ME 343 – Control Systems

Lecture 07

September 7, 2009

Impulse Response

Dirac's delta: $$\int_0^{\infty} u(\tau)\delta(t-\tau)d\tau = u(t)$$

Integration is a limit of a sum

$\Downarrow$

u(t) is represented as a sum of impulses

By superposition principle, we only need unit impulse response

$h(t-\tau)$ Response at t to an impulse applied at τ of amplitude $u(\tau)$

System Response: $u(t) \longrightarrow \boxed{h} \longrightarrow y(t)$

$$y(t) = \int_0^{\infty} u(\tau)h(t-\tau)d\tau$$

Impulse Response

t-domain: $u(t) \longrightarrow \boxed{h} \longrightarrow y(t)$

Impulse response

$$y(t) = \int_0^\infty u(\tau)h(t-\tau)d\tau \qquad u(t) = \delta(t) \Rightarrow y(t) = h(t)$$

The system response is obtained by convolving the input with the impulse response of the system.

Convolution: $\mathcal{L}\{\int_0^\infty u(\tau)h(t-\tau)d\tau\} = H(s)U(s)$

s-domain: $U(s) \longrightarrow \boxed{H} \longrightarrow Y(s)$

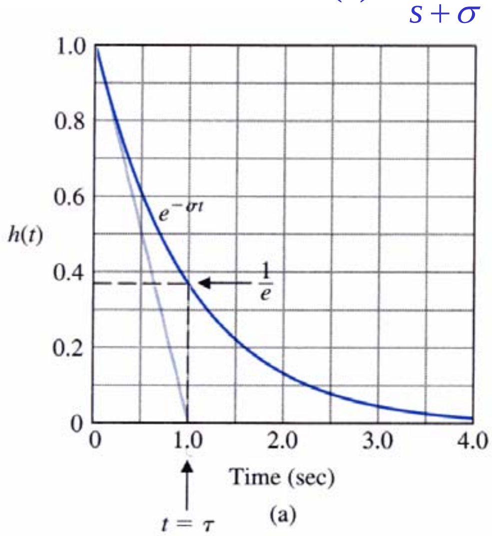
Impulse response

$$Y(s) = H(s)U(s) \qquad u(t) = \delta(t) \Rightarrow U(s) = 1 \Rightarrow Y(s) = H(s)$$

The system response is obtained by multiplying the transfer function and the Laplace transform of the input.

Time Response vs. Poles

Real pole: $H(s) = \dfrac{1}{s+\sigma} \Rightarrow h(t) = e^{-\sigma t}$ Impulse Response



$\sigma > 0$ Stable

$\sigma < 0$ Unstable

$\tau = \dfrac{1}{\sigma}$ Time Constant

Time Response vs. Poles

Real pole:

$$H(s)=\frac{\sigma}{s+\sigma}\Rightarrow h(t)=\sigma e^{-\sigma t}$$ Impulse Response

$$\tau=\frac{1}{\sigma}$$ Time Constant

$$Y(s)=\frac{\sigma}{s+\sigma}\frac{1}{s}\Rightarrow y(t)=1-e^{-\sigma t}$$ Step Response

Time Response vs. Poles

Complex poles: $$H(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$ Impulse Response

