ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 32:

- Causality and Differential Equations 5.1

APPLYING CAUSAL STROKES

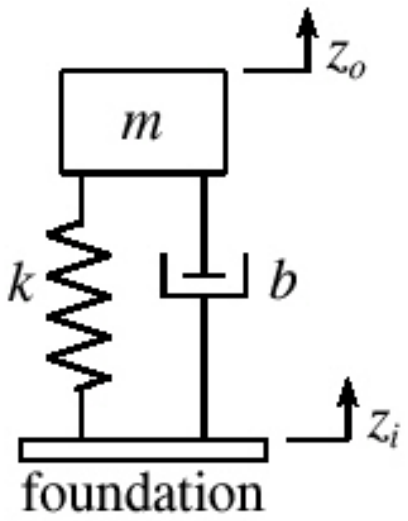
1. Mandatory strokes for effort and flow sources.
2. Resulting mandatory strokes through 0, 1, T, G
3. Apply integral causality to one of the remaining I, C
4. Apply steps 2 and 3 as many times as possible.

DIFF. EQS. FOR CAUSAL MODELS

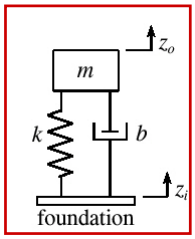
1. Annotate diagram in order of causal assignment.
   Input effort and flows first,
   with $\times$ for their conjugate variable.
2. Next a $\dot{p}$ or $\dot{q}$, circle it
   do integral causality to get p/I or q/I if linear
   or $f(p)$ or $e(q)$ if nonlinear
3. Propage efforts and flows through diagram
   use order of assignment from steps 1 and 2 (easier)
   causality determine output of propogation
4. Write first order differential equations
   bottled terms are on left sife
   right side determined by causality

