ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 36:

- Review – Test 2

ENERGY STORAGE: COMPLIANCE & INERTANCE

Power: $P = ef$

Energy: $E = \int P dt = \int ef\, dt$

Energy Storage Mechanisms

Compliance $\rightarrow$ Potential Energy

Store energy by virtue of a generalized displacement

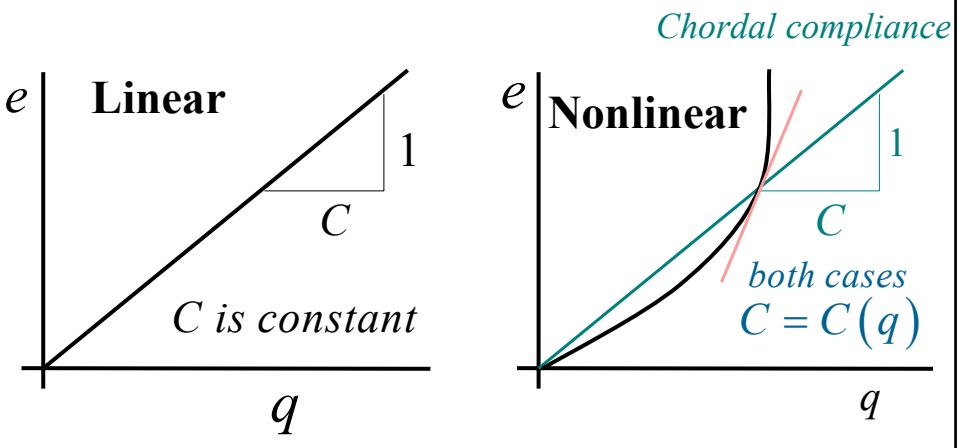
$$E = \int ef\, dt = \int e\dot{q} dt = \int e \frac{dq}{dt} dt = \int e dq, e = e(q)$$

Inertance $\rightarrow$ Kinetic Energy

Store energy by virtue of a generalized momentum

$$E = \int ef\, dt = \int \dot{p} f\, dt = \int \frac{dp}{dt} f\, dt = \int f\, dp, f = f(p)$$

