

## ME242 – MECHANICAL ENGINEERING SYSTEMS

### LECTURE 33:

- Over-casual and Under-casual Systems 5.1

## OVER-CASUAL SYSTEMS

Fewer differential equations than energy storage elements.

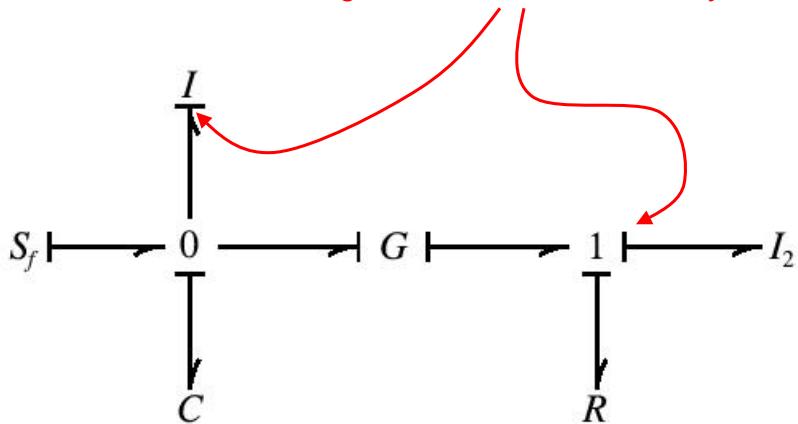
Using bond graphs produces differential equations  
with derivatives on both sides of first order equations.

One additional step required to get standard form.

## OVER-CASUAL SYSTEMS

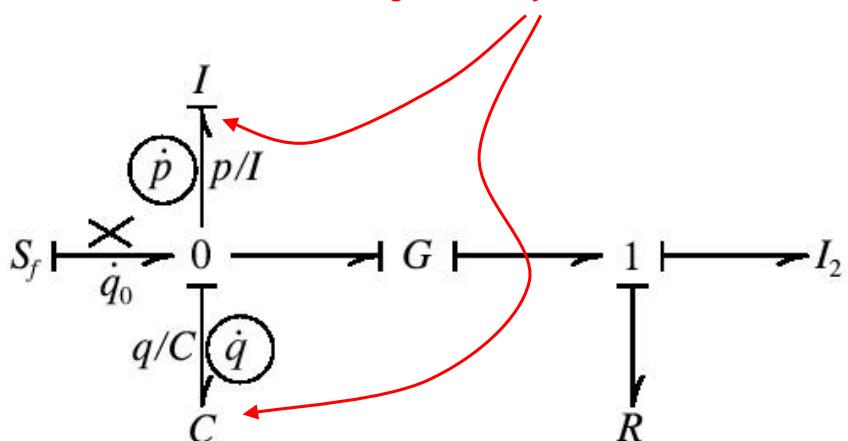
Case Study:

One integral and one derivative causality.



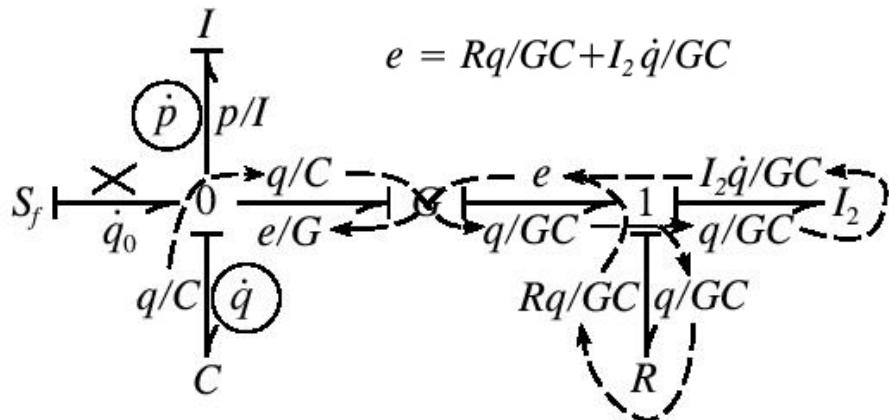
## OVER-CASUAL SYSTEMS

Do integral causality first.



Continue annotating like usual.

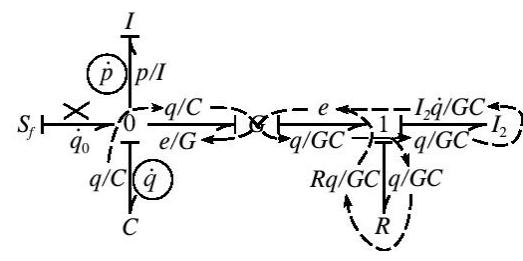
## OVER-CASUAL SYSTEMS



Next write state equations like usual.

