# ME242 – MECHANICAL ENGINEERING SYSTEMS

## LECTURE 31:

- Causality and Differential Equations 5.1



## STARTING BOND GRAPH

- Apply causality
- Fully annotate
- Write differential equations

## APPLYING CAUSAL STROKES

1. Mandatory strokes for effort and flow sources.
2. Resulting mandatory strokes through 0, 1, T, G
3. Apply integral causality to one of the remaining I, C
4. Apply steps 2 and 3 as many times as possible.



## THREE OUTCOMES POSSIBLE

1. Every bond assigned causality.
   Every compliance and inertance have integral causality.
   Models called **causal**

2. Every bond assigned causality.
   One or more compliance, inertance with differential causality.
   Models called **over-causal**

3. Some bonds note assigned causality.
   Models called **under-causal**

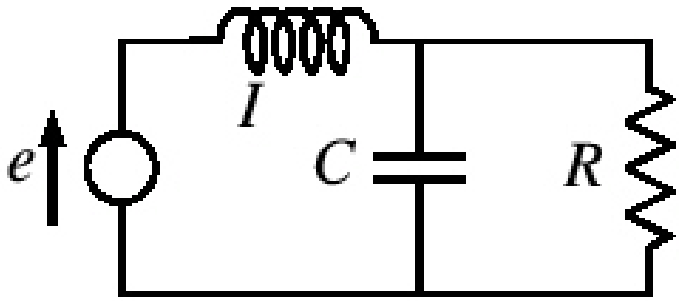
## DIFF. EQS. FOR CAUSAL MODELS

1. Annotate diagram in order of causal assignment.
   Input effort and flows first,
   with $\times$ for their conjugate variable.
2. Next a $\dot{p}$ or $\dot{q}$, circle it
   do integral causality to get $p/I$ or $q/I$ if linear
   or $f(p)$ or $e(q)$ if nonlinear
3. Propage efforts and flows through diagram
   use order of assignment from steps 1 and 2 (easier)
   causality determine output of propogation
4. Write first order differential equations
   bottled terms are on left sife
   right side determined by causality



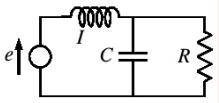
## DIFF. EQS. FOR CAUSAL MODELS

Example: Electrical System

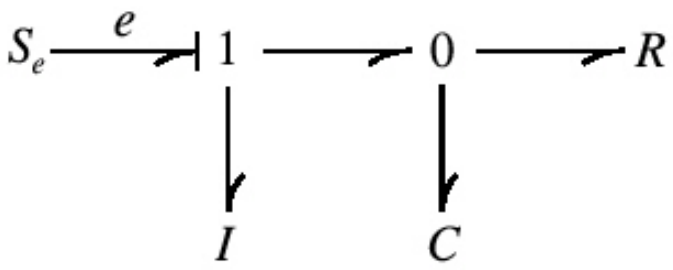



(a) circuit

## DIFF. EQS. FOR CAUSAL MODELS



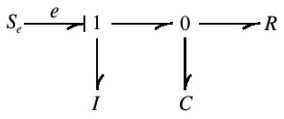

(a) circuit



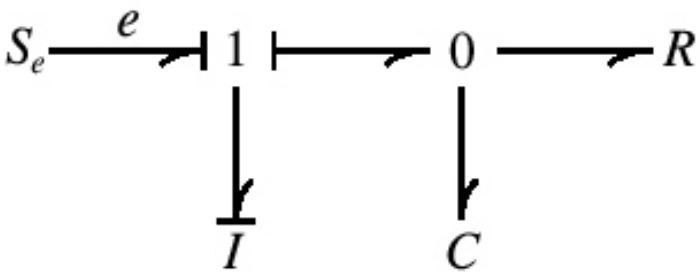

(b) bond graph with first causal stroke



## DIFF. EQS. FOR CAUSAL MODELS

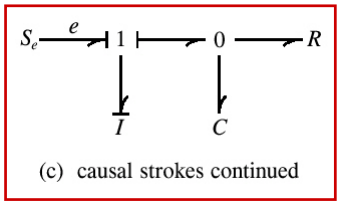



(b) bond graph with first causal stroke

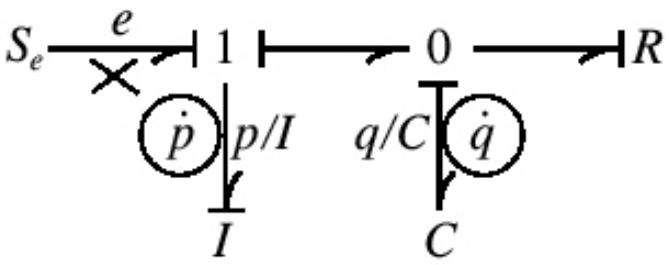



(c) causal strokes continued

## DIFF. EQS. FOR CAUSAL MODELS



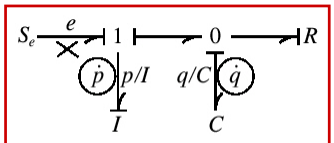

(c) causal strokes continued



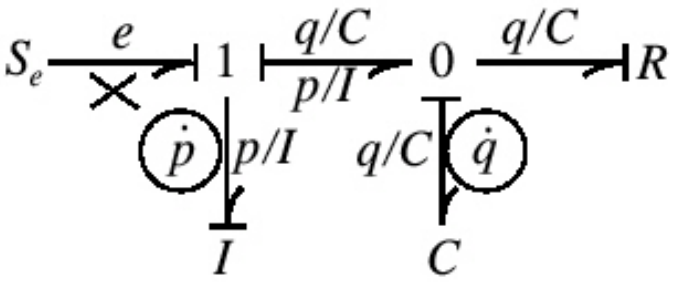

(d) step II completed



## DIFF. EQS. FOR CAUSAL MODELS

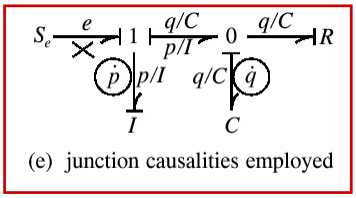



(d) step II completed




(e) junction causalities employed

## DIFF. EQS. FOR CAUSAL MODELS

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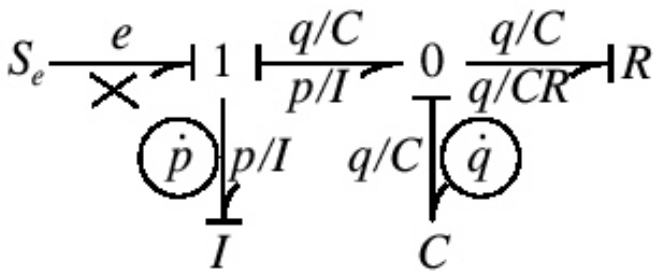
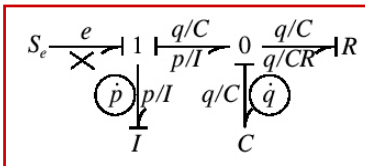



(f) final annotated bond graph



## DIFF. EQS. FOR CAUSAL MODELS




(f) final annotated bond graph

State Equations

$$\frac{dp}{dt} = e - \frac{1}{C}q$$

$$\frac{dq}{dt} = \frac{1}{I}p - \frac{1}{RC}q$$

If flow required of effort source needed

$$\dot{q}_f = p / I$$