LINEAR vs. NONLINEAR COMPLIANCE



Tangential compliance $\frac{1}{C} = \frac{\partial e(q)}{\partial q}$

CHARACTERISTIC

Given: $V = V(q)$

Find: *characteristic*

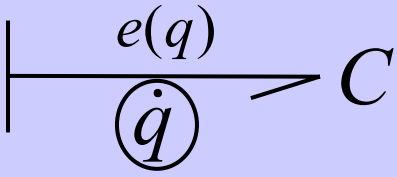
Recall: $V = \int e(q)\, dq$

Thus: $\frac{\partial V}{\partial q} = e(q)$

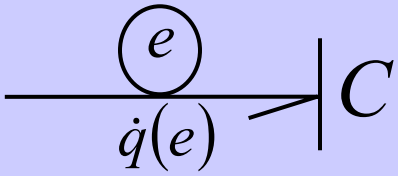
Governing way to obtain $e(q)$

COMPLIANCE - CAUSALITY

Integral



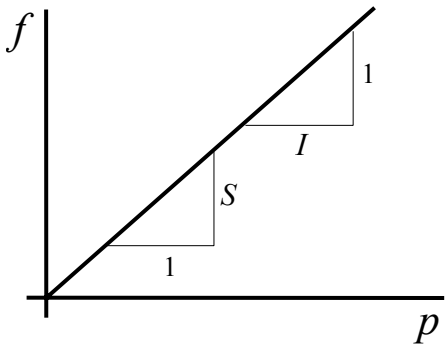
Derivative



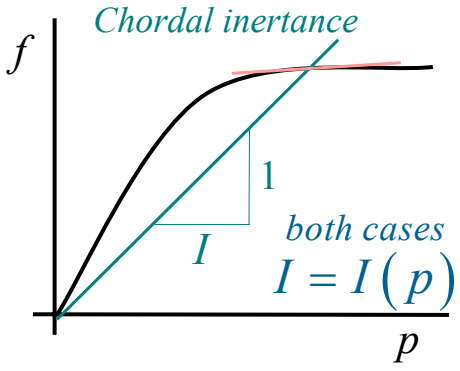
based on
$q = q(e)$

LINEAR vs. NONLINEAR INERTANCE

Linear



Nonlinear



$$\text{Tangential inertance}\ \frac{1}{I} = \frac{\partial f(p)}{\partial p}$$

CHARACTERISTIC

Given: $T = T(p)$

Find: *characteristic*

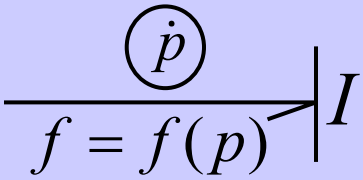
Recall: $T = \int f(p)\, dp$

Thus: $\dfrac{\partial T}{\partial p} = f(p)$

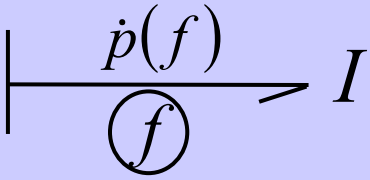
Governing way to obtain $f(p)$

INERTANCE - CAUSALITY

Integral



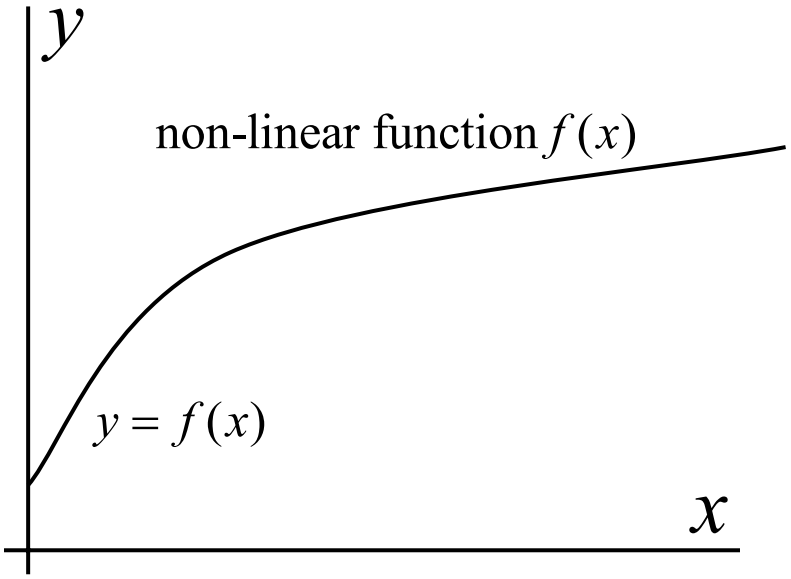
Derivative



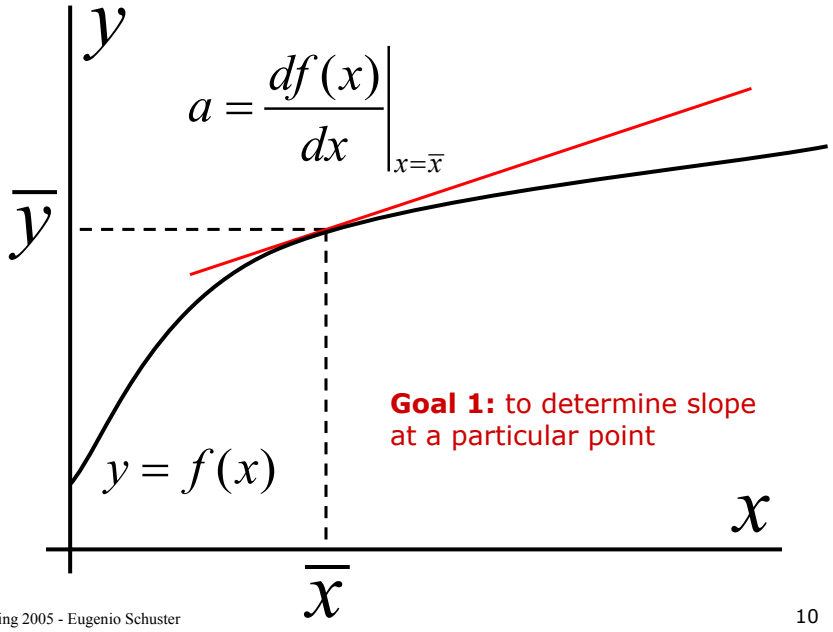
based on

$p = p(f)$

LINEARIZATION – FUNCTION OF ONE VARIABLE

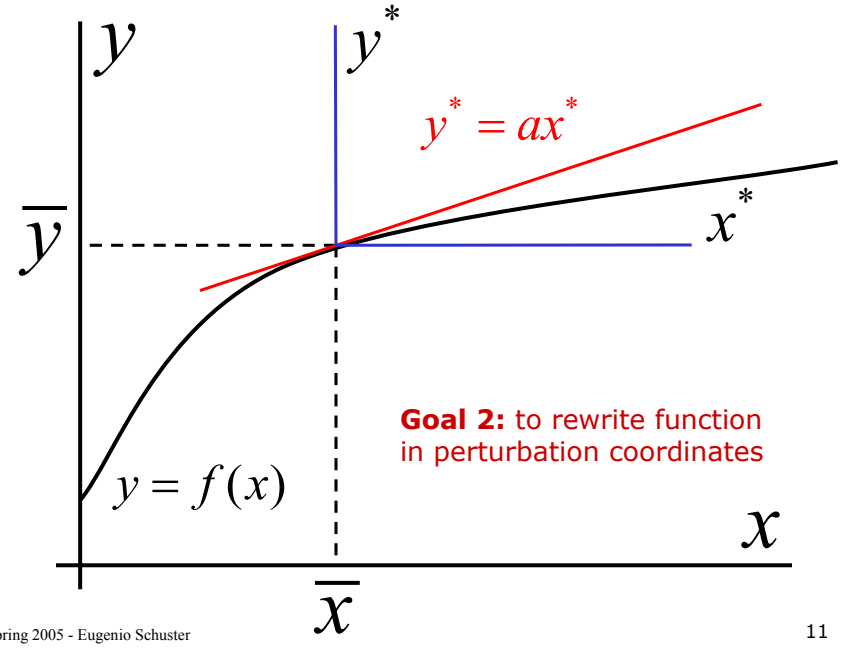


LINEARIZATION – FUNCTION OF ONE VARIABLE



Goal 1: to determine slope at a particular point

LINEARIZATION – FUNCTION OF ONE VARIABLE



LINEARIZATION – FUNCTION OF ONE VARIABLE

non-linear function $f(x)$ $\quad y = f(x)$

nominal value of x is $\overline{x}$

nominal value of y is $\overline{y}$

Nominal solution of $f(x)$ is $f(x)\big|_{x=\overline{x}} = f(\overline{x})$

Or $\quad \overline{y} = f(\overline{x})$

May be given $\overline{x}$ and have to solve for $\overline{y}$ or vice-versa

LINEARIZATION – FUNCTION OF ONE VARIABLE

$$y = f(x)$$

take Taylor series about nominal point

$$y = f(\bar{x}) + \left.\frac{df}{dx}\right|_{x=\bar{x}} (x-\bar{x}) + \left.\frac{d^2 f}{dx^2}\right|_{x=\bar{x}} \frac{(x-\bar{x})^2}{2} + \textit{higher order terms}$$

actual values	**pertubation values**
$x = \bar{x} + x^*$	$x^* = x - \bar{x}$
$y = \bar{y} + y^*$	$y^* = y - \bar{y}$

