ME 343 – Control Systems

Lecture 06

September 4, 2009

Impulse Response

Dirac's delta: $$\int_0^{\infty} u(\tau)\delta(t-\tau)d\tau = u(t)$$

Integration is a limit of a sum

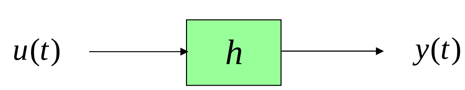
$\Downarrow$

u(t) is represented as a sum of impulses

By superposition principle, we only need unit impulse response

$h(t-\tau)$ Response at t to an impulse applied at τ of amplitude $u(\tau)$

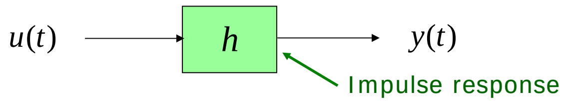
System Response:



$$y(t) = \int_0^{\infty} u(\tau)h(t-\tau)d\tau$$

Impulse Response

t-domain:

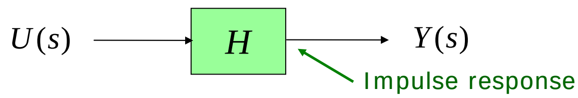


$$y(t) = \int_0^\infty u(\tau)h(t-\tau)d\tau \qquad u(t) = \delta(t) \Rightarrow y(t) = h(t)$$

The system response is obtained by convolving the input with the impulse response of the system.

Convolution: $\mathcal{L}\{\int_0^\infty u(\tau)h(t-\tau)d\tau\} = H(s)U(s)$

s-domain:

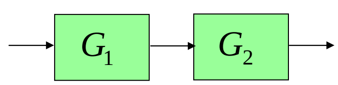


$$Y(s) = H(s)U(s) \qquad u(t) = \delta(t) \Rightarrow U(s) = 1 \Rightarrow Y(s) = H(s)$$

The system response is obtained by multiplying the transfer function and the Laplace transform of the input.

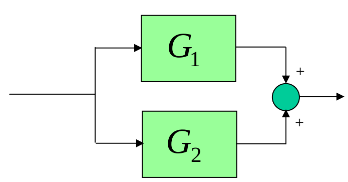
Block Diagrams

Series:



$$G = G_1 G_2$$

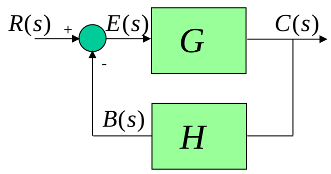
Parallel:



$$G = G_1 + G_2$$

Block Diagrams

Negative Feedback:



R	Reference input
$E = R - B$	Error signal
$C = GE$	Output
$B = HC$	Feedback signal

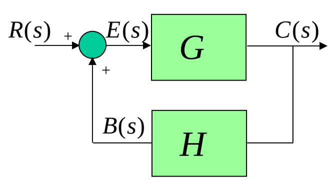
$$C = GR - GHC \Rightarrow (1+GH)C = GR \Rightarrow \frac{C}{R} = \frac{G}{(1+GH)}$$

$$E = R - HGE \Rightarrow (1+GH)E = R \Rightarrow \frac{E}{R} = \frac{1}{(1+GH)}$$

Rule: Transfer Function=Forward Gain/(1+Loop Gain)

Block Diagrams

Positive Feedback:



R	Reference input
$E = R + B$	Error signal
$C = GE$	Output
$B = HC$	Feedback signal

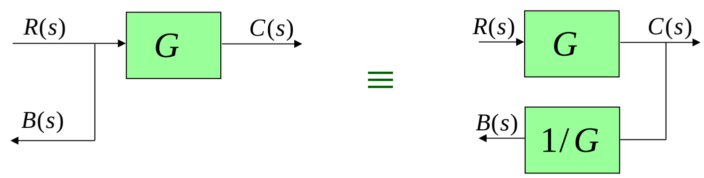
$$C = GR + GHC \Rightarrow (1-GH)C = GR \Rightarrow \frac{C}{R} = \frac{G}{(1-GH)}$$

$$E = R + HGE \Rightarrow (1-GH)E = R \Rightarrow \frac{E}{R} = \frac{1}{(1-GH)}$$