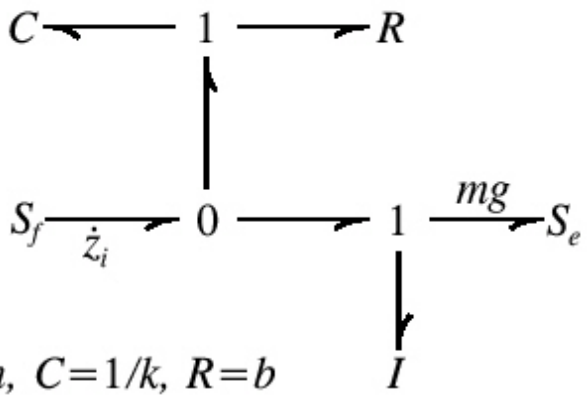
DIFF. EQS. FOR CAUSAL MODELS

Example: Mechanical System

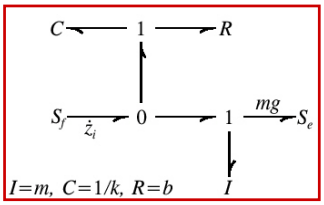


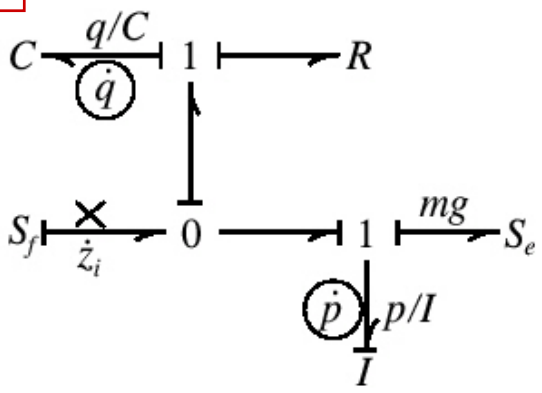
DIFF. EQS. FOR CAUSAL MODELS



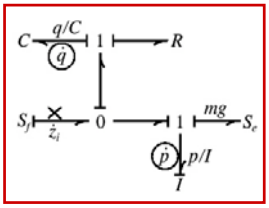


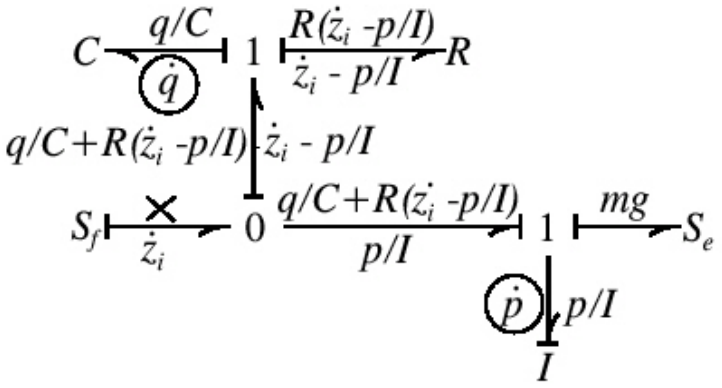
DIFF. EQS. FOR CAUSAL MODELS



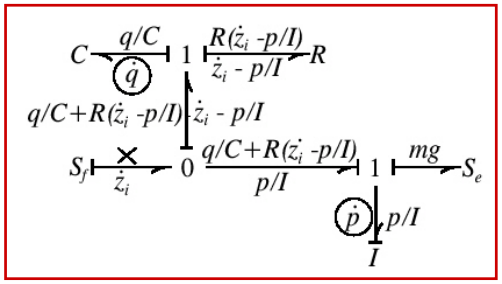


DIFF. EQS. FOR CAUSAL MODELS





DIFF. EQS. FOR CAUSAL MODELS



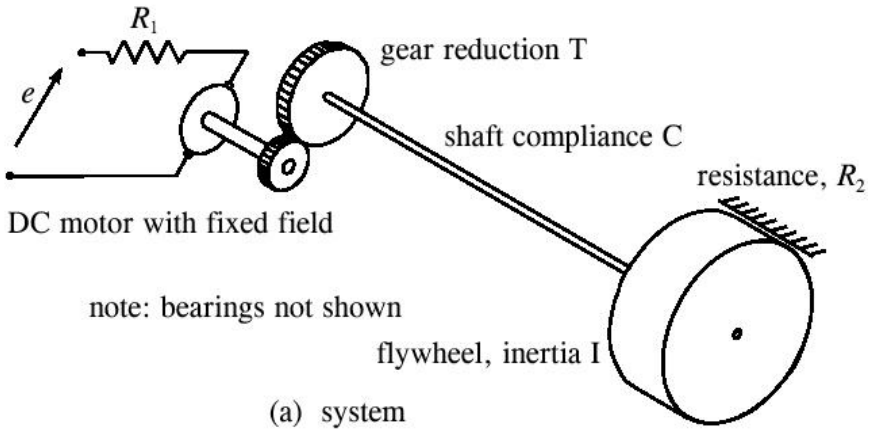
State Equations

$$\frac{dq}{dt} = \dot{z}_i(t) - \frac{1}{I}p$$

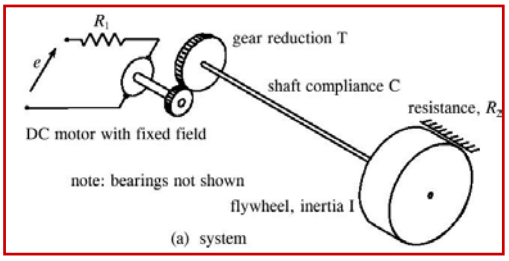
$$\frac{dp}{dt} = \frac{1}{C}q - \frac{R}{I}p + R\dot{z}_i(t) - mg$$

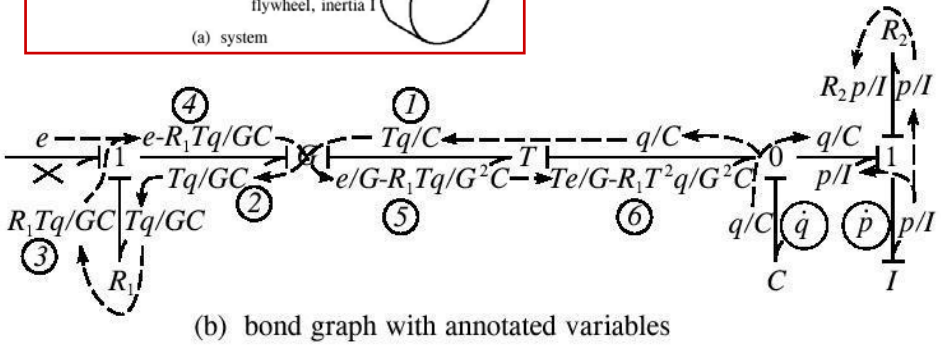
DIFF. EQS. FOR CAUSAL MODELS

Example: Transformer and Gyrators



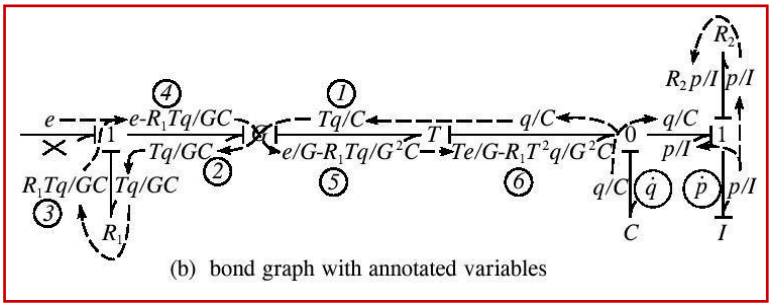
DIFF. EQS. FOR CAUSAL MODELS





(b) bond graph with annotated variables

DIFF. EQS. FOR CAUSAL MODELS



State Equations

$$\frac{dq}{dt} = \frac{T}{G}\left(e - \frac{R_1 T}{GC}q\right) - \frac{1}{I}p$$

$$\frac{dp}{dt} = \frac{1}{C}q - \frac{R_2}{I}p$$