LINEARIZATION – FUNCTION OF ONE VARIABLE

Taylor series in perturbation coordinates,

$$y - f(\bar{x}) = y - \bar{y} = y^* = \left.\frac{df}{dx}\right|_{x=\bar{x}} x^* + \left.\frac{d^2 f}{dx^2}\right|_{x=\bar{x}} \frac{x^{*2}}{2} + \textit{higher order terms}$$

Keeping only the linear (first order) terms

$$y^* = a x^* \qquad \text{where} \qquad a = \left.\frac{df}{dx}\right|_{x=\bar{x}}$$

LINEARIZATION – FUNCTION OF TWO VARIABLES

$$y = f(x,u)$$

Taylor's expansion (only first order terms):

$$y \simeq \overline{y} + \left.\frac{\partial f}{\partial x}\right|_{\substack{x=\overline{x}\\u=\overline{u}}} (x-\overline{x}) + \left.\frac{\partial f}{\partial u}\right|_{\substack{x=\overline{x}\\u=\overline{u}}} (u-\overline{u})$$

Defining: $x^* = x - \overline{x}, \quad y^* = y - \overline{y}, \quad u^* = u - \overline{u}$

We can write:

$$y^* \simeq c_x x^* + c_u u^*; \qquad c_x = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=\overline{x}\\u=\overline{u}}}; \qquad c_u = \left.\frac{\partial f}{\partial u}\right|_{\substack{x=\overline{x}\\y=\overline{y}}}$$

LINEARIZATION – FIRST ORDER ODEs

Start non - linear differential equation

(Single First Order Diff - E - Q)

$$\frac{dx}{dt} = f\left(x(t), u(t)\right)$$

Total, Nominal and Pertubation Variables

$$x(t) = \overline{x} + x^*(t)$$

$$u(t) = \overline{u} + u^*(t)$$

LINEARIZATION – FIRST ORDER ODEs

Find nominal solution(s)

$$f\left(\bar{x},\bar{u}\right)=0$$

Linearize function

$$\bar{a}=\left.\frac{\partial f}{\partial x}\right|_{\substack{x=\bar{x}\\u=\bar{u}}} \qquad \bar{b}=\left.\frac{\partial f}{\partial u}\right|_{\substack{x=\bar{x}\\u=\bar{u}}}$$

Linearize differential equation

$$\frac{dx^*}{dt}\approx \bar{a}x^*+\bar{b}u^*$$

LINEARIZATION – nth ORDER ODEs

State Variable Representation:

$$\frac{d\boldsymbol{x}}{dt}=\boldsymbol{f}\left(\boldsymbol{x},\boldsymbol{u}\right)$$

$$\boldsymbol{x}=\begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix},\boldsymbol{u}=\begin{bmatrix}u_1\\u_2\\\vdots\\u_m\end{bmatrix},\boldsymbol{f}=\begin{bmatrix}f_1\\f_2\\\vdots\\f_n\end{bmatrix}$$

LINEARIZATION – nth ORDER ODEs

State Variable Representation:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$

The equilibrium solution is given by

$$\bar{\boldsymbol{x}}, \bar{\boldsymbol{u}}: \quad 0 = \boldsymbol{f}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{u}})$$

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} f_1(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n, \bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m) \\ f_2(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n, \bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m) \\ \vdots \\ f_n(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n, \bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m) \end{bmatrix}$$

LINEARIZATION – nth ORDER ODEs

We define the perturbation variables as

$$\boldsymbol{x} = \bar{\boldsymbol{x}} + \boldsymbol{x}^*$$

$$\boldsymbol{u} = \bar{\boldsymbol{u}} + \boldsymbol{u}^*$$

Then we can write

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$

as (by Taylor series expansion)

$$\frac{d\boldsymbol{x}}{dt} = \cancel{\frac{d\bar{\boldsymbol{x}}}{dt}} + \frac{d\boldsymbol{x}^*}{dt} = \cancel{\boldsymbol{f}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{u}})} + \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right|_{\substack{\boldsymbol{x}=\bar{\boldsymbol{x}} \\ \boldsymbol{u}=\bar{\boldsymbol{u}}}} \boldsymbol{x}^* + \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}\right|_{\substack{\boldsymbol{x}=\bar{\boldsymbol{x}} \\ \boldsymbol{u}=\bar{\boldsymbol{u}}}} \boldsymbol{u}^* + HOT$$

LINEARIZATION – nth ORDER ODEs

After neglecting the HOT, we have

$$\frac{d\boldsymbol{x}^*}{dt} = A\boldsymbol{x}^* + B\boldsymbol{u}^*$$

$$A = \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right|_{\substack{\boldsymbol{x}=\bar{\boldsymbol{x}}\\ \boldsymbol{u}=\bar{\boldsymbol{u}}}} = \left[\begin{array}{cccc} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{array}\right]_{\substack{\boldsymbol{x}=\bar{\boldsymbol{x}}\\ \boldsymbol{u}=\bar{\boldsymbol{u}}}}, \quad B = \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}\right|_{\substack{\boldsymbol{x}=\bar{\boldsymbol{x}}\\ \boldsymbol{u}=\bar{\boldsymbol{u}}}} = \left[\begin{array}{cccc} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m} \end{array}\right]_{\substack{\boldsymbol{x}=\bar{\boldsymbol{x}}\\ \boldsymbol{u}=\bar{\boldsymbol{u}}}}$$

