ME 343 – Control Systems

Lecture 02

August 26, 2009

What is Control for?

Feedback control can help:

$\left.\begin{array}{l}\text{• reference following (tracking)}\\ \text{• equilibrium regulation}\\ \text{• disturbance rejection}\\ \text{• stability}\\ \text{• performance}\end{array}\right\}$ CHANGING DYNAMIC BEHAVIOR

First step in this design process: DYNAMIC MODEL

Model Classification

Model Representation $\longleftrightarrow$ **Control Technique**

Spatial Dependence	Lumped parameter system $f = f(t)$ Ordinary Diff. Eq. (ODE)	Distributed parameter system $f = f(t,x)$ Partial Diff. Eq. (PDE)
Linearity	Linear	Nonlinear
Temporal Representation	Continuous-time	Discrete-time
Domain Representation	Time	Frequency

Spatial Dependence

Distributed Parameter Systems

PDE $\boxed{n_e = n_e(t,r)}$

$$\frac{\partial n_e}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r} r\left(D_n \frac{\partial n_e}{\partial r} - n_e V_n\right) + S_n(t,r)$$

$$\left.\frac{\partial n_e}{\partial r}\right|_{r=0} = 0 \qquad n_e\big|_{r=a} = n_e^a$$

Lumped Parameter Systems

ODE $\boxed{n_e = n_e(t)}$

$$\frac{dn_e}{dt} = -\frac{1}{\tau_e} n_e + S_n(t)$$

$$n_e(0) = n_{e0}$$

Control: **Interior** **Boundary**

1. Reduction
2. Keep the PDE representation (problem specific)

Linearity: **Nonlinear/Linear**

Linear/Nonlinear Distributed Parameter Control

Linear/Nonlinear Lumped Parameter Control

Linearity

Nonlinear (ODE) Systems

$$y^{(n)} = g\left(t, y^{(n-1)}, \cdots, y, u^{(m-1)}, \cdots, u\right)$$

Linear (ODE) Systems

$$y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_0 y = b_{m-1} u^{(m-1)} + \cdots + b_0 u$$

Non-Autonomous $\quad \dot{x} = f(t,x,u)$, $\quad y = h(t,x,u)$

Autonomous $\quad \dot{x} = f(x,u)$, $\quad y = h(x,u)$

$x = [x_1 \ \cdots \ x_n]^T$ states
u input
y output

$\dot{x} = A(t)x + B(t)u$, $\quad y = C(t)x + D(t)u$ $\quad$ LTV

$\dot{x} = Ax + Bu$, $\quad y = Cx + Du$ $\quad$ LTI

1 Linearization → Linear Control

2 Keep the nonlinearities → Nonlinear Control

Output/State Feedback

Estimation: How to estimate states from input/output?

Linearity

Particular type of nonlinearities: Constraints

LTI $\quad \dot{x} = Ax + Bu$, $\quad y = Cx + Du$

$u, y \rightarrow sat(u), sat(y)$ input/output constraints

$\underline{x}_i < x_i < \overline{x}_i$ state constraints

1 A priori → Constraint is considered for control design

2 A posteriori → Constraint is NOT considered for control design

Anti-windup Techniques

Temporal Representation

Continuous-time Systems | **Discrete-time Systems**

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = b_{m-1}u^{(m-1)} + \cdots + b_0 u$$

Sampled-Data Systems

$$y[kT] + a_{k-1}y[(k-1)T] + \cdots + a_n y[(k-n)T] = b_o u[kT] + \cdots + b_m u[(k-m)T]$$

Sampling Time

LTI
$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$$x[k+1] = Ax[k] + Bu[k]$$
$$y[k] = Cx[k] + Du[k]$$
LTI

System Identification: How to create models from data?

Fault Detection and Isolation: How to detect faults from data?

