ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 27:

- Ideal Machines: Transformers and Gyrators   2.4

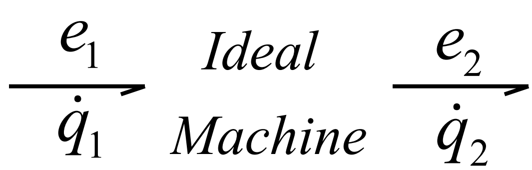
IDEAL MACHINES

$$\xrightarrow[\dot{q}_1]{e_1} \quad \textit{Ideal Machine} \quad \xrightarrow[\dot{q}_2]{e_2}$$

An ideal machine is a two port device that transmits work from one port to the other

- No energy is stored, generated or dissipated
- Entropy is not generated
- Can be run in either direction

IDEAL MACHINES



Power Conservation

$$e_1\dot{q}_1 = e_2\dot{q}_2$$

IDEAL MACHINES

Physical or Mechanical systems modeled as ideal machines

- Levers
- Gears
- Electric motors
- Piston pumps
- Electric Transformers

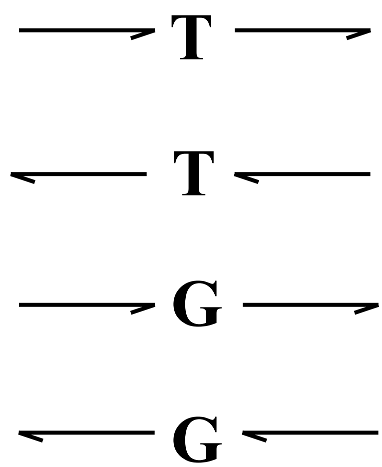
More accurate (and more complex) models of these devices might include other elements. Example: Real Electric Motor

IDEAL MACHINES

Two special cases: Two-port devices

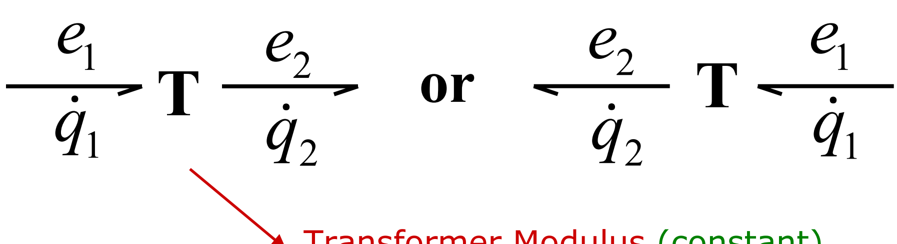
Transformers

Gyrators



IDEAL MACHINES - TRANSFORMER

Defining Condition: $\dot{q}_2 = T\dot{q}_1$



Transformer Modulus (constant)

The modulus of the Transformer, T, is defined as the ratio of the generalized velocity or flow on the bond with the outward power arrow to the generalized velocity or flow on the bond with the inward power convention arrow

IDEAL MACHINES - TRANSFORMER

Combining the Ideal Machine condition:

$$e_1\dot{q}_1 = e_2\dot{q}_2$$

with the Transformer condition:

$$\dot{q}_2 = T\dot{q}_1$$

yields an additional condition:

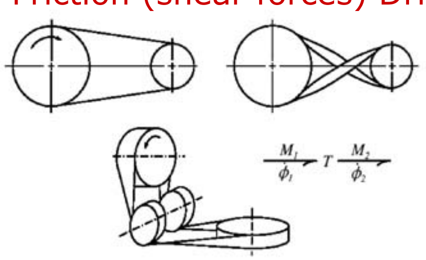
$$e_1 = Te_2$$

$$T = \frac{\dot{q}_2}{\dot{q}_1} = \frac{e_1}{e_2}$$

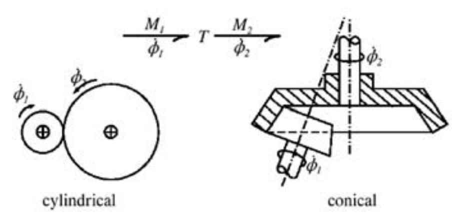
The ratio of the generalized forces of an ideal tranformer equals the inverse of the ratio of the respective generalized velocities