LINEARIZATION – PHASE PLANE

Local analysis around critical point → linearization

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Linearization around an equilibrium

The solution of the linearized equation can be written as:

eigenvector eigenvalue

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = c_1 \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} e^{\lambda_1 t} + c_2 \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} e^{\lambda_2 t}$$

LINEARIZATION – PHASE PLANE

Eigenvalues and Eigenvectors:

$$Av = \lambda v$$

Solution?:

$$Av = \lambda Iv$$

$$(\lambda I - A)v = 0$$

A non-trivial solution requires

$$\det(\lambda I - A) = 0 \quad \Rightarrow \lambda$$

Known λ we can compute v from

$$(\lambda I - A)v = 0 \quad \Rightarrow v$$

LINEARIZATION – STABILITY

$$\frac{dx}{dt} = f(x) \quad \Rightarrow \quad \frac{dx^*}{dt} = Ax^* \quad \Rightarrow \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = c_1 \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} e^{\lambda_1 t} + c_2 \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} e^{\lambda_2 t}$$

Linearization around an equilibrium

Solution

If the linearized system is strictly stable (i.e., if all eigenvalues of A are strictly in the left-half complex plane), then the equilibrium point is asymptotically stable (for the actual nonlinear system)

If the linearized system is unstable (i.e., if at least one eigenvalue of A is strictly in the right-half complex plane), then the equilibrium point is unstable (for the actual nonlinear system)

If the linearized system is marginally stable (i.e., if all eigenvalues of A are in the left-half complex plane, but at least one of them is on the imaginary axis), then one cannot conclude anything from the linear approximation (the equilibrium point may be stable, asymptotically stable or unstable for the actual nonlinear system)

LINEARIZATION – 2D PHASE PLANE

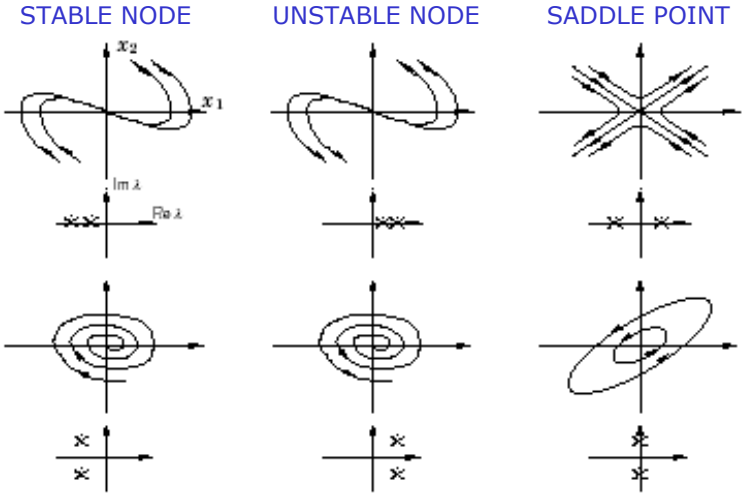
Linear System:

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = c_1 \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} e^{\lambda_1 t} + c_2 \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} e^{\lambda_2 t}$$

Classification of the equilibrium

If λ_1 and λ_2 are negative real	STABLE NODE
If λ_1 and λ_2 are positive real	UNSTABLE NODE
If λ_1 is positive real and λ_2 is negative real	SADDLE POINT
If λ_1 and λ_2 are imaginary conjugates	CENTER POINT
If λ_1 and λ_2 are complex conjugates	
Real part negative	STABLE FOCUS
Real part positive	UNSTABLE FOCUS

LINEARIZATION – 2D PHASE PLANE



SIMPLE CIRCUITS

Creation of simple multi-element bond graphs

Key Concepts:

- Each element has two sides
- Elements interconnect by simple junctions
- Graph simplified by removal of ground
- Remove two ported multiports

Slight Differences:

- Electrical and Fluid very similar
- Mechanical slightly different

ELECTRIC CIRCUITS

Identify all nodes and all elements
(*there are two nodes for each element*)

1. Represent each electrical junction (node) with a 0 junction
   - Represent each element with I, R or C
     (*each element gets a bond to it*)
2. Connect each element's bond to a 1 junction
   - Connect each 1 junction to 2 0 junctions
3. Discard all bonds for $e = 0$ (ground) and $i = 0$
4. Eliminate all junctions with only two bonds

FLUID CIRCUITS

Identify all nodes and all elements
(*there are two nodes for each element*)

1. Represent each node with a 0 junction
   - Represent each element with I, R or C
     (*each element gets a bond to it*)
2. Connect each element's bond to a 1 junction
   - Connect each 1 junction to 2 0 junctions
3. Discard all bonds for $e = 0$ (ground) and $i = 0$
4. Eliminate all junctions with only two bonds

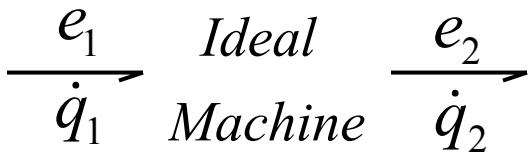
Just like electrical circuits!

