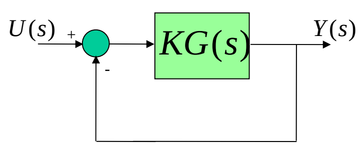
ME 343 – Control Systems

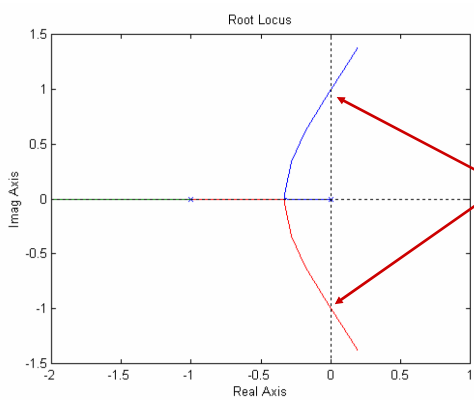
Lecture 24

October 19, 2009

Neutral Stability



$$G(s) = \frac{1}{s(s+1)^2}$$



Root locus condition:

$$|KG(s)| = 1, \quad \angle G(s) = -180^\circ$$

At points of neutral stability RL condition hold for $s=j\omega$

$$|KG(j\omega)| = 1, \quad \angle G(j\omega) = -180^\circ$$

Stability: At $\angle G(j\omega) = -180^\circ$

$|KG(j\omega)| < 1$ If ↑K leads to instability

$|KG(j\omega)| > 1$ If ↓K leads to instability

Stability Margins

The GAIN MARGIN (GM) is the factor by which the gain can be raised before instability results.

$$|GM| < 1 \left(|GM|_{dB} < 0 \right) \Rightarrow$$ UNSTABLE SYSTEM

GM is equal to $1/|KG(j\omega)|$ $\left(-|KG(j\omega)|_{dB}\right)$ at the frequency where $\angle G(j\omega) = -180^\circ$.

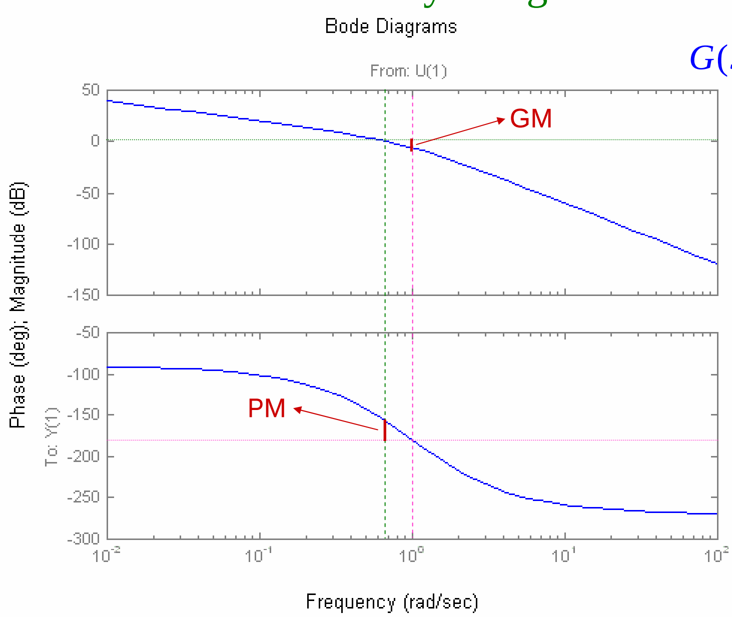
The PHASE MARGIN (PM) is the value by which the phase can be raised before instability results.

$$PM < 0 \Rightarrow$$ UNSTABLE SYSTEM

PM is the amount by which the phase of $G(j\omega)$ exceeds -180° when $|KG(j\omega)| = 1$ $\left(|KG(j\omega)|_{dB} = 0\right)$

Stability Margins

$$G(s) = \frac{1}{s(s+1)^2}$$



Frequency Response

$$u(t) = A\cos(\omega t + \phi) \longrightarrow \boxed{G(s)} \longrightarrow y_{ss} = |G(j\omega)| A\cos(\omega t + \phi + \angle G(j\omega))$$

Stable Transfer Function

$$G(j\omega) = |G(j\omega)| e^{j\angle G(j\omega)}$$ BODE plots

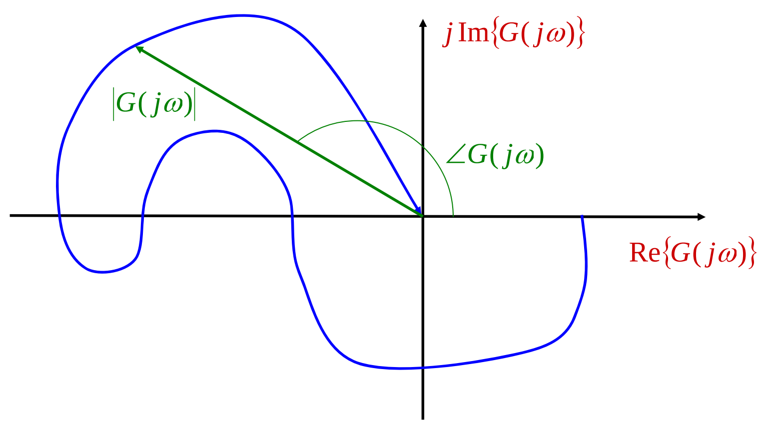
$$G(j\omega) = \mathrm{Re}\{G(j\omega)\} + j\,\mathrm{Im}\{G(j\omega)\}$$ NYQUIST plots

Nyquist Diagrams

$$G(j\omega) = \mathrm{Re}\{G(j\omega)\} + j\,\mathrm{Im}\{G(j\omega)\} = |G(j\omega)| e^{j\angle G(j\omega)}$$

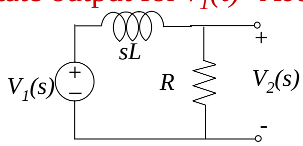
How are the Bode and Nyquist plots related?

