ME 343 – Control Systems

Lecture 19

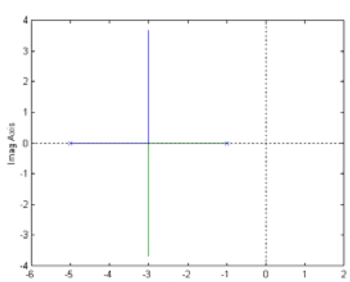
October 7, 2009

Root Locus - Compensators

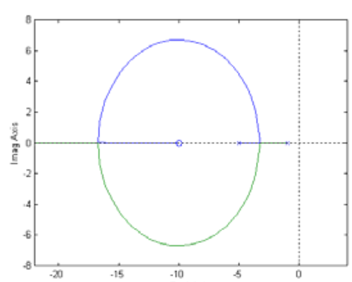
Example: $$L(s) = G(s) = \frac{1}{(s+1)(s+5)}$$

Can we place the closed loop pole at $s_o = -7 + i5$ only varying K?
NO. We need a COMPENSATOR.

$$L(s) = G(s) = \frac{1}{(s+1)(s+5)}$$



$$L(s) = D(s)G(s) = (s+10)\frac{1}{(s+1)(s+5)}$$



The zero attracts the locus!!!

Root Locus – Phase lead compensator

Pure derivative control is not normally practical because of the amplification of the noise due to the differentiation and must be approximated:

$$D(s) = \frac{s+z}{s+p}, \qquad p > z$$

Phase lead COMPENSATOR

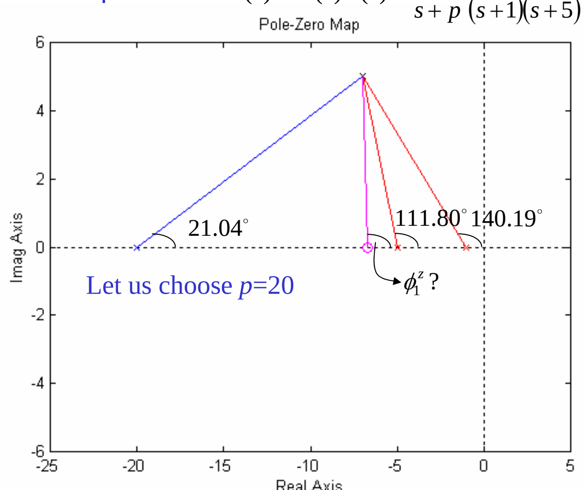
When we study frequency response we will understand why we call "Phase Lead" to this compensator.

$$L(s) = D(s)G(s) = \frac{s+z}{s+p}\frac{1}{(s+1)(s+5)}, \qquad p > z$$

How do we choose z and p to place the closed loop pole at s_o=-7+i5?

Root Locus – Phase lead compensator

Example: $L(s) = D(s)G(s) = \frac{s+z}{s+p}\frac{1}{(s+1)(s+5)}, \qquad p > z$



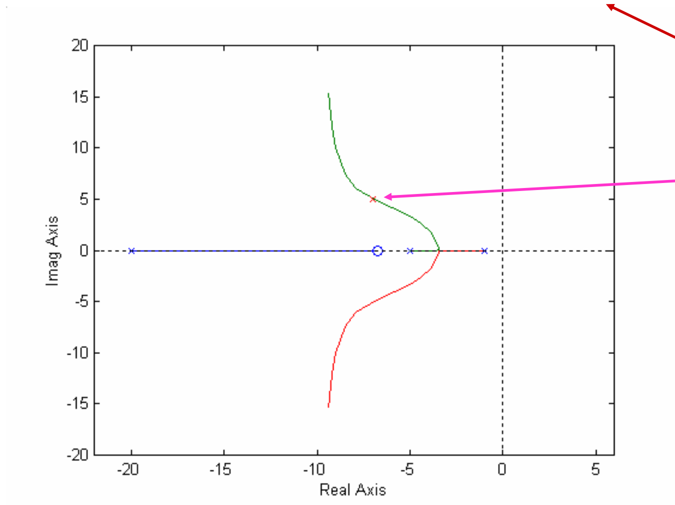
Phase lead COMPENSATOR

$$\angle L(s) = \phi^{K_p} + \left(\phi_1^z + \phi_2^z + \cdots + \phi_m^z\right) - \left(\phi_1^p + \phi_2^p + \cdots + \phi_n^p\right) = 180^\circ$$

$$\phi_1^z = 180^\circ + 140.19^\circ + 111.80^\circ + 21.04^\circ = 453.03^\circ = 93.03^\circ \Rightarrow z = -6.735$$