MECHANICAL CIRCUITS

Identify all nodes and all elements
(*there are two nodes for each element*)

1. Represent each mech. junction with a 1 junction
   - **Place all I's on 1 junctions**
2. Connect each R, C elements on a 0 junction
   - Connect each 0 junction to 2 1 junctions
3. Coalesce bonded junctions of same type
4. Add in S, S_e S_f as needed.
5. Discard all bonds for $e = 0$ (ground) and $f = 0$
6. Eliminate all junctions with only two bonds

IDEAL MACHINES



An ideal machine is a two port device that transmits work from one port to the other

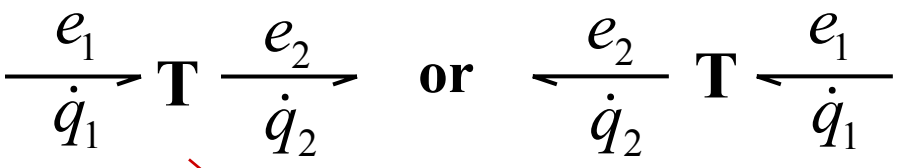
- No energy is stored, generated or dissipated
- Entropy is not generated
- Can be run in either direction

Power Conservation

$$e_1\dot{q}_1 = e_2\dot{q}_2$$

IDEAL MACHINES - TRANSFORMER

Defining Condition: $\dot{q}_2 = T\dot{q}_1$



Transformer Modulus (constant)

The modulus of the Transformer, T, is defined as the ratio of the generalized velocity or flow on the bond with the outward power arrow to the generalized velocity or flow on the bond with the inward power convention arrow

IDEAL MACHINES - TRANSFORMER

Combining the Ideal Machine condition:

$$e_1\dot{q}_1 = e_2\dot{q}_2$$

with the Transformer condition:

$$\dot{q}_2 = T\dot{q}_1$$

yields an additional condition:

$$e_1 = Te_2$$

$$T = \frac{\dot{q}_2}{\dot{q}_1} = \frac{e_1}{e_2}$$

The ratio of the generalized forces of an ideal tranformer equals the inverse of the ratio of the respective generalized velocities

IDEAL MACHINES - GYRATORS

Defining Condition: $e_2 = G\dot{q}_1$

$$\frac{e_1}{\dot{q}_1} \rightarrow \mathbf{G} \rightarrow \frac{e_2}{\dot{q}_2} \quad \mathbf{or} \quad \frac{e_2}{\dot{q}_2} \leftarrow \mathbf{G} \leftarrow \frac{e_1}{\dot{q}_1}$$

Gyrator Modulus (constant)

The modulus of the Gyrator, G, is defined as the ratio of the effort on one of the bonds – either one – to the flow on the other bond

IDEAL MACHINES - GYRATOR

Combining the Ideal Machine condition:

$$e_1\dot{q}_1 = e_2\dot{q}_2$$

with the Gyrator condition:

$$e_2 = G\dot{q}_1$$

yields an additional condition:

$$e_1 = G\dot{q}_2$$

$$G = \frac{e_2}{\dot{q}_1} = \frac{e_1}{\dot{q}_2}$$

The ratio of the generalized forces of an ideal tranformer equals the inverse of the ratio of the respective generalized velocities

CASCADED TRANSFORMER

$$\frac{e_1}{\dot{q}_1} \quad \mathbf{T}_1 \quad \frac{e_2}{\dot{q}_2} \quad \mathbf{T}_2 \quad \frac{e_3}{\dot{q}_3}$$

The definition of transformer requires that

$$\dot{q}_3 = T_2\dot{q}_2 = T_2T_1\dot{q}_1$$

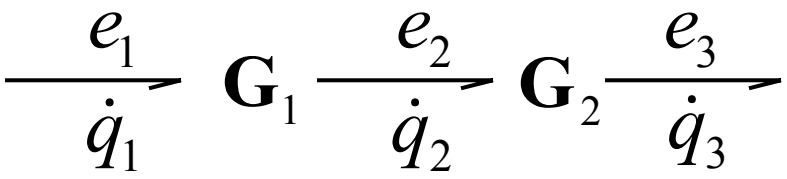
$$e_1 = T_1e_2 = T_1T_2e_3$$

Then,

$$\frac{e_1}{\dot{q}_1} \quad \mathbf{T} \quad \frac{e_3}{\dot{q}_3}$$

$$T = T_1T_2$$

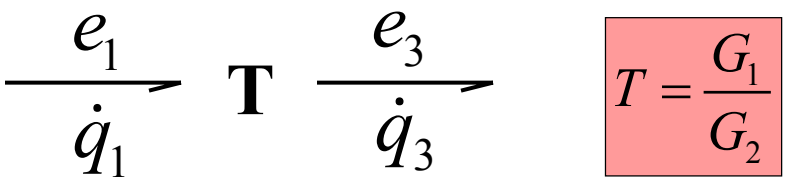
CASCADED GYRATORS



The definition of transformer requires that

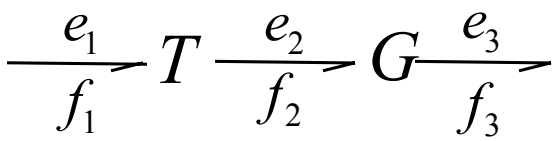
$$\dot{q}_3 = \frac{e_2}{G_2} = \frac{G_1}{G_2}\dot{q}_1 \quad e_1 = G_1\dot{q}_2 = \frac{G_1}{G_2}e_3$$

