ME 343 – Control Systems

Lecture 01

August 24, 2009

ME 343 Control Systems

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Office hours: Monday/Wednesday/Friday after class or by arrangement

Web page: http://www.lehigh.edu/~eus204/Teaching/ME343/ME343.html

E-mail list: Make sure to be in the mailing list!!!

Class time: Monday/Wednesday/Friday 11:10AM to 12:00PM, Mohler 110

What do I expect you to know?

Prerequisites: ME 242 or ME 245 or ECE 125 (MATH 205 + MECH 102)

- Mathematics
  - Solution of sets of linear equations
    - $Ax=b$ equations for vector x given matrix A and vector b
  - Solution of constant coefficient linear ordinary differential equations
    - Laplace transform introduction
    - Initial conditions and forced response
  - Complex analysis
    - Numbers, poles, zeros
- Dynamic Systems
  - First-order vector ordinary differential equations
  - Linearity and linearization
  - Laplace transform
  - Transfer function and frequency response

I will help you as we go by doing examples

which should refresh your memory

How will the course be examined?

Weekly homework (ten assignments)	15%
Three in-class midterms (See schedule on web page)	55%
Final exam (See schedule on web page)	30%

Homework:

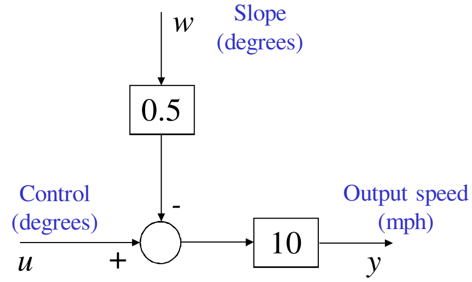
- Hand in one week before due date
- NO lateness (must be returned on due date before class)
- Homework assignments will be graded based on level of effort
- Solutions of ALL the problems on the web site
- Solutions have to be comprehensible to be marked highly
- Follow the Homework Guidelines posted in the web page

What is control?

For any analysis we need a mathematical MODEL of the system

Model → Relation between gas pedal and speed:
10 mph change in speed per each degree rotation of gas pedal
Disturbance → Slope of road:
5 mph change in speed per each degree change of slope

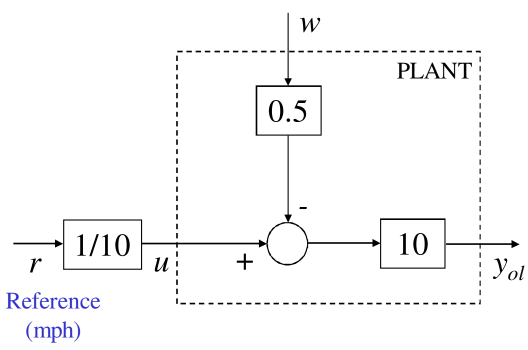
Block diagram for the cruise control plant:



$$y = 10(u - 0.5w)$$

What is control?

Open-loop cruise control:



$$u = \frac{r}{10}$$

$$y_{ol} = 10(u - 0.5w) = 10(\frac{r}{10} - 0.5w) = r - 5w$$

$$e_{ol} = r - y_{ol} = 5w$$

$$e_{ol}[\%] = \frac{r - y_{ol}}{r} = 500\frac{w}{r}$$

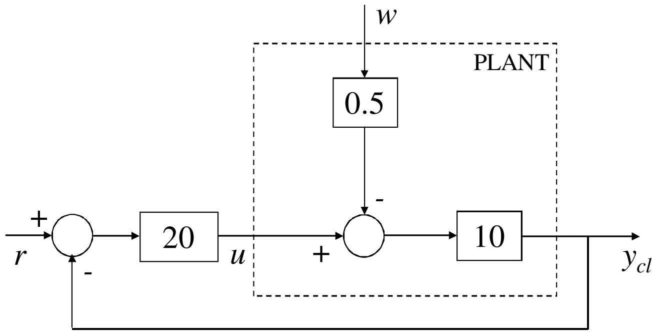
$r = 65, w = 0 \Rightarrow e_{ol} = 0$

$r = 65, w = 1 \Rightarrow e_{ol} = 5\text{mph}, e_{ol} = 7.69\%$

OK when:
1- Plant is known exactly
2- There is no disturbance

What is control?