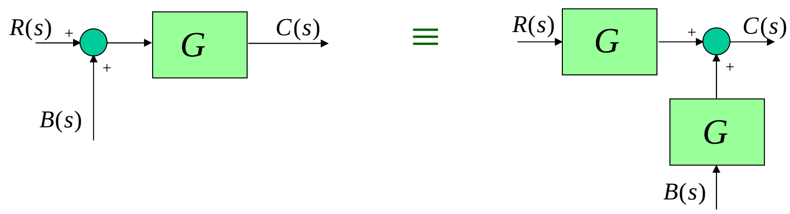
Rule: Transfer Function=Forward Gain/(1-Loop Gain)

Block Diagrams

Moving through a branching point:

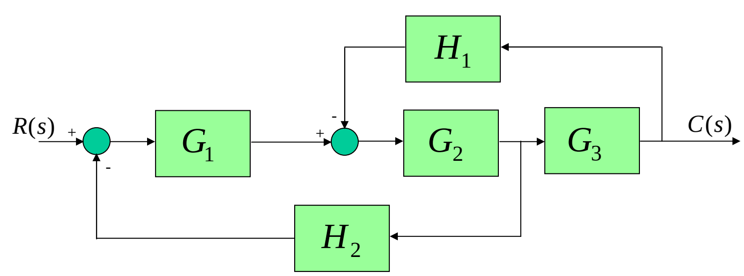


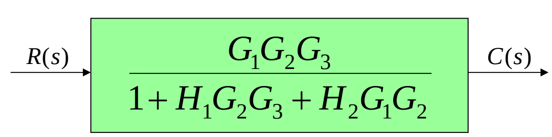
Moving through a summing point:



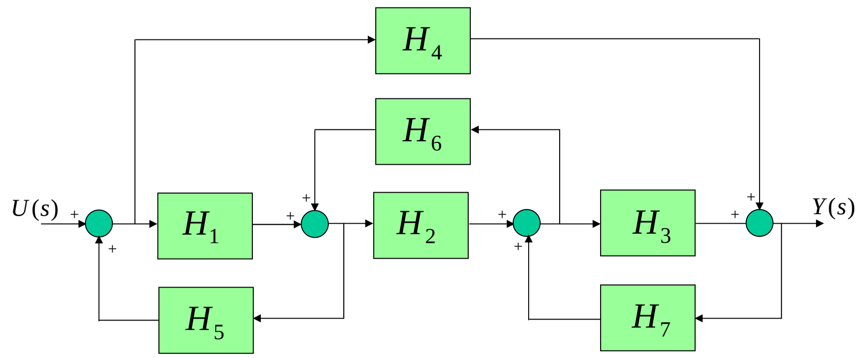
Block Diagrams

Example:

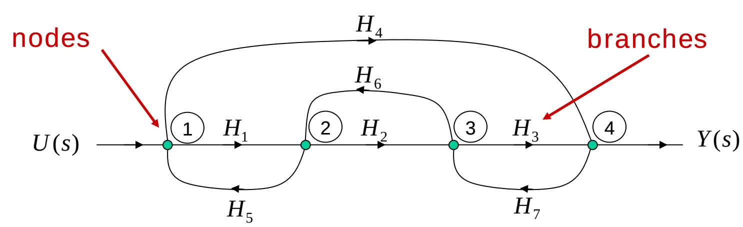




Mason's Rule



Signal Flow Graph



Mason's Rule

Path: a sequence of connected branches in the direction of the signal flow without repetition
Loop: a closed path that returns to its starting node
Forward path: connects input and output

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{\Delta}\sum_i G_i \Delta_i$$

G_i = gain of the ith forward path

Δ = the system determinant

$= 1 - \sum$ (all loop gains)

$+ \sum$ (gain products of all possible two loops that do not touch)

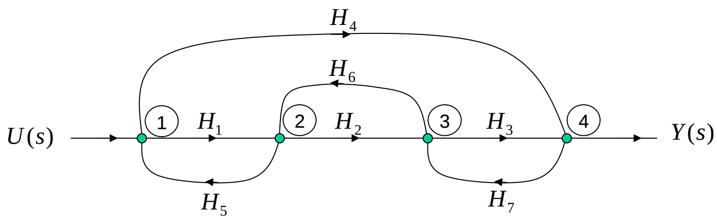
$- \sum$ (gain products of all possible three loops that do not touch)

$+ \cdots$

Δ_i = value of Δ for the part of the graph that does not touch the ith forward path

Mason's Rule

Example:



$$\frac{Y(s)}{U(s)} = \frac{H_1H_2H_3 + H_4 - H_4H_2H_6}{1 - H_1H_5 - H_2H_6 - H_3H_7 - H_4H_7H_6H_5 + H_1H_5H_3H_7}$$