$$=\frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}$$

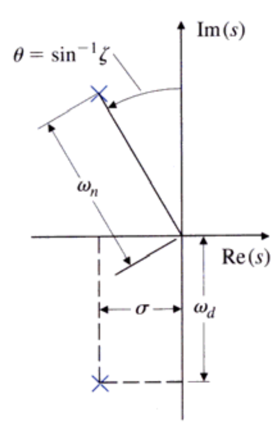
ω_n : Undamped natural frequency

ζ : Damping ratio

$$H(s)=\frac{\omega_n^2}{(s+\sigma+j\omega_d)(s+\sigma-j\omega_d)}$$

$$=\frac{\omega_n^2}{(s+\sigma)^2+\omega_d^2}$$

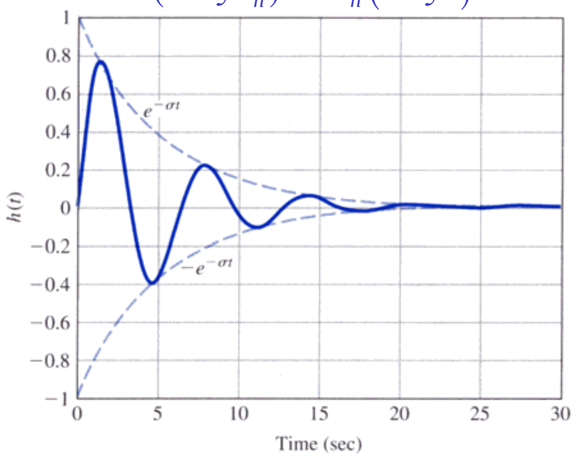
$$\sigma=\zeta\omega_n,\omega_d=\omega_n\sqrt{1-\zeta^2}$$



Time Response vs. Poles

Complex poles:

$$H(s)=\frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)} \rightarrow h(t)=\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\sigma t}\sin(\omega_d t)$$



Impulse Response

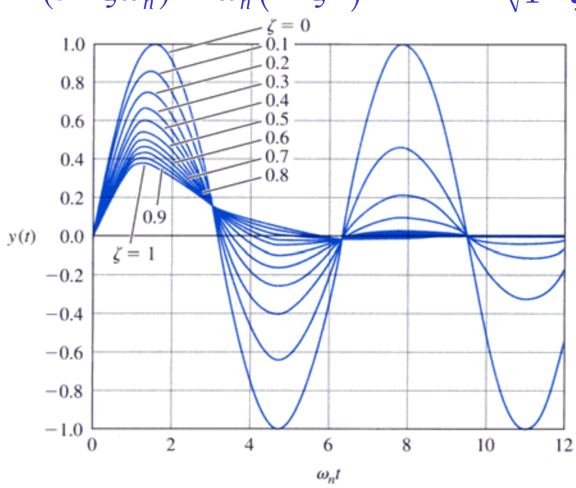
$\sigma > 0$ Stable

$\sigma < 0$ Unstable

Time Response vs. Poles

Complex poles:

$$H(s)=\frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)} \rightarrow h(t)=\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\sigma t}\sin(\omega_d t)$$

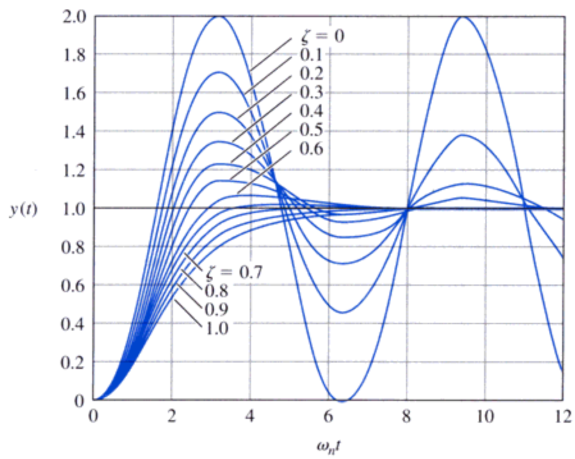


Impulse Response

Time Response vs. Poles

Complex poles:

$$Y(s)=\frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}\frac{1}{s}\rightarrow y(t)=1-e^{-\sigma t}\left[\cos(\omega_d t)+\frac{\sigma}{\omega_d}\sin(\omega_d t)\right]$$



Step Response

Time Response vs. Poles

Complex poles:

$$H(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$=\frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}$$

CASES:

$\zeta=0: s^2+\omega_n^2$ Undamped

$\zeta<1: (s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)$ Underdamped

$\zeta=1: (s+\omega_n)^2$ Critically damped

$\zeta>1: \left[s+\left(\zeta+\sqrt{\zeta^2-1}\right)\omega_n\right]\left[s+\left(\zeta-\sqrt{\zeta^2-1}\right)\omega_n\right]$ Overdamped

Time Response vs. Poles