Closed-loop cruise control:



$$u = 20(r - y_{cl})$$

$$y_{cl} = 10(u - 0.5w) = \frac{200}{201} r - \frac{5}{201} w$$

$$e_{cl} = r - y_{cl} = \frac{1}{201} r + \frac{5}{201} w$$

$$e_{cl}[\%] = \frac{r - y_{cl}}{r} = \frac{1}{201} + \frac{5}{201} \frac{w}{r}$$

$$r = 65, w = 0 \Rightarrow e_{cl} = \frac{1}{201} \% = 0.5\%$$

$$r = 65, w = 1 \Rightarrow e_{cl} = \frac{1}{201} + \frac{5}{201} \frac{5}{65} = 0.69\%$$

What is control?

Feedback control can help:

- reference following (tracking)
- disturbance rejection
- changing dynamic behavior

LARGE gain is essential but there is a STABILITY limit

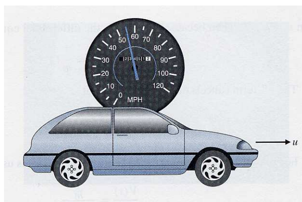
"The issue of how to get the gain as large as possible to reduce the errors due to disturbances and uncertainties without making the system become unstable is what much of feedback control design is all about"

First step in this design process: DYNAMIC MODEL

Dynamic Models

Real world is more complex than static maps → We need Dynamic Models

MECHANICAL SYSTEMS: $F = ma$ Newton's law



$$m\ddot{x} = u - b\dot{x}$$

$v = \dot{x}$ velocity

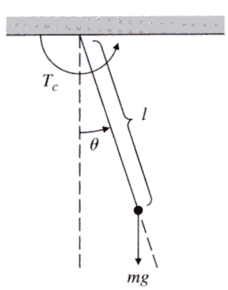
$a = \dot{v} = \ddot{x}$ acceleration

$$\dot{v} + \frac{b}{m} v = \frac{u}{m} \xrightarrow{\text{Laplace Transform}} \frac{V_o}{U_o} = \frac{1/m}{s + b/m}$$

Transfer Function $\frac{d}{dt} \rightarrow s$

Dynamic Models

MECHANICAL SYSTEMS: $F = I\alpha$ Newton's law



$$ml^2\ddot{\theta} = -lmg\sin\theta + T_c$$

$\omega = \dot{\theta}$ angular velocity

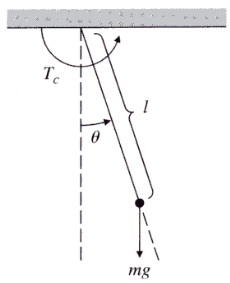
$\alpha = \dot{\omega} = \ddot{\theta}$ angular acceleration

$I = ml^2$ moment of inertia

$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2} \xrightarrow{\sin\theta \approx \theta} \ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$$

Linearization

Dynamic Models



$$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$$

Reduce to first order equations:

$$x_1 = \theta \qquad \dot{x}_1 = x_2$$

$$x_2 = \dot{\theta} \qquad \dot{x}_2 = -\frac{g}{l}x_1 + \frac{T_c}{ml^2}$$

$$x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, u \equiv \frac{T_c}{ml^2} \Rightarrow \dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

State Variable Representation

General case:

$$\dot{x} = Fx + Gu$$

$$y = Hx + Ju$$

Dynamic Models

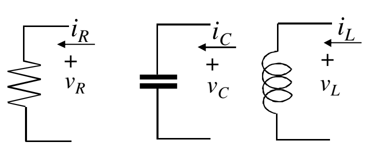
ELECTRICAL SYSTEMS:

Kirchoff's Current Law (KCL):

The algebraic sum of currents entering a node is zero at every instant

Kirchoff's Voltage Law (KVL)

The algebraic sum of voltages around a loop is zero at every instant



Resistors:

$$v_R(t) = Ri_R(t) \Leftrightarrow i_R(t) = Gv_R(t)$$

Capacitors:

$$i_C(t) = C\frac{dv_C(t)}{dt} \Leftrightarrow v_C(t) = \frac{1}{C}\int_0^t i_C(\tau)d\tau + v_C(0)$$

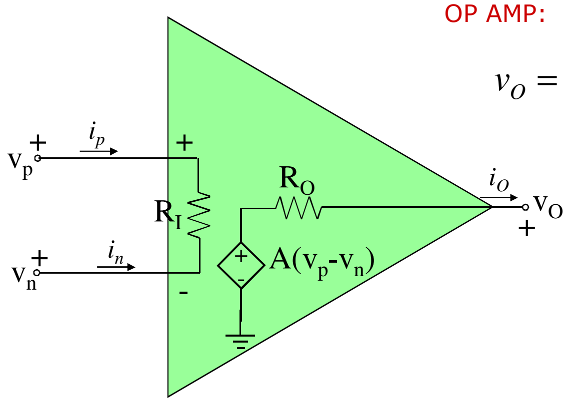
Inductors:

$$v_L(t) = L\frac{di_L(t)}{dt} \Leftrightarrow i_L(t) = \frac{1}{L}\int_0^t v_L(\tau)d\tau + i_L(0)$$

Dynamic Models

ELECTRICAL SYSTEMS:

OP AMP:



$$v_O = A(v_p - v_n), A \to \infty$$

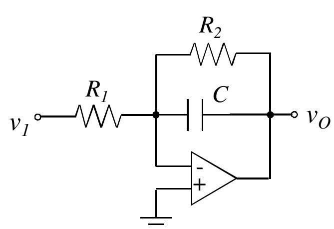
$$v_p = v_n$$

$$i_p = i_n = 0$$

To work in the linear mode we need FEEDBACK!!!

Dynamic Models

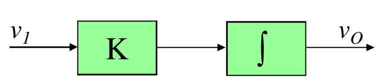
ELECTRICAL SYSTEMS:



KCL:

$$\frac{dv_O}{dt} = \frac{1}{R_2 C} v_O - \frac{1}{R_1 C} v_I$$

$$R_2 = \infty \text{ (OC)} \Rightarrow v_O(t) = v_O(0) - \frac{1}{R_1 C}\int_0^t v_I(\mu)d\mu$$

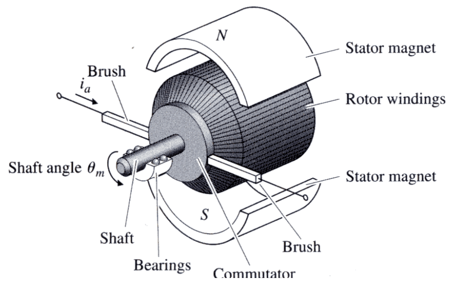


$$K = -\frac{1}{RC}$$

Inverting integrator

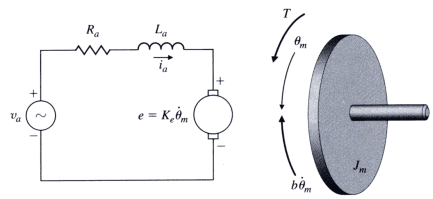
Dynamic Models

ELECTRO-MECHANICAL SYSTEMS: DC Motor



torque → $T = K_t i_a$ ← armature current

emf → $e = K_e \dot{\theta}_m$ ← shaft velocity



(a) (b)

$$J_m \ddot{\theta}_m = -b\dot{\theta}_m + T$$

$$-v_a + R_a i_a + L\frac{di_a}{dt} + e = 0$$

Obtain the State Variable Representation

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Dynamic Models

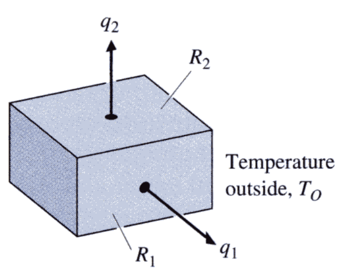
HEAT-FLOW:

Heat Flow, Temperature Difference

$$q = \frac{1}{R}(T_1 - T_2)$$

$$\dot{T} = \frac{1}{C}q$$

Thermal capacitance, Thermal resistance



$$\dot{T}_I = \frac{1}{C_I}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)(T_o - T_I)$$

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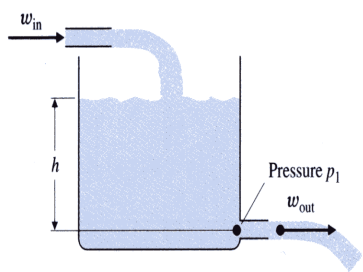
Dynamic Models

FLUID-FLOW: Mass rate → Mass Conservation law

$$\dot{m} = w_{in} - w_{out}$$

Inlet mass flow — Outlet mass flow



$$\dot{m} = \rho A \dot{h} \Rightarrow \dot{h} = \frac{1}{\rho A}(w_{in} - w_{out})$$

A: area of the tank
ρ: density of fluid
h: height of water

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Model Representation ⟷ **Control Technique**

Spatial Dependence	Lumped parameter system $f = f(t)$ Ordinary Diff. Eq. (ODE)	Distributed parameter system $f = f(t,x)$ Partial Diff. Eq. (PDE)
Linearity	Linear	Nonlinear
Temporal Representation	Continuous-time	Discrete-time
Domain Representation	Time	Frequency

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Topics covered:

Continuous Time Control

Classical Control (Transfer Function – Frequency Domain)

- Modeling. ODEs. Linearization.
- Laplace transform. Transfer functions.
- Block diagrams. Mason's Rule.
- Time response specifications.
- Effects of zeros and poles.
- Stability via Routh-Hurwitz.
- Feedback: Disturbance rejection, Sensitivity, Steady-state tracking.
- PID controllers and Ziegler-Nichols tuning procedure.
- Actuator saturation and integrator wind-up.
- Time Delays
- Root locus.
- Frequency response--Bode and Nyquist diagrams.
- Stability Margins.
- Design of dynamic compensators.

Modern Control (State Space – Time Domain)

Discrete-time Control

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Text: *Feedback Control of Dynamic Systems,*
5th Edition, G.F. Franklin, J.D. Powel and A. Emami-Naeini
Prentice Hall 2006.

We will work through Chapters 1-8 in order

What is beyond ME 343 – Control Systems?

ME 389 – CONTROL SYSTEMS LABORATORY (Spring)

ME 350 – ADVANCED TOPICS IN CONTROLS (Spring)

ME 310 PROJECTS

CONTROL EXPERIMENTS

Controls Lab Upgrade
Wind Tunnel

- Inverted Pendulum System

CONTROL PROJECTS

- Microprocessor Chip Temperature Control
- Stabilization of Neoclassical Tearing Modes in Fusion Plasmas via Extremum Seeking
- Tokamak Plasma Equilibrium Controllability Limitations Due to Delays

ROBO-SOCCER

2008/2009 ME 310 Projects

CONTROL EXPERIMENTS

Controls Lab Upgrade

- Design and Construction of Flexible Inverted Pendulum System
- Setup of 3 DOF (degree-of-freedom) helicopter
- Setup of magnetic levitation system
- Design and construction of remotely operated experiments

Wind Tunnel Upgrade

- Design and construction of feedback controller for wing drag minimization

CONTROL PROJECTS

- Advanced control design

ROBO-SOCCER

- Global mapping system (vision systems)
- High-level planning system (AI)
- Holonomic drivetrain and chassis design