IDEAL MACHINES – TRANSFORMER - EXAMPLES

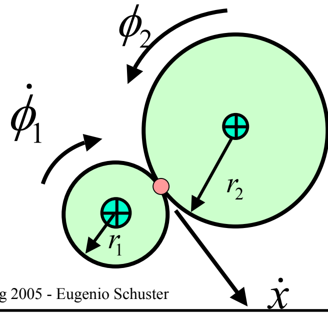
Friction (shear forces) Drives:



Pulley Drives



Rolling Contact Drives

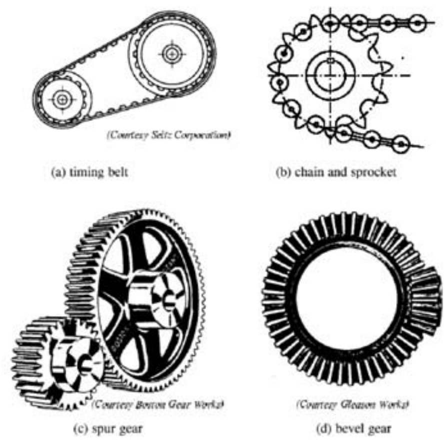


as $\dot{x} = r_1\dot{\phi}_1 = r_2\dot{\phi}_2$

$$T = \frac{\dot{\phi}_2}{\dot{\phi}_1} = \frac{r_1}{r_2} = T$$

IDEAL MACHINES – TRANSFORMER - EXAMPLES

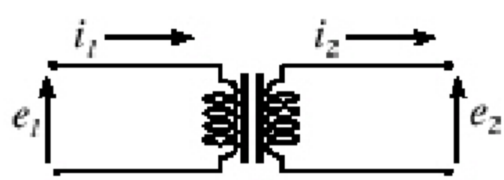
Positive Action (normal forces) Drives:



Toothed Drives

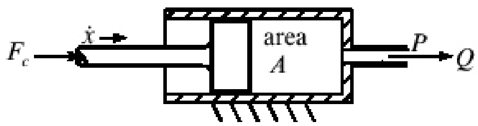
IDEAL MACHINES – TRANSFORMER - EXAMPLES

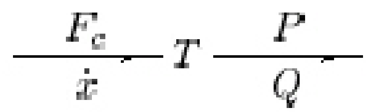
Electric Transformer:



IDEAL MACHINES – TRANSFORMER - EXAMPLES

Positive-displacement mechanical to fluid transducer:





piston-and-cylinder / ram

Conservation of Energy (or Power Balance)

$$F_c \dot{x} = PQ$$

Conservation of Mass (or Kinematic Constraint)

$$Q = A\dot{x}$$

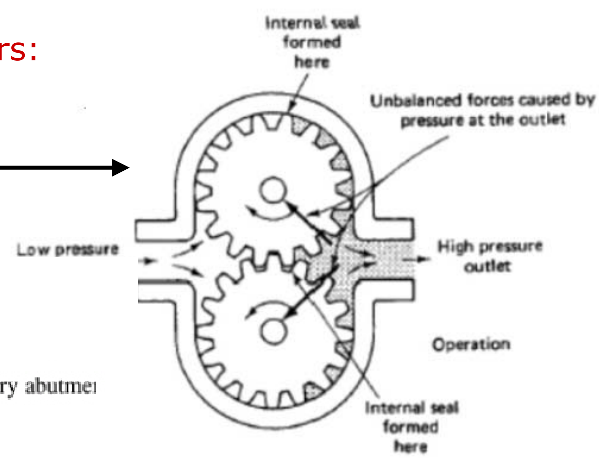
Cons. of Momentum (or Force Equilibrium)

