ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 28:

- Systems with Transformers and Gyrators 2.5

SYSTEMS WITH TRANSFORMER AND GYRATORS

Objective 1: Examine implication of two or more cascaded ideal machines (two-port elements)

$$\longrightarrow \mathbf{T}_1 \longrightarrow \mathbf{T}_2 \longrightarrow$$

$$\longrightarrow \mathbf{T}_1 \longrightarrow \mathbf{G}_2 \longrightarrow$$

$$\longrightarrow \mathbf{G}_1 \longrightarrow \mathbf{T}_2 \longrightarrow$$

$$\longrightarrow \mathbf{G}_1 \longrightarrow \mathbf{G}_2 \longrightarrow$$

SYSTEMS WITH TRANSFORMER AND GYRATORS

Objective 2: Examine implication of an ideal machine (two-port element) in sequence with a source or a resistance (a one-port element)

$\longrightarrow$ **T** $\longrightarrow$ **R**

$\longrightarrow$ **G** $\longrightarrow$ **R**

S $\longrightarrow$ **T** $\longrightarrow$

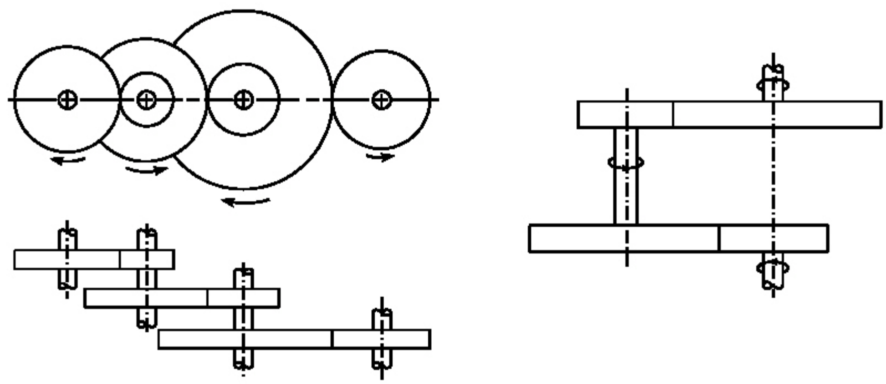
S $\longrightarrow$ **G** $\longrightarrow$

SYSTEMS WITH TRANSFORMER AND GYRATORS

We are interested in analyzing the implications associated with:

- Effective modulus
- Effective Load Characteristic
- Effective Source Characteristic
- Source-load matching / power transfered

CASCADED TRANSFORMER



CASCADED TRANSFORMER

$$\xrightarrow[\dot{q}_1]{e_1} \mathbf{T}_1 \xrightarrow[\dot{q}_2]{e_2} \mathbf{T}_2 \xrightarrow[\dot{q}_3]{e_3}$$

The definition of transformer requires that

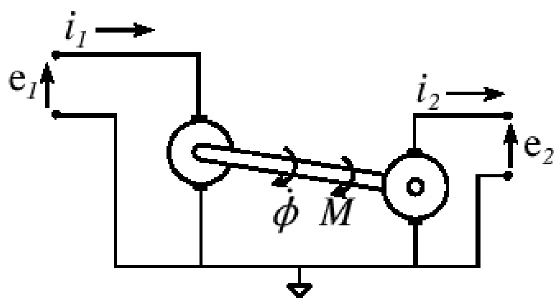
$$\dot{q}_3 = T_2 \dot{q}_2 = T_2 T_1 \dot{q}_1$$

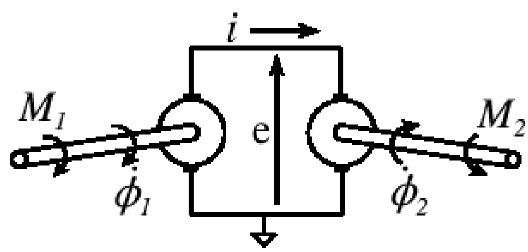
$$e_1 = T_1 e_2 = T_1 T_2 e_3$$

Then,

$$\xrightarrow[\dot{q}_1]{e_1} \mathbf{T} \xrightarrow[\dot{q}_3]{e_3} \qquad \boxed{T = T_1 T_2}$$

CASCADED GYRATORS





CASCADED GYRATORS

$$\xrightarrow[\dot{q}_1]{e_1} \mathbf{G}_1 \xrightarrow[\dot{q}_2]{e_2} \mathbf{G}_2 \xrightarrow[\dot{q}_3]{e_3}$$

The definition of transformer requires that

$$\dot{q}_3 = \frac{e_2}{G_2} = \frac{G_1}{G_2}\dot{q}_1 \qquad e_1 = G_1\dot{q}_2 = \frac{G_1}{G_2}e_3$$

