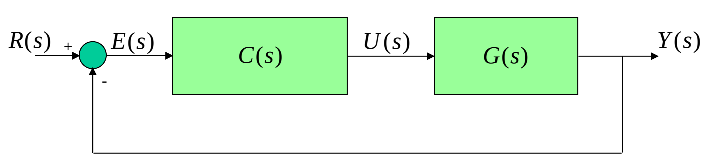
ME 343 – Control Systems

Lecture 11

September 16, 2009

Steady-state Tracking

The Unity Feedback Case



$$\frac{E(s)}{R(s)} = \frac{1}{1 + C(s)G(s)}$$

Test Inputs:

$$r(t) = \frac{t^k}{k!}1(t)$$

$$R(s) = \frac{1}{s^{k+1}}$$

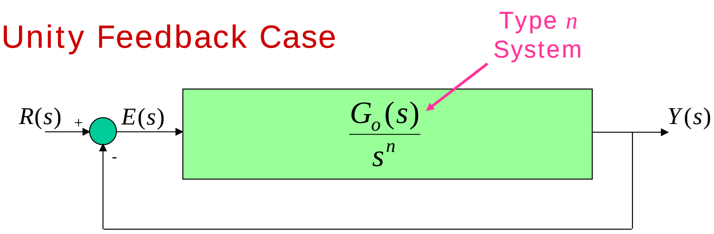
$k=0$: step (position)

$k=1$: ramp (velocity)

$k=2$: parabola (acceleration)

Steady-state Tracking

The Unity Feedback Case



$$C(s)G(s)=\frac{G_o(s)}{s^n},E(s)=\frac{1}{1+\frac{G_o(s)}{s^n}}R(s),R(s)=\frac{1}{s^{k+1}}$$

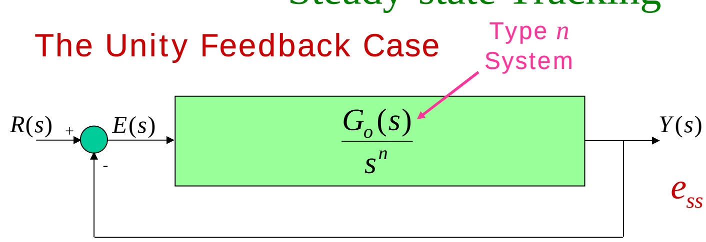
Steady State Error:

Final Value Theorem

$$e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to 0}sE(s)=\lim_{s\to 0}s\frac{1}{1+\frac{G_o(s)}{s^n}}\frac{1}{s^{k+1}}=\lim_{s\to 0}\frac{s^n}{s^n+G_o(s)}\frac{1}{s^k}=\lim_{s\to 0}\frac{s^{n-k}}{s^n+G_o(0)}$$

Steady-state Tracking

The Unity Feedback Case



$$e_{ss}=\lim_{s\to 0}\frac{s^{n-k}}{s^n+G_o(0)}$$

Steady State Error:

Type (n)	Input (k): Step (k=0)	Ramp (k=1)	Parabola (k=2)
Type 0	$\frac{1}{1+G_o(0)}=\frac{1}{1+\lim_{s\to 0}C(s)G(s)}=\frac{1}{1+K_p}$	∞	∞
Type 1	0	$\frac{1}{G_o(0)}=\frac{1}{\lim_{s\to 0}sC(s)G(s)}=\frac{1}{K_v}$	∞
Type 2	0	0	$\frac{1}{G_o(0)}=\frac{1}{\lim_{s\to 0}s^2C(s)G(s)}=\frac{1}{K_a}$

Steady-state Tracking

$$K_p = \lim_{s\to 0} C(s)G(s) \qquad n=0 \qquad \text{Position Constant}$$

$$K_v = \lim_{s\to 0} sC(s)G(s) \qquad n=1 \qquad \text{Velocity Constant}$$

$$K_a = \lim_{s\to 0} s^2C(s)G(s) \qquad n=2 \qquad \text{Acceleration Constant}$$

n: Degree of the poles of $CG(s)$ at the origin (the number of integrators in the loop with unity gain feedback)

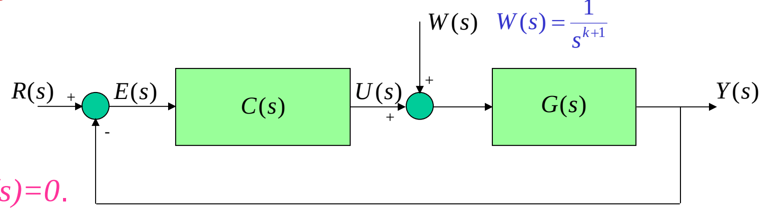
- Applying integral control to a plant with no zeros at the origin makes the system type ≥ I
- All this is true ONLY for unity feedback systems
- Since in Type I systems e_{ss}=0 for any $CG(s)$, we say that the system type is a robust property.

Steady-state Disturbance Rejection

The Unity Feedback Case

$$w(t) = \frac{t^k}{k!}1(t)$$

$$W(s) = \frac{1}{s^{k+1}}$$



Set $r=0$.

Want $Y(s)/W(s)=0$.

$$\frac{Y(s)}{W(s)} = \frac{G(s)}{1+C(s)G(s)} = T(s) = s^n T_o(s)$$

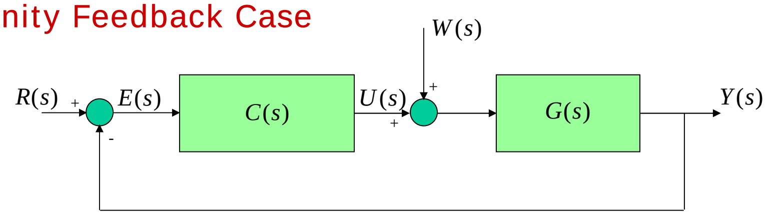
Steady State Error: $e=r-y=-y$

Final Value Theorem

$$-e_{ss} = y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} sT(s)\frac{1}{s^{k+1}} = \lim_{s\to 0} T_o(s)\frac{s^n}{s^k}$$

Steady-state Disturbance Rejection

The Unity Feedback Case



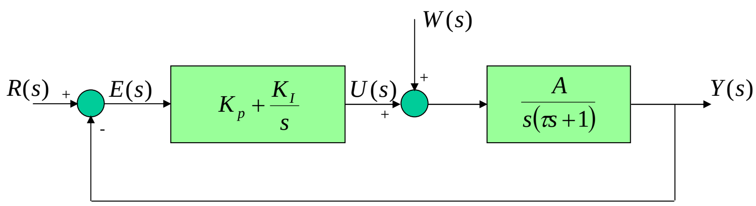
Steady State Output:

Type (n)	Step ($k=0$)	Ramp ($k=1$)	Parabola ($k=2$)
		Disturbance (k)	
Type 0	*	∞	∞
Type 1	0	*	∞
Type 2	0	0	*

$0 < * < \infty$

Steady-state Disturbance Rejection

Example:



$$K_I \neq 0 \quad \Rightarrow \quad \text{type 1 to } w$$

$$K_P \neq 0, K_I = 0 \quad \Rightarrow \quad \text{type 0 to } w$$

Steady-state Tracking

Example: FPE 4.16