They are two way to represent the same information



Nyquist Diagrams

Find the steady state output for $v_1(t)=A\cos(\omega t+\phi)$



Compute the s-domain transfer function T(s)

Voltage divider $\quad T(s)=\dfrac{R}{sL+R}$

Compute the frequency response

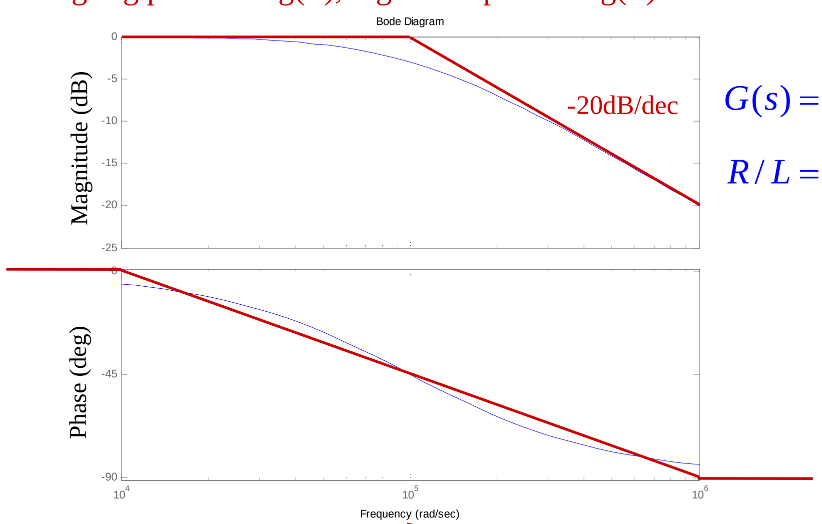
$$|T(j\omega)|=\frac{R}{\sqrt{R^2+(\omega L)^2}},\qquad \angle T(j\omega)=-\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Compute the steady state output

$$v_{2SS}(t)=\frac{AR}{\sqrt{R^2+(\omega L)^2}}\cos\left[\omega t+\phi-\tan^{-1}(\omega L/R)\right]$$

Bode Diagrams

- Log-log plot of mag(T), log-linear plot of arg(T) versus ω



$$G(s)=\frac{10}{s+10}$$

$$R/L=10$$

$[\omega]=rad/\sec,\ \omega=2\pi f,\ [f]=Hz$

Nyquist Diagrams

$$T(j\omega) = \frac{R}{R + j\omega L} = \frac{R}{R + j\omega L}\frac{R - j\omega L}{R - j\omega L} = \frac{R^2 - j\omega RL}{R^2 + \omega^2 L^2}$$

$$|T(j\omega)| = \frac{R}{\sqrt{R^2 + (\omega L)^2}}, \qquad \angle T(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\mathrm{Re}\{T(j\omega)\} = \frac{R^2}{R^2 + \omega^2 L^2}, \qquad \mathrm{Im}\{T(j\omega)\} = -\frac{\omega RL}{R^2 + \omega^2 L^2}$$

1- $\omega \to 0 : |T(j\omega)| \to 1, \quad \angle T(j\omega) \to 0 \qquad T(j\omega) = 1$

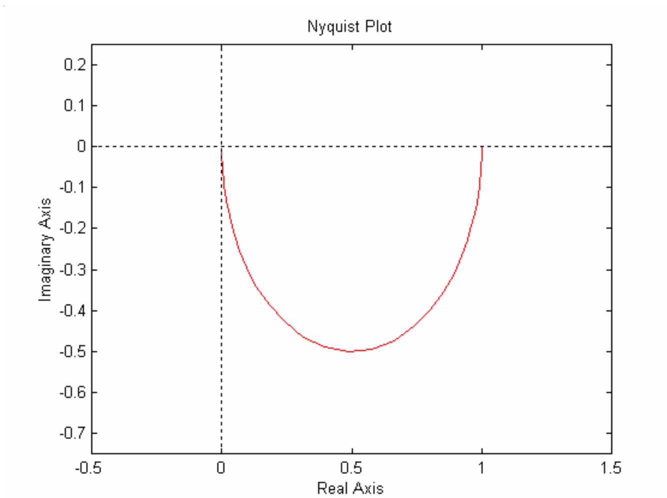
2- $\omega \to \infty : |T(j\omega)| \to 0, \quad \angle T(j\omega) \to -90^\circ \qquad T(j\omega) \xrightarrow[\omega\to\infty]{} -j\frac{R}{\omega L} \xrightarrow[\omega\to\infty]{} 0$

3- $\mathrm{Re}\{T(j\omega)\} = 0 \Leftrightarrow \omega = \infty$

4- $\mathrm{Im}\{T(j\omega)\} = 0 \Leftrightarrow \omega = 0, \omega = \infty$

Nyquist Diagrams

$\mathrm{Im}\{G(j\omega)\}$ vs. $\mathrm{Re}\{G(j\omega)\}$



$$G(s) = \frac{10}{s + 10}$$

$$R / L = 10$$

Nyquist Diagrams

General procedure for sketching Nyquist Diagrams:

- Find $G(j0)$
- Find $G(j\infty)$
- Find ω^* such that $\mathrm{Re}\{G(j\omega^*)\}=0$; $\mathrm{Im}\{G(j\omega^*)\}$ is the intersection with the imaginary axis.
- Find ω^* such that $\mathrm{Im}\{G(j\omega^*)\}=0$; $\mathrm{Re}\{G(j\omega^*)\}$ is the intersection with the real axis.
- Connect the points