Then,

$$\xrightarrow[\dot{q}_1]{e_1} \mathbf{T} \xrightarrow[\dot{q}_3]{e_3} \qquad \boxed{T = \frac{G_1}{G_2}}$$

TRANSFORMER – GYRATORS PAIRS

$$\xrightarrow[f_1]{e_1} T \xrightarrow[f_2]{e_2} G \xrightarrow[f_3]{e_3}$$

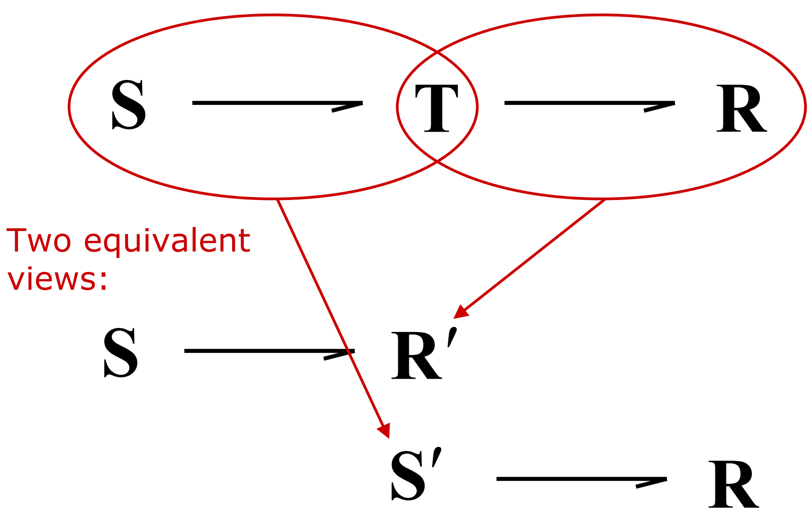
Gyrator: $e_3 = Gf_2$ Transformer: $f_2 = Tf_1 \Rightarrow e_3 = GTf_1$

Transformer: $e_1 = Te_2$ Gyrator: $e_2 = Gf_3 \Rightarrow e_1 = TGf_3$

Then,

$$\xrightarrow[f_1]{e_1} G' \xrightarrow[f_3]{e_3} \qquad \text{with } \boxed{G' = TG}$$

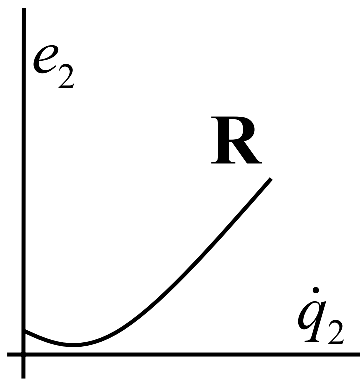
SOURCE - TRANSFORMER – RESISTOR



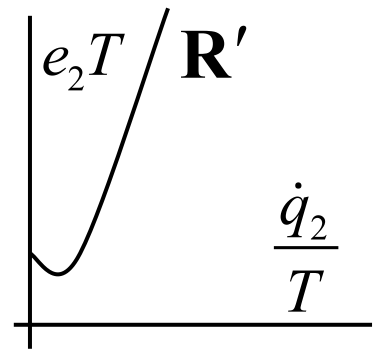
EFFECTIVE RESISTANCE (AS SEEN BY SOURCE)

$$\mathbf{S} \xrightarrow[\dot{q}_1]{e_1} \mathbf{T} \xrightarrow[\dot{q}_2]{e_2} \mathbf{R}$$

(example; T=2)



$\Rightarrow$

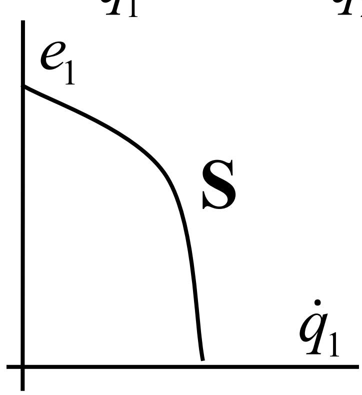


$$\mathbf{S} \longrightarrow \mathbf{R}'$$

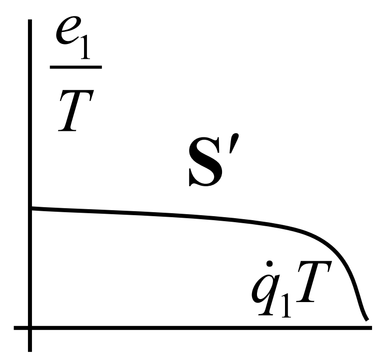
EFFECTIVE SOURCE (AS SEEN BY RESISTANCE)

$$\mathbf{S} \xrightarrow[\dot{q}_1]{e_1} \mathbf{T} \xrightarrow[\dot{q}_2]{e_2} \mathbf{R}$$

(example; T=2)