Root Locus – Phase lead compensator

Example: $L(s) = D(s)G(s) = \frac{s+6.735}{s+20} \frac{1}{(s+1)(s+5)}$



Root Locus – Phase lead compensator

Selecting z and p is a trial an error procedure. In general:

- The zero is placed in the neighborhood of the closed-loop natural frequency, as determined by rise-time or settling time requirements.
- The poles is placed at a distance 5 to 20 times the value of the zero location. The pole is fast enough to avoid modifying the dominant pole behavior.

The exact position of the pole p is a compromise between:

- Noise suppression (we want a small value for p)
- Compensation effectiveness (we want large value for p)

Root Locus – Phase lag compensator

Example: $$L(s) = D(s)G(s) = \frac{s+6.735}{s+20}\frac{1}{(s+1)(s+5)}$$

$$K_p = \lim_{s\to 0} L(s) = \lim_{s\to 0} D(s)G(s) = \lim_{s\to 0}\frac{s+6.735}{s+20}\frac{1}{(s+1)(s+5)} = 6.735\times 10^{-2}$$

What can we do to increase K_p? Suppose we want $K_p=10$.

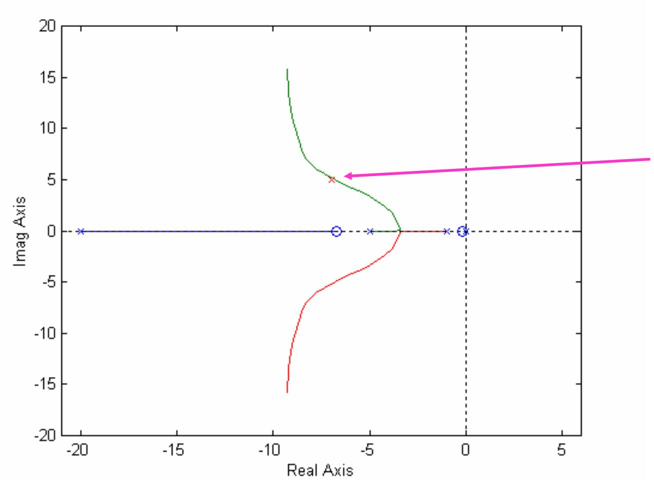
$$L(s) = D(s)G(s) = \frac{s+z}{s+p}\frac{s+6.735}{s+20}\frac{1}{(s+1)(s+5)}, \qquad p<z$$

Phase lag COMPENSATOR

We choose: $$\frac{z}{p} = \frac{1}{6.735}\times 10^{3} = 148.48$$

Root Locus – Phase lag compensator

Example: $$L(s) = D(s)G(s) = \frac{s+0.14848}{s+0.001}\frac{s+6.735}{s+20}\frac{1}{(s+1)(s+5)}$$



$s_o = -6.94 + i5.03$

$K = 18.31$

Root Locus – Phase lag compensator

Selecting z and p is a trial an error procedure. In general:

- The ratio zero/pole is chosen based on the error constant specification.
- We pick z and p small to avoid affecting the dominant dynamic of the system (to avoid modifying the part of the locus representing the dominant dynamics)
- Slow transient due to the small p is almost cancelled by an small z. The ratio zero/pole cannot be very big.

The exact position of z and p is a compromise between:

- Steady state error (we want a large value for z/p)
- The transient response (we want the pole p placed far from the origin)

Root Locus - Compensators

Phase lead compensator: $$D(s) = \frac{s+z}{s+p}, \quad z < p$$

Phase lag compensator: $$D(s) = \frac{s+z}{s+p}, \quad z > p$$

We will see why we call “phase lead” and “phase lag” to these compensators when we study frequency response