ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 30:

- Systems with Mechanical Constraints 4.2

MECHANICAL CONSTRAINTS

Kinematic Constraints: Govern details on how efforts and flows are related

Two Approaches:

1. Write Displacement constraints and then take derivative to get velocity constraints
2. Write velocity constraint directly

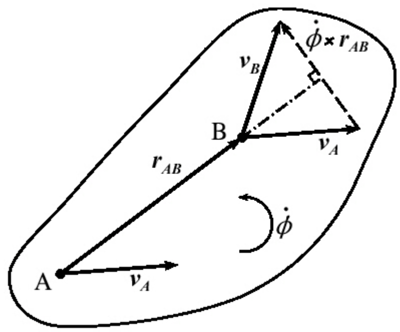
Either approach can be used
Use the one that is more natural

MECHANICAL CONSTRAINTS

1. The vector velocities for arbitrary points A and B on a rigid body are related by

$$\mathbf{v}_B = \mathbf{v}_A + \dot{\phi} \times \mathbf{r}_{AB},$$

where $\mathbf{r}_{AB}$ is a geometric vector from point A to point B, and $\dot{\phi}$ is the angular velocity vector for the body.

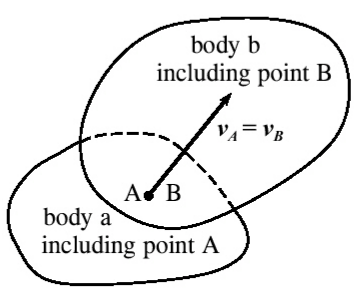


(a) two points on a body

MECHANICAL CONSTRAINTS

2. Point A on one member and point B on another member have the same velocities if both points are located coextensively at a pinned or swivel joint between the members, *i.e.*

$$\mathbf{v}_A = \mathbf{v}_B.$$

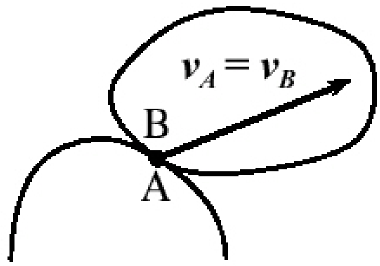


(b) pinned joint between two bodies

parallel to surfaces at contact

MECHANICAL CONSTRAINTS

3. Two instantaneously contacting points A and B which belong to separate members in rolling contact also satisfy equation (4.6). (The accelerations of these two points are different, however, unlike the corresponding points for pinned members.)

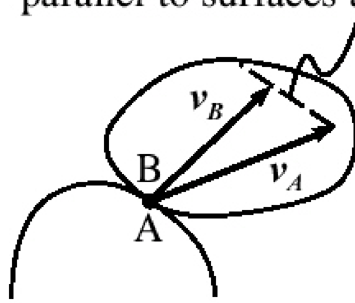


(c) rolling contact between two bodies

MECHANICAL CONSTRAINTS

4. Two instantaneously contacting points A and B which belong to separate members in sliding contact have zero relative velocity in the direction normal to the surfaces in contact. That is, if $\mathbf{n}$ is a vector normal to the surfaces of contact,

$$(\mathbf{v}_A - \mathbf{v}_B) \cdot \mathbf{n} = 0 \tag{4.7}$$



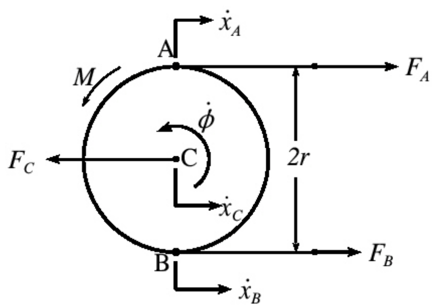
(c) sliding contact between two bodies

APPROACH TO MODELING

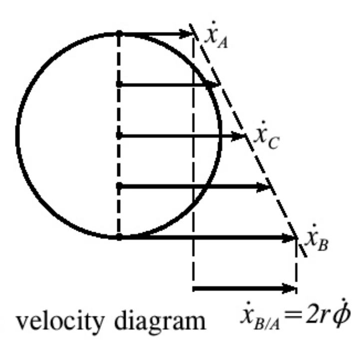
1. Identify critical velocities.
   ---body mass centers
   ---connection points between bodies
2. Label critical velocities on physical model
3. Place each at its own 1 junction
4. Find constraint relationships between them
5. Place on bond diagram
6. Add in I, C, R and S as needed

