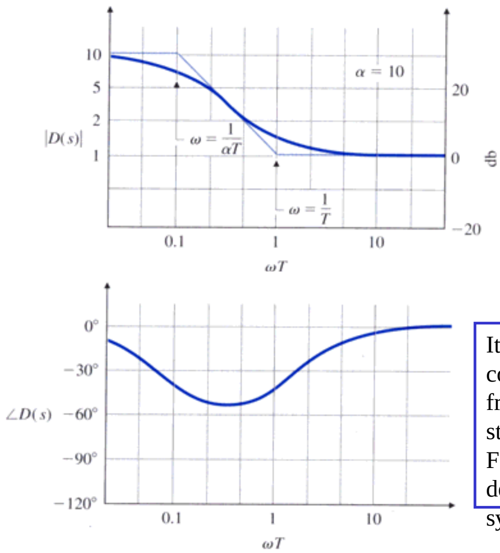
$$\log\omega_{MAX}=\frac{1}{2}\left[\log\left(\frac{1}{T}\right)+\log\left(\frac{1}{\alpha T}\right)\right]$$

It is a high-pass filter and approximates PD control. It is used whenever substantial improvement in damping is needed. It tends to increase the speed of response of a system for a fixed low-frequency gain.

Frequency Response – Phase Lead Compensators

1. Determine the open-loop gain K to satisfy error or bandwidth requirements:
   - To meet error requirement, pick K to satisfy error constants (K_p, K_v, K_a) so that e_{ss} specification is met.
   - To meet bandwidth requirement, pick K so that the open-loop crossover frequency is a factor of two below the desired closed-loop bandwidth.
2. Determine the needed phase lead → α based on the PM specification. $\alpha=\frac{1-\sin\phi_{MAX}}{1+\sin\phi_{MAX}}$
3. Pick ω_{MAX} to be at the crossover frequency.
4. Determine the zero and pole of the compensator.
   $z=1/T=\omega_{MAX}\,\alpha^{1/2}$ $\qquad$ $p=1/\alpha T=\omega_{MAX}\,\alpha^{1/2}$
5. Draw the compensated frequency response and check PM.
6. Iterate on the design. Add additional compensator if needed.

Frequency Response – Phase Lag Compensators



$$D(s) = \alpha \frac{Ts+1}{\alpha Ts+1}, \qquad \alpha > 1$$

It is a low-pass filter and approximates PI control. It is used to increase the low frequency gain of the system and improve steady state response for fixed bandwidth. For a fixed low-frequency gain, it will decrease the speed of response of the system.

Frequency Response – Phase Lag Compensators

1. Determine the open-loop gain K that will meet the PM requirement without compensation.

2. Draw the Bode plot of the uncompensated system with crossover frequency from step 1 and evaluate the low-frequency gain.

3. Determine α to meet the low frequency gain error requirement.

4. Choose the corner frequency $\omega=1/T$ (the zero of the compensator) to be one decade below the new crossover frequency ω_c.

5. The other corner frequency (the pole of the compensator) is then $\omega=1/\alpha T$.

6. Iterate on the design