ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 34:

- Over-casual and Under-casual Systems 5.1

UNDER-CASUAL SYSTEMS

Causality insufficient to define all efforts and flows.

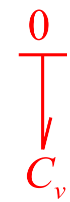
Virtual compliances or/and inertances needed.

Using bond graphs produces some algebraic equations.

UNDER-CASUAL SYSTEMS

Standard procedure leaves bonds with no strokes.

Add virtual compliance to 0 junction.
(or/and)
Add virtual inertance to 1 junction.

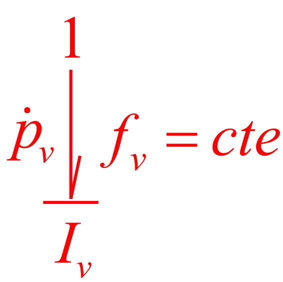


These elements have zero modulii $\left(C_v = 0, I_v = 0\right)$

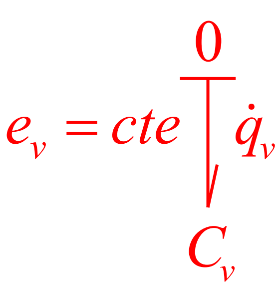
and zero initial state. $\left(q_v = 0, p_v = 0\right)$

This is the "algebraic-reduction method"

UNDER-CASUAL SYSTEMS



$$p_v = I_v f_v = 0 \Rightarrow \dot{p}_v = 0$$



$$q_v = C_v e_v = 0 \Rightarrow \dot{q}_v = 0$$

UNDER-CASUAL SYSTEMS

Application of the method results in

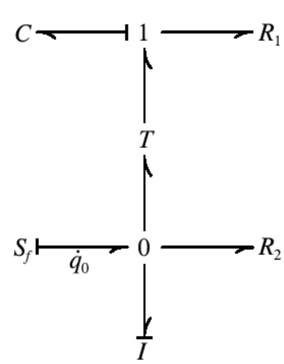
Semi-Explicit Form

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{u}, t)$$

UNDER-CASUAL SYSTEMS

Case Study:

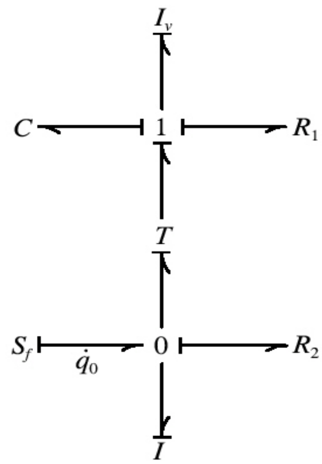


(a) steps 1-4 for placing causal strokes

Is this under-casual?

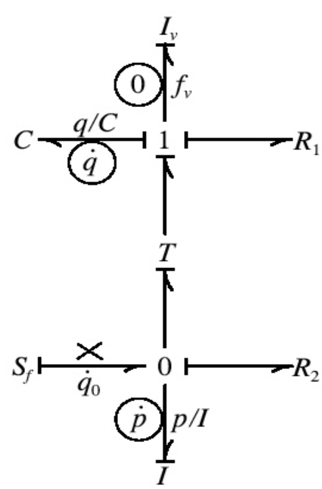
Where can we place virtual storage devices?

UNDER-CASUAL SYSTEMS



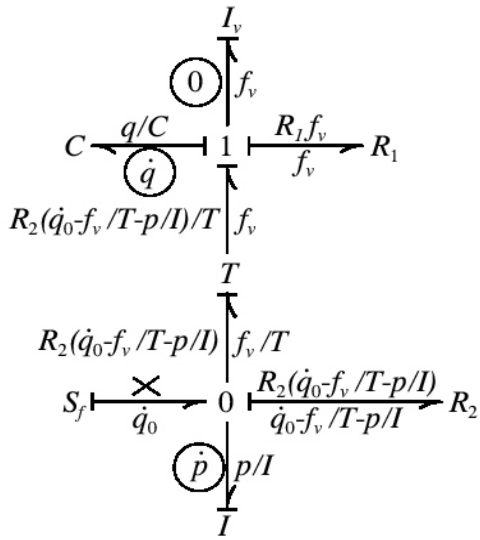
(b) step 5 with virtual inertance

UNDER-CASUAL SYSTEMS



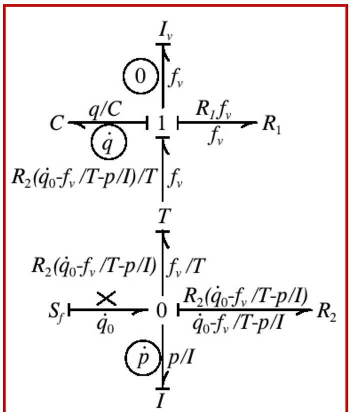
(c) steps I-II of procedure to write
the differential equations