$\Rightarrow$



$$\mathbf{S}' \longrightarrow \mathbf{R}$$

SOURCE - TRANSFORMER – RESISTOR

CASE STUDY: Automobile Drive Train

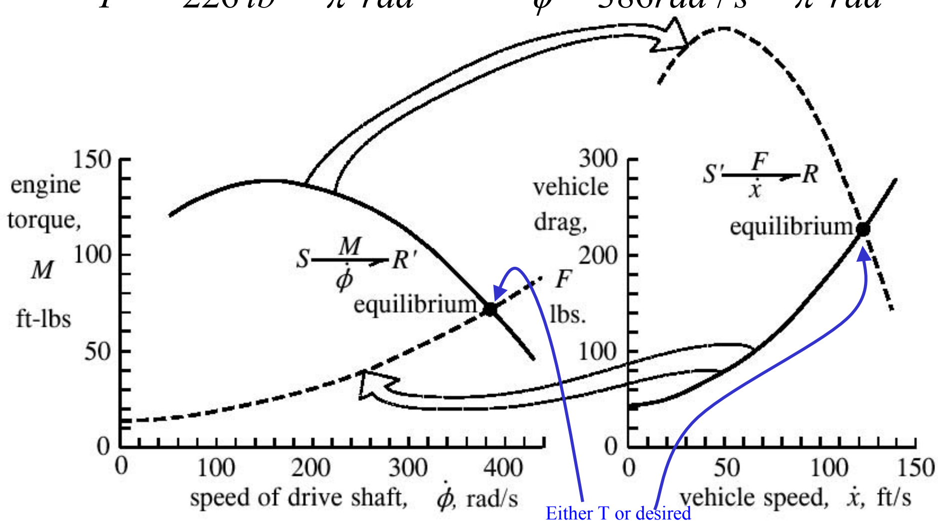
$$\text{ENGINE} \xrightarrow[\dot{\phi}]{M} \text{DRIVETRAIN} \longrightarrow \text{WHEELS} \xrightarrow[\dot{x}]{F} \text{VEHICLE}$$

$$S \xrightarrow[\dot{\phi}]{M} T_d \longrightarrow T_w \xrightarrow[\dot{x}]{F} R$$

$$S \xrightarrow[\dot{\phi}]{M} T \xrightarrow[\dot{x}]{F} R$$

SOURCE - TRANSFORMER – RESISTOR

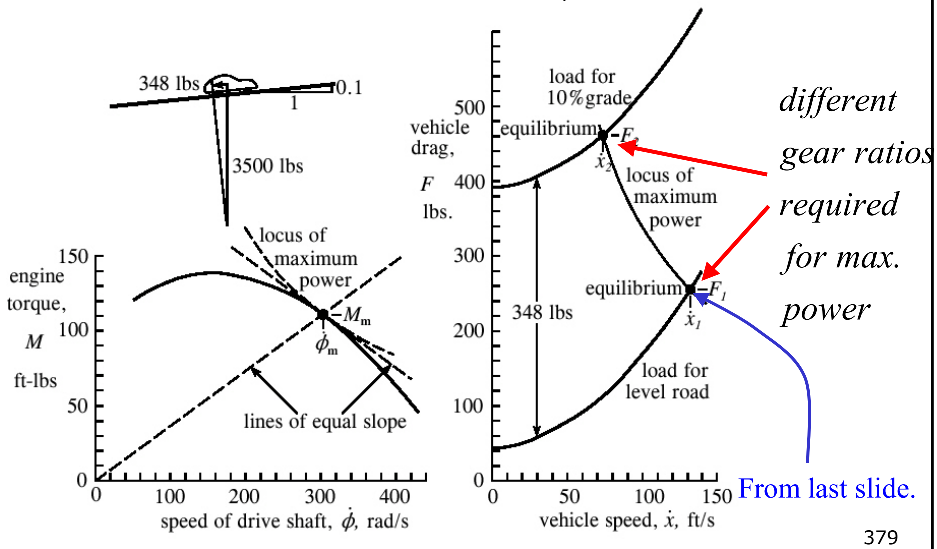
$$T = \frac{M}{F} = \frac{72\,ft\,lb}{226\,lb} = \frac{1}{\pi}\frac{ft}{rad} \qquad T = \frac{\dot{x}}{\dot{\phi}} = \frac{123\,ft/s}{386\,rad/s} = \frac{1}{\pi}\frac{ft}{rad}$$