$\boxed{\text{System Identification}}$

Domain Representation

Continuous-time Systems | **Discrete-time Systems**

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = b_{m-1}u^{(m-1)} + \cdots + b_0 u$$

$$y[k] + a_{k-1}y[k-1] + \cdots + a_n y[k-n] = b_o u[k] + \cdots + b_m u[k-m]$$

$\boxed{\text{Modern Control}}$

$\boxed{\text{Classical Control}}$

Laplace Transform

Z Transform

$$(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)Y(s) = (b_{m-1}s^{m-1} + \cdots + b_1 s + b_0)U(s)$$

$$(1 + a_1 z + \cdots + a_{n-1}z^{n-1} + a_n z^{-n})Y(s) = (b_o + \cdots + b_m z^{-m})U(s)$$

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$
$$= K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$$T(z) = \frac{Y(z)}{U(z)} = \frac{b_o + \cdots + b_m z^{-m}}{1 + a_1 z + \cdots + a_{n-1}z^{n-1} + a_n z^{-n}}$$
$$= K\frac{(z-z_1)(z-z_2)\cdots(z-z_m)}{(z-p_1)(z-p_2)\cdots(z-p_n)}$$

T	TF
p_i	poles
z_i	zeros
K	gain

$s = j\omega$

$z = e^{j\omega T}$

$$T(j\omega) = |T(j\omega)|e^{\angle T(j\omega)}$$

Frequency Response

$$T(e^{j\omega T}) = |T(e^{j\omega T})|e^{\angle T(e^{j\omega T})}$$

Optimality

Continuous-time Systems

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = b_{m-1}u^{(m-1)} + \cdots + b_0 u$$

$$\dot{x} = A(t)x + B(t)u$$
$$y = C(t)x + D(t)u$$

Discrete-time Systems

$$y[k] + a_{k-1}y[k-1] + \cdots + a_n y[k-n] = b_o u[k] + \cdots + b_m u[k-m]$$

$$x_{k+1} = A_k x_k + B_k u_k$$
$$y_k = C_k x_k + D_k u_k$$

$$\min_{u_k} \frac{1}{2} x_N^T S_N x_N + \frac{1}{2}\sum_{k=0}^{N-1}\left(x_k^T Q_k x_k + u_k^T R_k u_k\right)$$

$$\min_{u(t)} \frac{1}{2} x^T(T) S_T x(T) + \frac{1}{2}\int_0^T \left(x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)\right)dt$$

Optimal Control

Model Uncertainties

How to deal with uncertainties in the model?

A Non-model-based control

- PID
- Extremum Seeking

B Model-based control

- 1 Robust Control → Design for a family of plants
- 2 Adaptive Control → Update model (controller) in real time

Robust & Adaptive Control

Model Classification

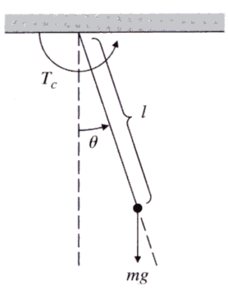
Model Representation ⟷ **Control Technique**

Spatial Dependence	Lumped parameter system $f = f(t)$ Ordinary Diff. Eq. (ODE)	Distributed parameter system $f = f(t, x)$ Partial Diff. Eq. (PDE)
Linearity	Linear	Nonlinear
Temporal Representation	Continuous-time	Discrete-time
Domain Representation	Time	Frequency

Dynamic Model

MECHANICAL SYSTEM:

$T = I\alpha$ Newton's law



damping coefficient

$$I\alpha = -lmg\sin\theta - b\omega + T_c$$

$\omega = \dot{\theta}$ angular velocity

$\alpha = \dot{\omega} = \ddot{\theta}$ angular acceleration

$I = ml^2$ moment of inertia

$$\ddot{\theta} = -\frac{b}{ml^2}\dot{\theta} - \frac{g}{l}\sin\theta + \frac{T_c}{ml^2}$$

Which are the equilibrium points when T_c=0?

Stable

At equilibrium: $\ddot{\theta} = \dot{\theta} = 0 \Rightarrow 0 = -\frac{g}{l}\sin\theta \Rightarrow \theta = 0, \pi$

Unstable

Open loop simulations: pend_par.m, pendol01.mdl