Then,



$$T = \frac{G_1}{G_2}$$

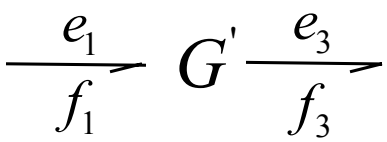
TRANSFORMER – GYRATORS PAIRS



Gyrator: $e_3 = Gf_2$ Transformer: $f_2 = Tf_1 \Rightarrow e_3 = GTf_1$

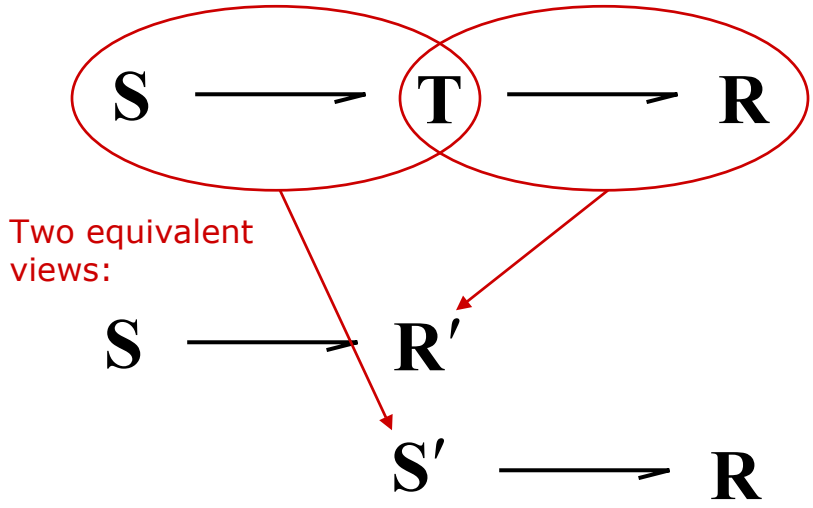
Transformer: $e_1 = Te_2$ Gyrator: $e_2 = Gf_3 \Rightarrow e_1 = TGf_3$

Then,

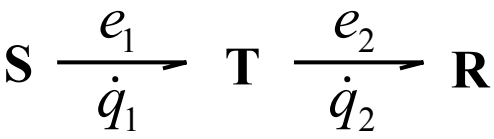


with $G' = TG$

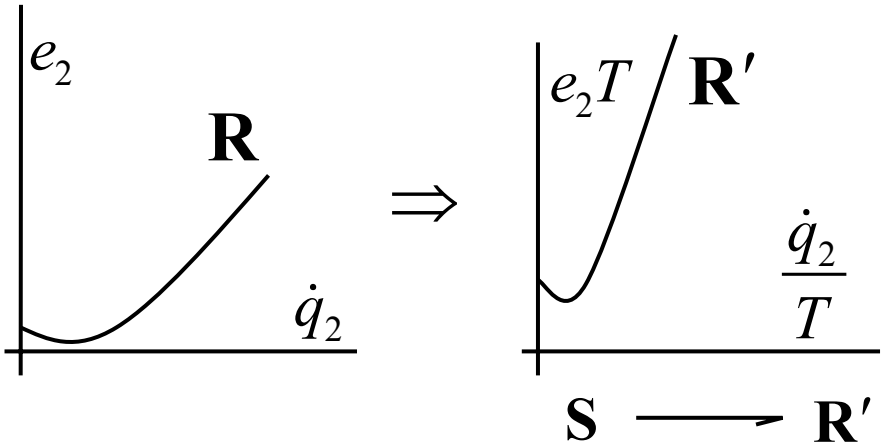
SOURCE - TRANSFORMER – RESISTOR



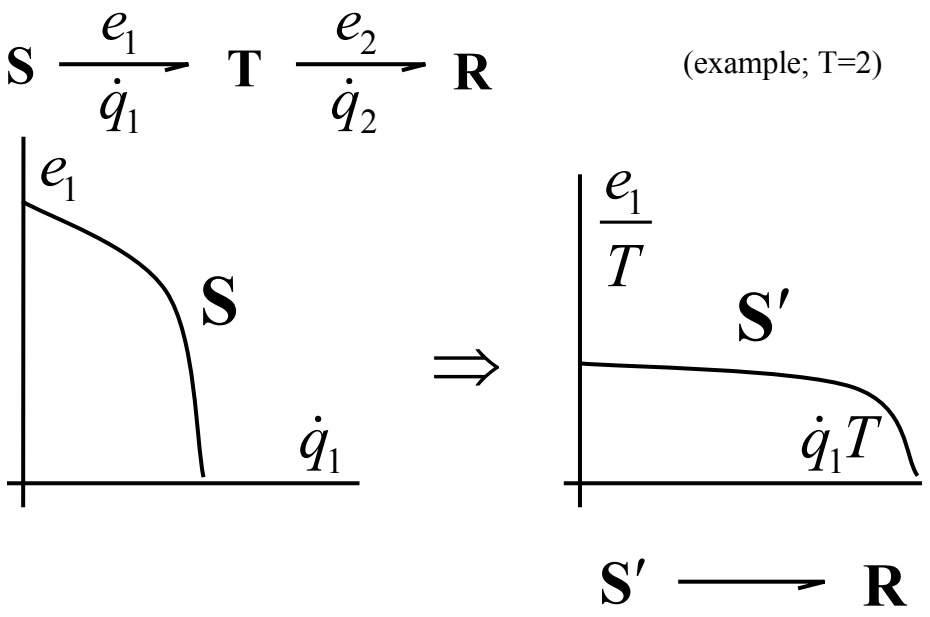
EFFECTIVE RESISTANCE (AS SEEN BY SOURCE)