## OVER-CASUAL SYSTEMS

$$\frac{dp}{dt} = \frac{1}{C}q$$



$$\frac{dq}{dt} = \dot{q}_0 - \frac{1}{I}p - \frac{1}{G}e = \dot{q}_0 - \frac{1}{I}p - \frac{I_2}{G^2C} \frac{dq}{dt} - \frac{R}{G^2C}q$$

*the derivative term appears on both sides of the equation*

*Now rewrite in standard form.*

## OVER-CASUAL SYSTEMS

$$\frac{dp}{dt} = \frac{1}{C}q$$

$$\frac{dq}{dt} = \dot{q}_0 - \frac{1}{I}p - \frac{1}{G}e = \dot{q}_0 - \frac{1}{I}p - \frac{I_2}{G^2C} \frac{dq}{dt} - \frac{R}{G^2C}q$$



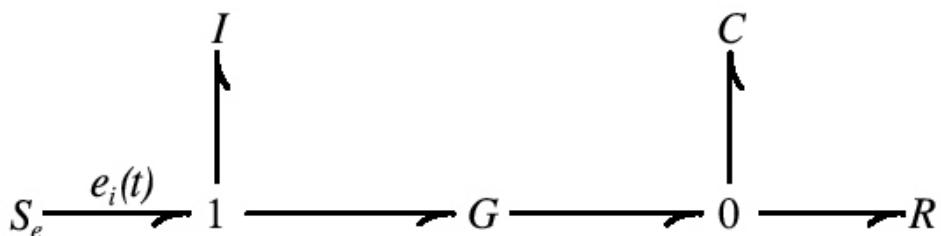
$$\frac{dp}{dt} = \frac{1}{C}q$$

$$\left(1 + \frac{I_2}{G^2C}\right) \frac{dq}{dt} = \dot{q}_0 - \frac{1}{I}p - \frac{R}{G^2C}q$$

Need to divide by LHS coefficient for simulation.

## OVER-CASUAL SYSTEMS

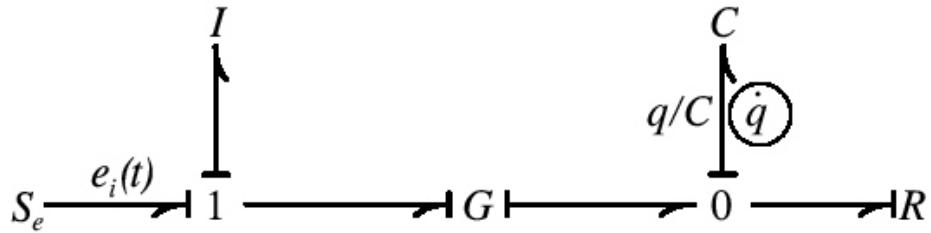
Example:



Is this over causal?

Which storage device do I set as integral causality?  
(Either is o.k. --- it is your choice.)

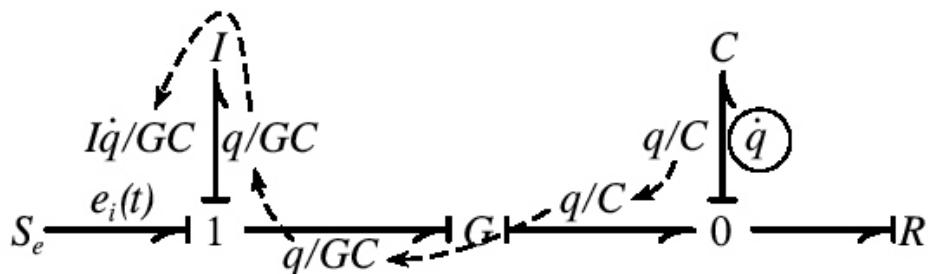
## OVER-CASUAL SYSTEMS



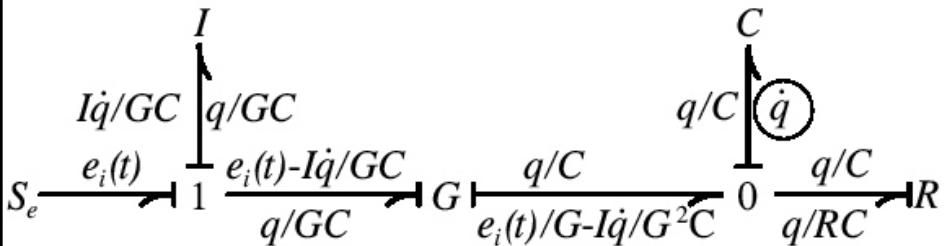
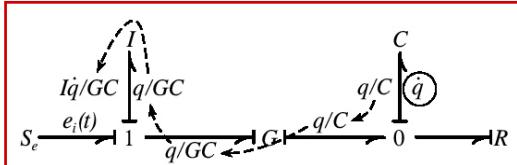
OK now what?

(Do the next steps then check back.)

## OVER-CASUAL SYSTEMS

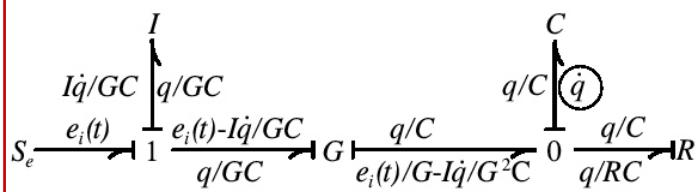


## OVER-CASUAL SYSTEMS



Next write state equations like usual.

## OVER-CASUAL SYSTEMS



$$\frac{dq}{dt} = \frac{1}{G}e_i(t) - \frac{I}{G^2C} \frac{dq}{dt} - \frac{1}{RC}q$$

Now rewrite in standard form.

## OVER-CASUAL SYSTEMS

$$\frac{dq}{dt} = \frac{1}{G}e_i(t) - \frac{I}{G^2C} \frac{dq}{dt} - \frac{1}{RC}q$$



$$\left(1 + \frac{I}{G^2C}\right) \frac{dq}{dt} = \frac{1}{g}e_i(t) - \frac{1}{RC}q$$

Need to divide by LHS coefficient for simulation.