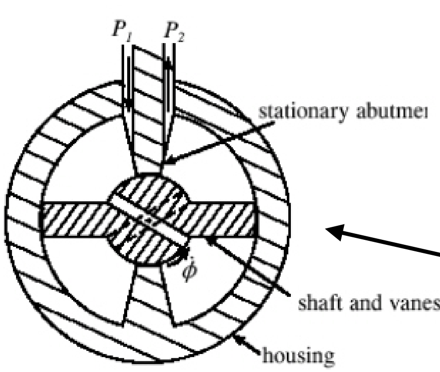
$$F_c = PA$$

IDEAL MACHINES – TRANSFORMER - EXAMPLES

Pumps and Actuators:

Pump →

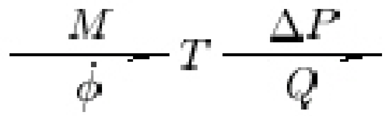




← *Actuator*

IDEAL MACHINES – TRANSFORMER - EXAMPLES

Pump:



Conservation of Energy $\quad M\dot{\phi} = \Delta P Q$

(or Power Balance)

Radian Displacement ↓

Conservation of Mass $\quad Q = D\dot{\phi}$

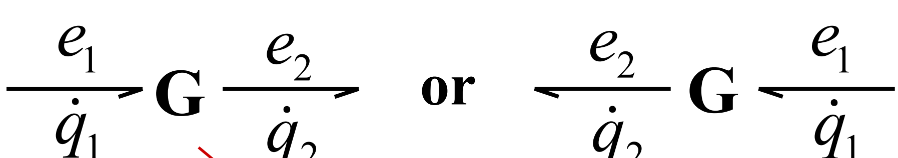
(or Kinematic Constraint)

Cons. of Momentum $\quad M = D\Delta P$

(or Force Equilibrium)

IDEAL MACHINES - GYRATORS

Defining Condition: $\quad e_2 = G\dot{q}_1$



Gyrator Modulus (constant)

The modulus of the Gyrator, G, is defined as the ratio of the effort on one of the bonds – either one – to the flow on the other bond

IDEAL MACHINES - GYRATOR

Combining the Ideal Machine condition:

$$e_1\dot{q}_1 = e_2\dot{q}_2$$

with the Gyrator condition:

$$e_2 = G\dot{q}_1$$

yields an additional condition:

$$e_1 = G\dot{q}_2$$

$$G = \frac{e_2}{\dot{q}_1} = \frac{e_1}{\dot{q}_2}$$

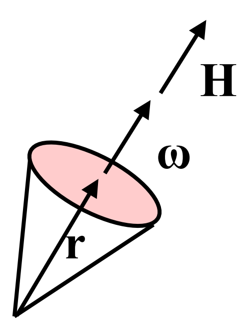
The ratio of the generalized forces of an ideal tranformer equals the inverse of the ratio of the respective generalized velocities

IDEAL MACHINES – GYRATOR - EXAMPLES

Spinning Top (A type of Gyroscope):

Top Spinning with angular velocity ω

Has angular momentum H = Iω



IDEAL MACHINES – GYRATOR - EXAMPLES

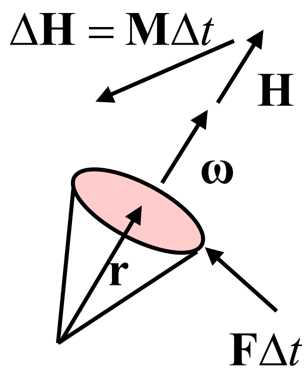
Apply a perpendicular impulsive force $\mathbf{F}\Delta t$