MECHANICAL CONSTRAINTS

Example: Pulley System



(a) physical configuration



(b) velocity diagram

$$\dot{x}_A = \dot{x}_C - r\dot{\phi},$$
$$\dot{x}_B = \dot{x}_C + r\dot{\phi}.$$

MECHANICAL CONSTRAINTS

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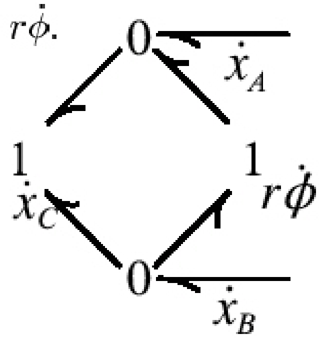
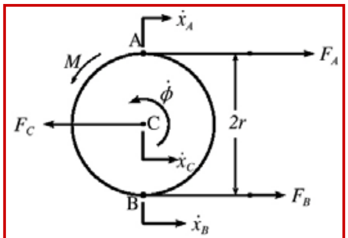
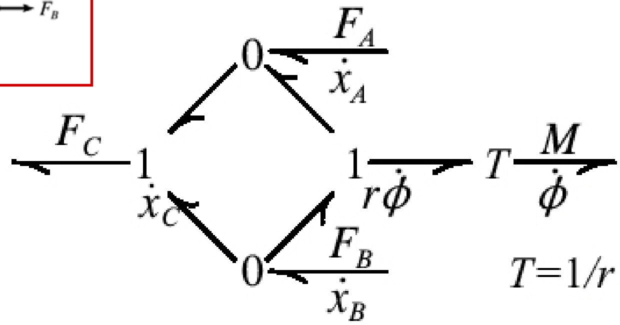


(c) bond graph with kinematical constraints

MECHANICAL CONSTRAINTS

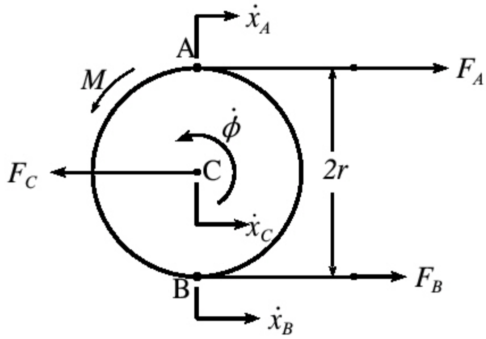


(a) physical configuration

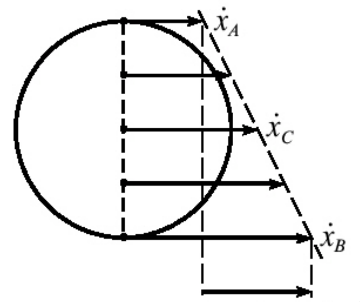


(d) bond graph with forces added

MECHANICAL CONSTRAINTS



(a) physical configuration



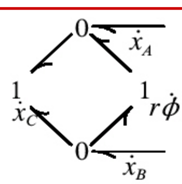
(b) velocity diagram

$$\dot{x}_C = \frac{1}{2}(\dot{x}_A + \dot{x}_B),$$
$$r\dot{\phi} = \frac{1}{2}(\dot{x}_B - \dot{x}_A).$$

alternative to

$$\dot{x}_A = \dot{x}_C - r\dot{\phi},$$
$$\dot{x}_B = \dot{x}_C + r\dot{\phi}.$$

MECHANICAL CONSTRAINTS



$$\dot{x}_A = \dot{x}_C - r\dot{\phi},$$
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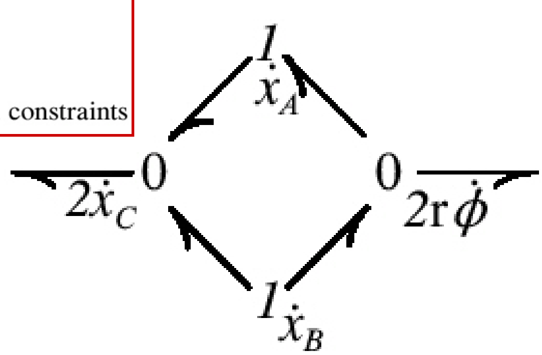
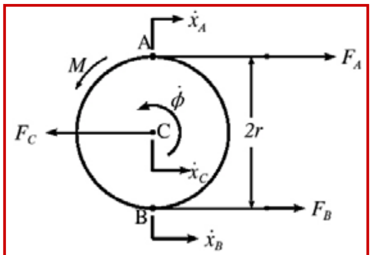
(c) bond graph with kinematical constraints