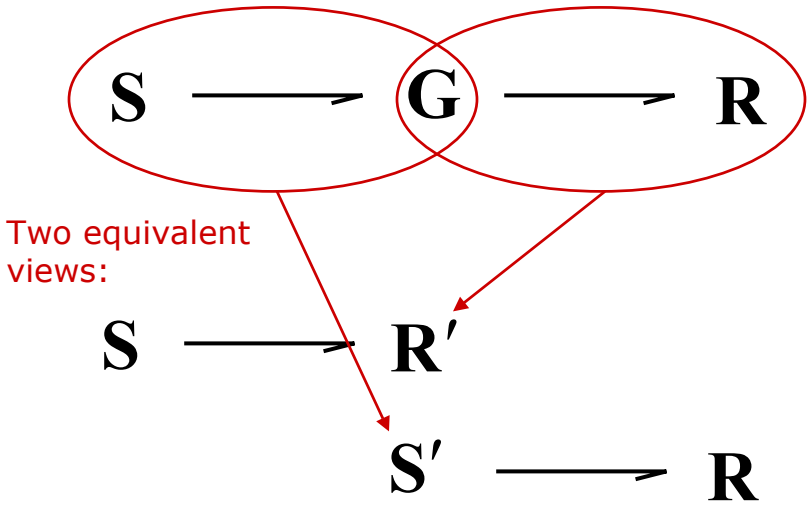
(example; T=2)



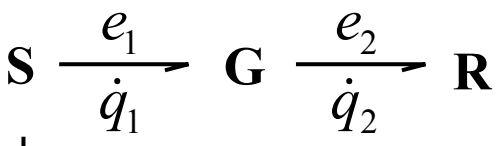
EFFECTIVE SOURCE (AS SEEN BY RESISTANCE)



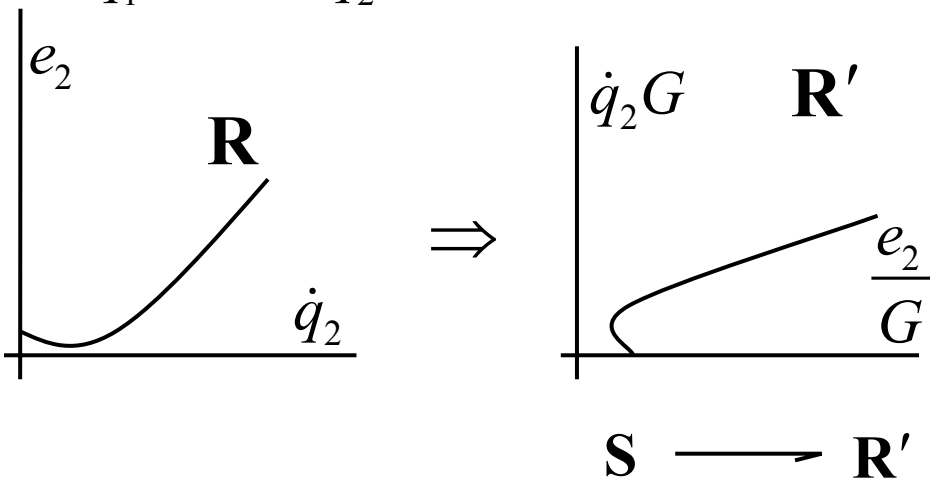
SOURCE - GYRATOR – RESISTOR



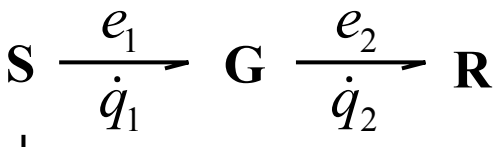
EFFECTIVE RESISTANCE (AS SEEN BY SOURCE)



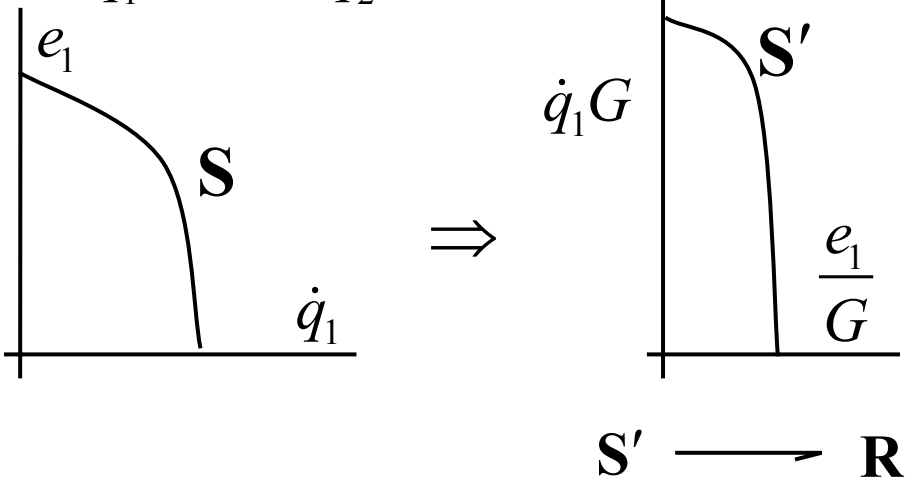
(example; G=.5)



EFFECTIVE SOURCE (AS SEEN BY RESISTANCE)



(example; G=2)



MECHANICAL CONSTRAINTS

Kinematic Constraints: Govern details on how efforts and flows are related

Two Approaches:

1. Write Displacement constraints and then take derivative to get velocity constraints
2. Write velocity constraint directly

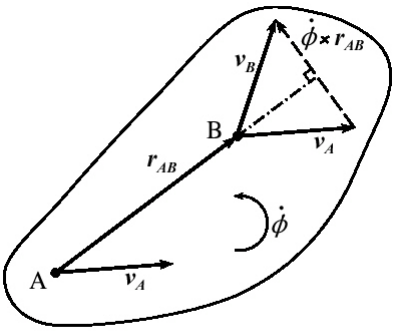
Either approach can be used
Use the one that is more natural

MECHANICAL CONSTRAINTS

1. The vector velocities for arbitrary points A and B on a rigid body are related by

$$\mathbf{v}_B = \mathbf{v}_A + \dot{\phi} \times \mathbf{r}_{AB},$$

where $\mathbf{r}_{AB}$ is a geometric vector from point A to point B, and $\dot{\phi}$ is the angular velocity vector for the body.

