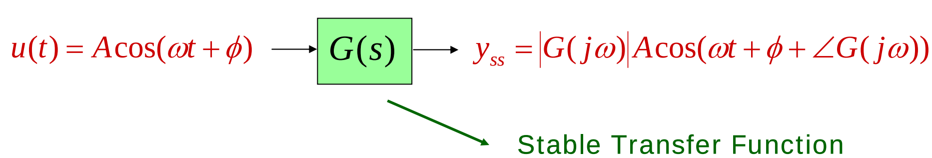
ME 343 – Control Systems

Lecture 23

October 16, 2009

Frequency Response



- After a transient, the output settles to a sinusoid with an amplitude magnified by $|G(j\omega)|$ and phase shifted by $\angle G(j\omega)$.
- Since all signals can be represented by sinusoids (Fourier series and transform), the quantities $|G(j\omega)|$ and $\angle G(j\omega)$ are extremely important.
- Bode developed methods for quickly finding $|G(j\omega)|$ and $\angle G(j\omega)$ for a given $G(s)$ and for using them in control design.

Bode Diagrams

$$G(s) = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$$G(s) = |K|\frac{r_1^z r_2^z \cdots r_m^z}{r_1^p r_2^p \cdots r_n^p} e^{i\left[\phi^K + \left(\phi_1^z+\phi_2^z+\cdots+\phi_m^z\right)-\left(\phi_1^p+\phi_2^p+\cdots+\phi_n^p\right)\right]}$$

The magnitude and phase of $G(s)$ when $s=j\omega$ is given by:

Nonlinear in the magnitudes

$$|G(j\omega)| = |K|\frac{r_1^z r_2^z \cdots r_m^z}{r_1^p r_2^p \cdots r_n^p},$$

$$\angle G(j\omega) = \phi^K + \left(\phi_1^z+\phi_2^z+\cdots+\phi_m^z\right)-\left(\phi_1^p+\phi_2^p+\cdots+\phi_n^p\right)$$

Linear in the phases

Bode Diagrams

Why do we express $|G(j\omega)|$ in decibels?

$$|G(j\omega)|_{dB} = 20\log|G(j\omega)|$$

$$|G(j\omega)| = |K|\frac{r_1^z r_2^z \cdots r_m^z}{r_1^p r_2^p \cdots r_n^p} \Rightarrow |G(j\omega)|_{dB} = ?$$

By properties of the logarithm we can write:

$$20\log|G(s)| = 20\log|K| + \left(20\log r_1^z + 20\log r_m^z + \cdots + 20\log r_m^z\right) - \left(20\log r_1^p + 20\log r_2^p + \cdots + 20\log r_n^p\right)$$

The magnitude and phase of $G(s)$ when $s=j\omega$ is given by:

Linear in the magnitudes (dB)

$$|G(s)|_{dB} = |K|_{dB} + \left(r_1^z\big|_{dB} + r_2^z\big|_{dB} + \cdots + r_m^z\big|_{dB}\right) - \left(r_1^p\big|_{dB} + r_2^p\big|_{dB} + \cdots + r_n^p\big|_{dB}\right)$$

$$\angle G(s) = \phi^K + \left(\phi_1^z+\phi_2^z+\cdots+\phi_m^z\right)-\left(\phi_1^p+\phi_2^p+\cdots+\phi_n^p\right)$$

Linear in the phases

General Transfer Function (Bode Diagrams)

$$G(j\omega) = K_o (j\omega)^m (j\omega\tau + 1)^n \left[\left(\frac{j\omega}{\omega_n} \right)^2 + 2\zeta \frac{j\omega}{\omega_n} + 1 \right]^q$$

The magnitude (dB) (phase) is the sum of the magnitudes (dB) (phases) of each one of the terms. We learn how to plot each term, we learn how to plot the whole magnitude and phase Bode Plot.

Classes of terms:

1- $G(j\omega) = K_o$

2- $G(j\omega) = (j\omega)^m$

3- $G(j\omega) = (j\omega\tau + 1)^n$

4- $G(j\omega) = \left[\left(\frac{j\omega}{\omega_n} \right)^2 + 2\zeta \frac{j\omega}{\omega_n} + 1 \right]^q$

Bode Diagrams

Example: $$G(s) = \frac{2000(s + 0.5)}{s(s + 10)(s + 50)}$$

Frequency Response: Poles/Zeros in the RHP

- Same $|G(j\omega)|$.
- The effect on $\angle G(j\omega)$ is opposite than the stable case.

An unstable pole behaves like a stable zero
An "unstable" zero behaves like a "stable" pole

Example: $$G(s) = \frac{1}{s-2}$$

This frequency response cannot be found experimentally but can be computed and used for control design.