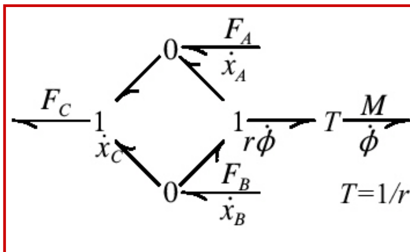
$$\dot{x}_C = \frac{1}{2}(\dot{x}_A + \dot{x}_B),$$
$$r\dot{\phi} = \frac{1}{2}(\dot{x}_B - \dot{x}_A).$$

(e) alternate to (c)

MECHANICAL CONSTRAINTS

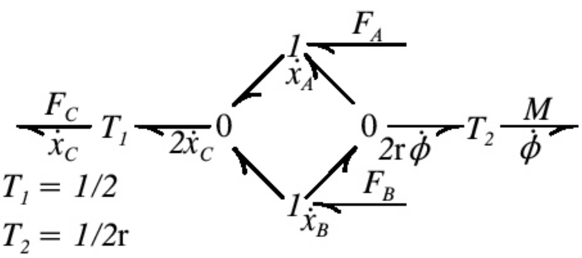


(a) physical configuration



(d) bond graph with forces added

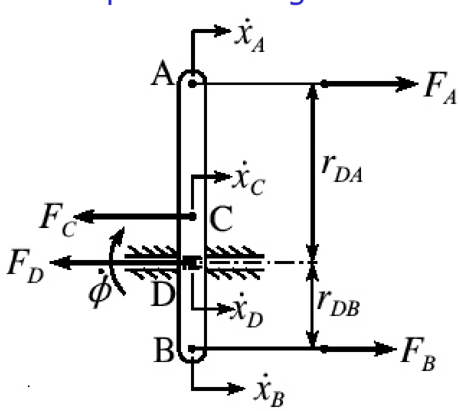
$$\dot{x}_C = \frac{1}{2}(\dot{x}_A + \dot{x}_B),$$
$$r\dot{\phi} = \frac{1}{2}(\dot{x}_B - \dot{x}_A).$$



(f) alternate to (d)

MECHANICAL CONSTRAINTS

Example: Floating Lever



(small deflections assumed)

$T_A = r_{DB}/r_{AB}$ $\quad T_C = 1/2$

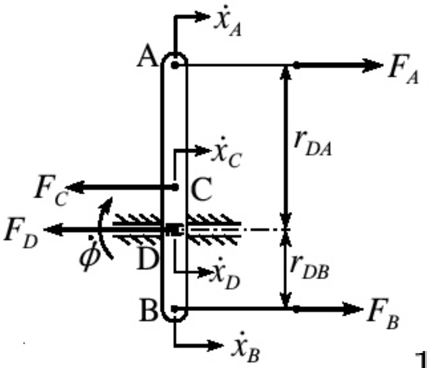
$T_B = r_{DA}/r_{AB}$ $\quad T_\phi = 1/r_{AB}$

write down the "3" constraints

use $\dot{x}_A$ and $\dot{x}_B$ as the independent variables

use $\dot{x}_C$ and $\dot{x}_D$ and $\dot{\phi}$ as the dependent variables

MECHANICAL CONSTRAINTS



(small deflections assumed)

$T_A = r_{DB}/r_{AB}$ $\quad T_C = 1/2$

$T_B = r_{DA}/r_{AB}$ $\quad T_\phi = 1/r_{AB}$

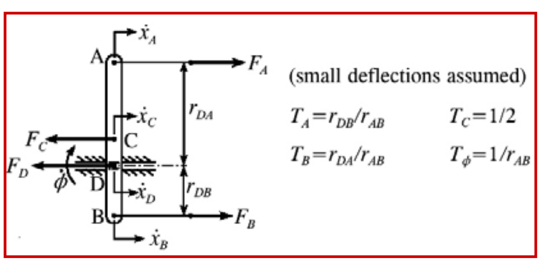
$$\dot{x}_C = \frac{1}{2}(\dot{x}_A + \dot{x}_B),$$

$$r\dot{\phi} = \frac{1}{2}(\dot{x}_B - \dot{x}_A).$$

$$\dot{x}_D = T_A \dot{x}_A + T_B \dot{x}_B;$$

next
convert constraints
into bond graph

MECHANICAL CONSTRAINTS



$$\dot{x}_C = \frac{1}{2}(\dot{x}_A + \dot{x}_B),$$

$$r\dot{\phi} = \frac{1}{2}(\dot{x}_B - \dot{x}_A).$$

$$\dot{x}_D = T_A \dot{x}_A + T_B \dot{x}_B;$$