UNDER-CASUAL SYSTEMS



(d) after step IV

UNDER-CASUAL SYSTEMS



(d) after step IV

$$\frac{dq}{dt} = f_v$$

$$\frac{dp}{dt} = R_2\left(\dot{q}_0 - \frac{f_v}{T} - \frac{p}{I},\right)$$

$$0 = \frac{R_2}{T}\left(\dot{q}_0 - \frac{f_v}{T} - \frac{p}{I}\right) - \frac{q}{C} - R_1 f_v$$

UNDER-CASUAL SYSTEMS

$$\frac{dq}{dt} = f_v$$

$$\frac{dp}{dt} = R_2\left(\dot{q}_0 - \frac{f_v}{T} - \frac{p}{I},\right)$$

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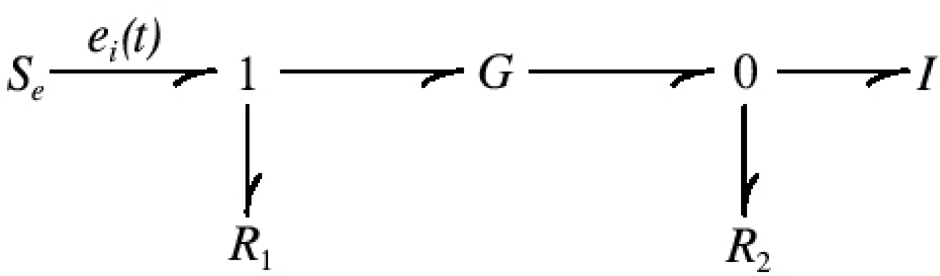
$$\Downarrow$$

$$\frac{dq}{dt} = \frac{T}{1 + R_1T^2/R_2}\left[\dot{q}_0 - \frac{1}{I}p - \frac{T}{R_1C}q\right],$$

$$\frac{dp}{dt} = \frac{T}{1 + R_1T^2/R_2}\left[R_1T\dot{q}_0 - \frac{R_1T}{I}p + \frac{1}{C}q\right]$$

UNDER-CASUAL SYSTEMS

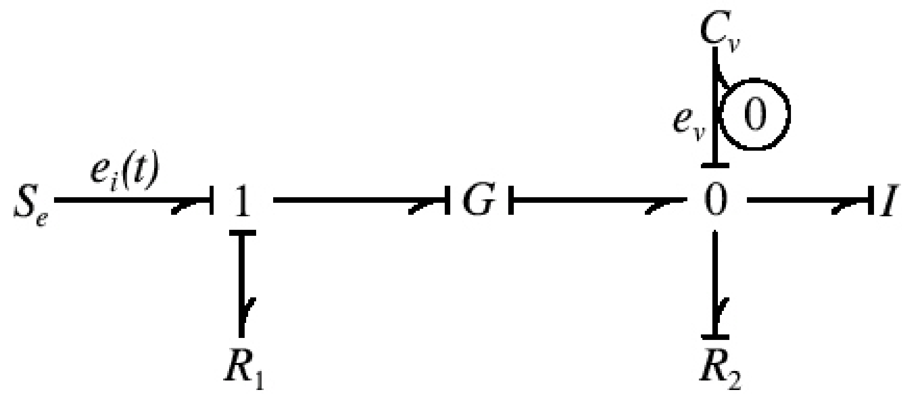
Example:



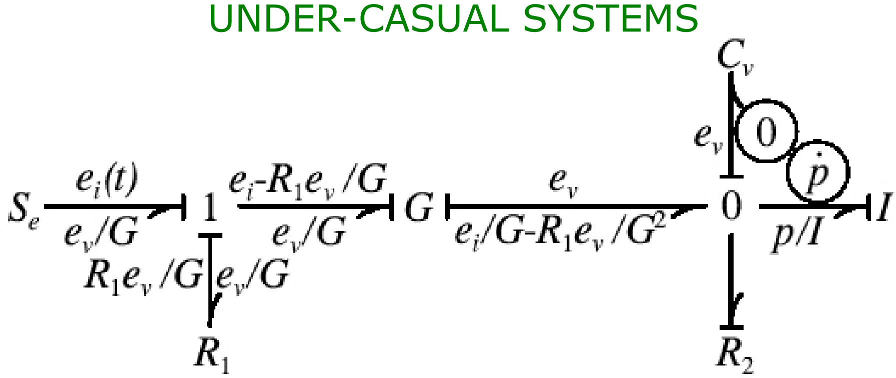
Is this under-casual?

Where can we place virtual storage devices?

UNDER-CASUAL SYSTEMS



UNDER-CASUAL SYSTEMS



$$\frac{dp}{dt} = e_v = \frac{1}{R_1/G^2 - 1/R_2}\left(\frac{1}{G}e_i(t) - \frac{1}{I}p\right)$$