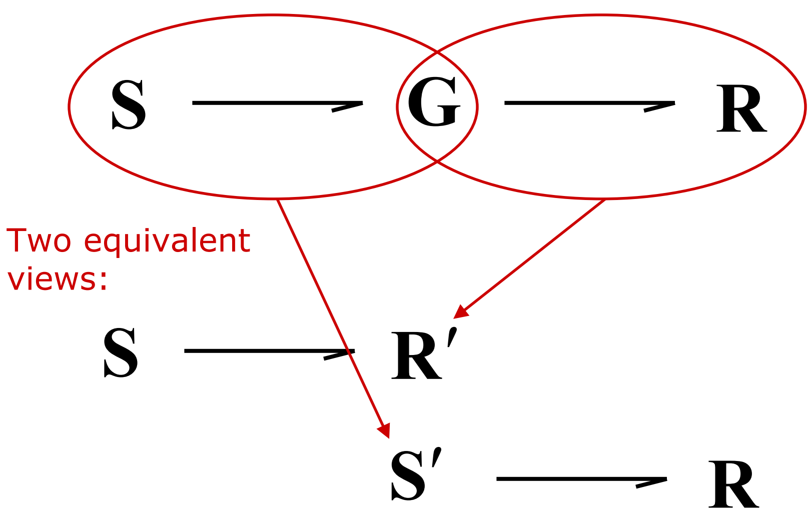
Either T or desired equilibrium given.

SOURCE - TRANSFORMER – RESISTOR

$$T = \frac{M}{F} = \frac{111.5\,ft\,lb}{461\,lb} = .242\frac{ft}{rad} \qquad T = \frac{\dot{x}}{\dot{\phi}} = \frac{73.5\,ft/s}{304\,rad/s} = .242\frac{ft}{rad}$$



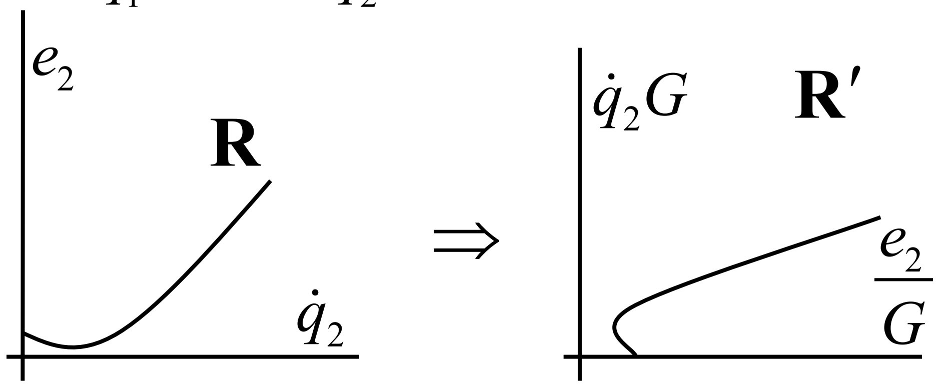
SOURCE - GYRATOR – RESISTOR



EFFECTIVE RESISTANCE (AS SEEN BY SOURCE)

$$\mathbf{S} \xrightarrow[\dot{q}_1]{e_1} \mathbf{G} \xrightarrow[\dot{q}_2]{e_2} \mathbf{R}$$

(example; G=.5)

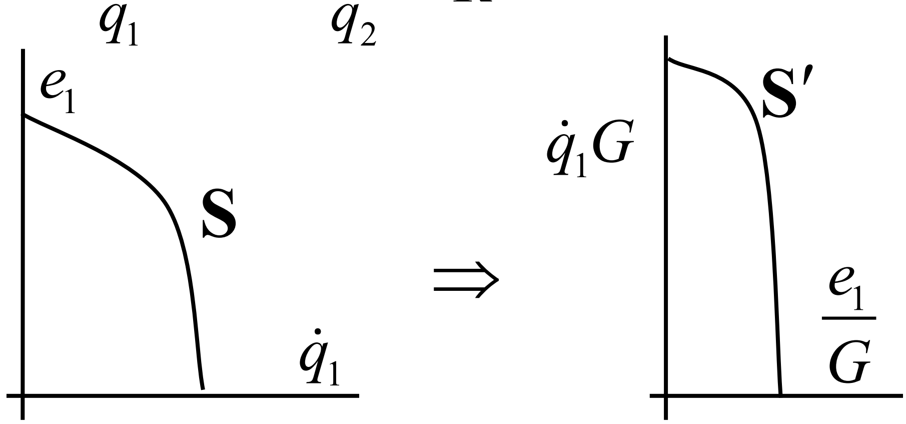


$$\mathbf{S} \longrightarrow \mathbf{R}'$$

EFFECTIVE SOURCE (AS SEEN BY RESISTANCE)

$$\mathbf{S} \xrightarrow[\dot{q}_1]{e_1} \mathbf{G} \xrightarrow[\dot{q}_2]{e_2} \mathbf{R}$$

(example; G=2)



$$\mathbf{S}' \longrightarrow \mathbf{R}$$

SOURCE - GYRATOR – RESISTOR

CASE STUDY: DC Motor