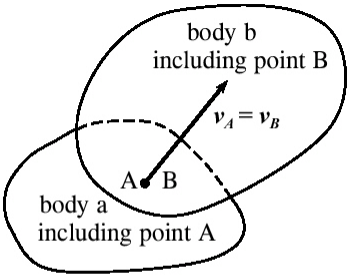


(a) two points on a body

MECHANICAL CONSTRAINTS

2. Point A on one member and point B on another member have the same velocities if both points are located coextensively at a pinned or swivel joint between the members, *i.e.*

$$\mathbf{v}_A = \mathbf{v}_B.$$

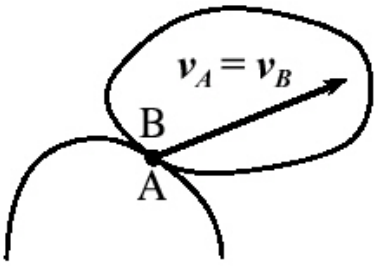


(b) pinned joint between two bodies

parallel to surfaces at contact

MECHANICAL CONSTRAINTS

3. Two instantaneously contacting points A and B which belong to separate members in rolling contact also have same velocity . (The accelerations of these two points are different, however, unlike the corresponding points for pinned members.)



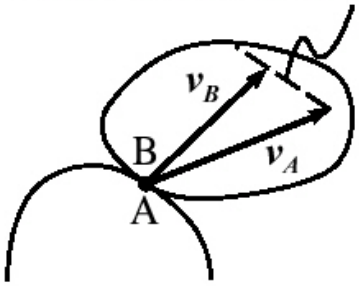
(c) rolling contact between two bodies

MECHANICAL CONSTRAINTS

4. Two instantaneously contacting points A and B which belong to separate members in sliding contact have zero relative velocity in the direction normal to the surfaces in contact. That is, if **n** is a vector normal to the surfaces of contact,

$$(\mathbf{v}_A - \mathbf{v}_B) \cdot \mathbf{n} = 0$$

parallel to surfaces at contact



(c) sliding contact between two bodies

APPROACH TO MODELING

1. Identify critical velocities.
   ---body mass centers
   ---connection points between bodies
2. Label critical velocities on physical model
3. Place each at its own 1 junction
4. Find constraint relationships between them
5. Place on bond diagram
6. Add in I, C, R and S as needed

0-Junction

APPLYING CAUSAL STROKES

1. Mandatory strokes for effort and flow sources.
2. Resulting mandatory strokes through 0, 1, T, G
3. Apply integral causality to one of the remaining I, C
4. Apply steps 2 and 3 as many times as possible.

THREE OUTCOMES POSSIBLE

1. Every bond assigned causality.
   Every compliance and inertance have integral causality.
   Models called **causal**

2. Every bond assigned causality.
   One or more compliance, inertance with differential causality.
   Models called **over-causal**

3. Some bonds note assigned causality.
   Models called **under-causal**

DIFF. EQS. FOR CAUSAL MODELS

1. Annotate diagram in order of causal assignment.
   Input effort and flows first,
   with $\times$ for their conjugate variable.
2. Next a $\dot{p}$ or $\dot{q}$, circle it
   do integral causality to get p/I or q/I if linear
   or $f(p)$ or $e(q)$ if nonlinear
3. Propage efforts and flows through diagram
   use order of assignment from steps 1 and 2 (easier)
   causality determine output of propogation
4. Write first order differential equations
   bottled terms are on left sife
   right side determined by causality

OVER-CASUAL SYSTEMS

Fewer differential equations than energy storage elements.

Using bond graphs produces differential equations
with derivatives on both sides of first order equations.

One additionial step required to get standard form.

UNDER-CASUAL SYSTEMS

Causality insufficient to define all efforts and flows.

Virtual compliances or/and inertances needed.

Using bond graphs produces some algebraic equations.

UNDER-CASUAL SYSTEMS

Standard procedure leaves bonds with no strokes.

Add virtual compliance to 0 junction.

(or/and)

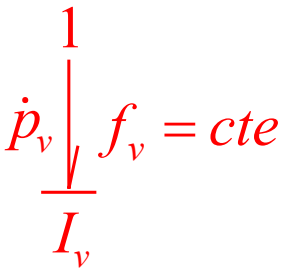
Add virtual inertance to 1 junction.

These elements have zero modulii $(C_v = 0, I_v = 0)$

and zero initial state. $(q_v = 0, p_v = 0)$

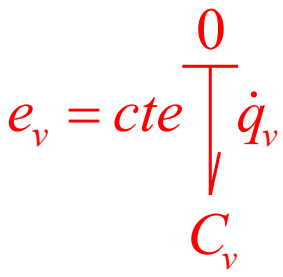
This is the "algebraic-reduction method"

UNDER-CASUAL SYSTEMS



$$p_v = I_v f_v = 0 \Rightarrow \dot{p}_v = 0$$

$$\boxed{I_v = 0}$$



$$q_v = C_v e_v = 0 \Rightarrow \dot{q}_v = 0$$

$$\boxed{C_v = 0}$$

UNDER-CASUAL SYSTEMS

Application of the method results in

Semi-Explicit Form

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{dt}} = \mathbf{f}\left(\mathbf{x}, \mathbf{u}, t\right)$$

$$\mathbf{0} = \mathbf{g}\left(\mathbf{x}, \mathbf{u}, t\right)$$