Causes an impulsive moment perpendicular to F,

$\mathbf{M}\Delta t = \mathbf{r} \times \mathbf{F}\Delta t$

The moment equals the change in angular momentum

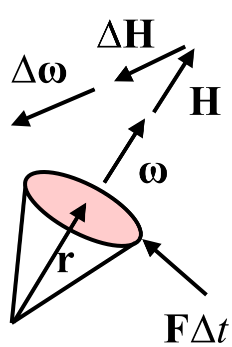
$\mathbf{M}\Delta t = \Delta\mathbf{H}$



IDEAL MACHINES – GYRATOR - EXAMPLES

To get a $\Delta\mathbf{H}$ requires a $\Delta\omega$

The $\Delta\omega$ is perpendicular to both ω and F

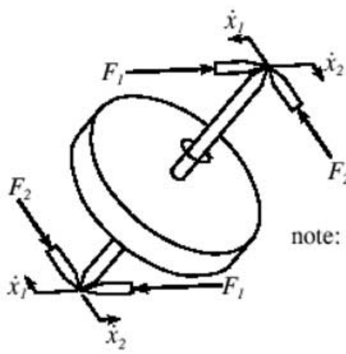


If $\mathbf{F}\Delta t$ is in direction 1 and r is indirection 3

then $\Delta\omega$ is indirection 2

IDEAL MACHINES – GYRATOR - EXAMPLES

Gyroscope:



$$\xrightarrow[\dot{x}_1]{F_1} G \xrightarrow[\dot{x}_2]{F_2}$$

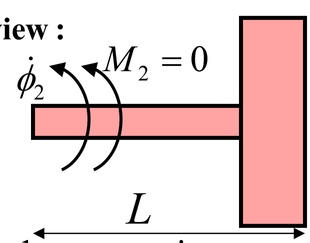
note: axes for x_1 and x_2 are normal

IDEAL MACHINES – GYRATOR - EXAMPLES

Gyroscope:

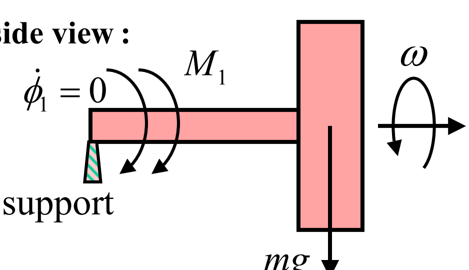
2.25 The shaft of a rapidly spinning rotor (or gyroscope) is horizontal and is supported at a distance L from its center of mass. The axis of the shaft is observed to precess about a vertical axis at a steady rate $\dot{\phi}$. The axis remains horizontal. The rotor has mass m and spins at ω rad/s. Knowing that this device can be represented by a gyrator with modulus $I\omega$, where $I = mr_g^2$ is the mass moment of inertia and r_g is the radius of gyration, determine the rate of precession.

top view :



$\dot{\phi}_2$ is the precession rate

side view :



$$\xrightarrow[\dot{\phi}_1]{M_1} G \xrightarrow[\dot{\phi}_2]{M_2}$$

$$G = I\omega = mr_g^2\omega$$

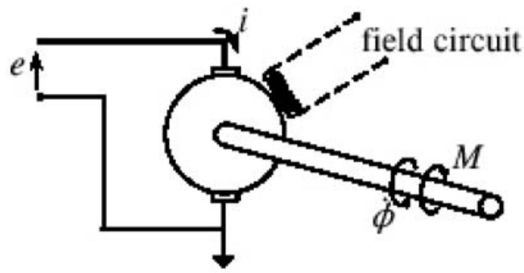
$$M_1 = Lmg$$

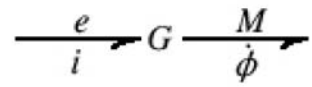
$$M_2 = G\dot{\phi}_1 = 0$$

$$\dot{\phi}_2 = \frac{M_1}{G} = \frac{Lmg}{mr_g^2\omega} = \frac{Lg}{r_g^2\omega}$$

IDEAL MACHINES – GYRATOR - EXAMPLES

Electric Motor / Generator (Tachometer)





IDEAL MACHINES – GYRATOR - EXAMPLES

DC motor with N coils of radius r rotating in magnetic field B:

$$F = 2\pi r N B i \qquad M = G_2 i$$

$$F = G_1 i \qquad G_1 = 2\pi r N B \qquad G_2 = 2\pi r^2 N B$$

$$M = rF \qquad T_1 = 1/r \qquad e = G_2 \dot{\phi}$$