ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 18:

- Nonlinear Compliances and Inertances 3.6

ENERGY STORAGE: COMPLIANCE & INERTANCE

Power: $P = ef$

Energy: $E = \int P dt = \int ef\, dt$

Energy Storage Mechanisms

Compliance → Potential Energy

Store energy by virtue of a generalized displacement

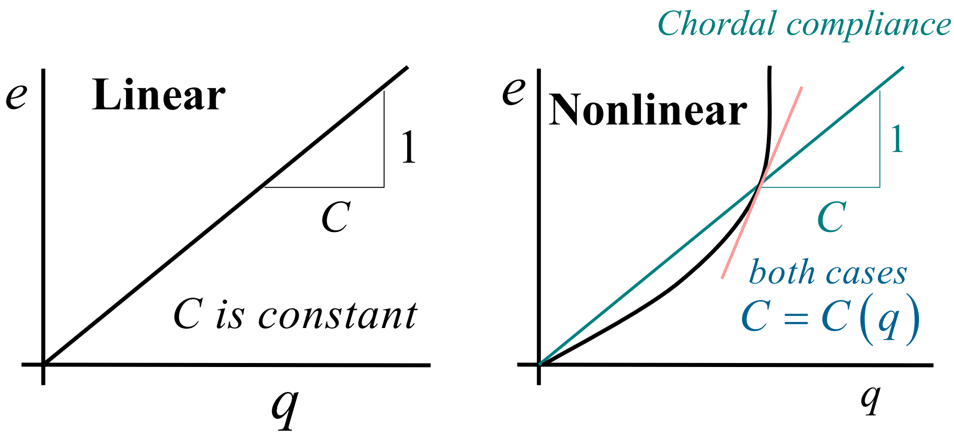
$$E = \int ef\, dt = \int e\dot{q} dt = \int e\frac{dq}{dt} dt = \int e dq, e = e(q)$$

Inertance → Kinetic Energy

Store energy by virtue of a generalized momentum

$$E = \int ef\, dt = \int \dot{p} f\, dt = \int \frac{dp}{dt} f\, dt = \int f\, dp, f = f(p)$$

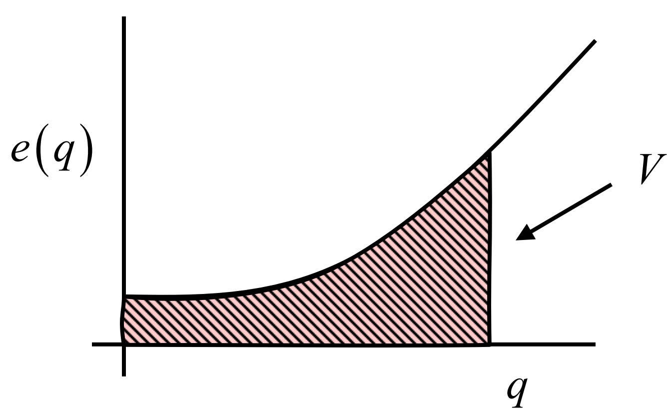
LINEAR vs. NONLINEAR COMPLIANCE



Tangential compliance $\frac{1}{C} = \frac{\partial e(q)}{\partial q}$

POTENTIAL ENERGY STORAGE

$$Potential\ Energy = \int e(q)\,dq = V$$



CHARACTERISTIC

Given: $V = V(q)$

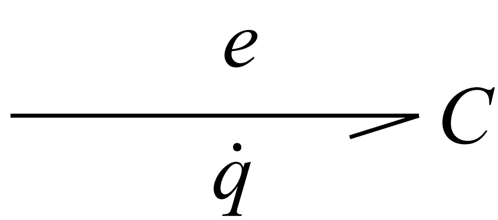
Find: *characteristic*

Recall: $V = \int e(q)\, dq$

Thus: $$\frac{\partial V}{\partial q} = e(q)$$

Governing way to obtain $e(q)$

COMPLIANCE ENERGY STORAGE



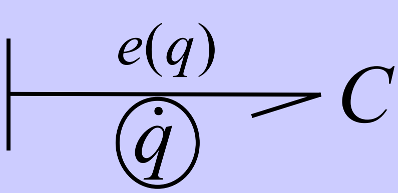
where

$e = e(q)$ *and* $q = q(e)$

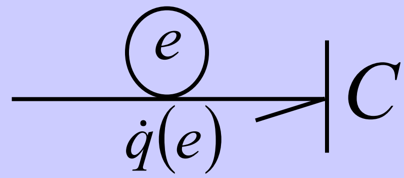
alternative description of compliance characteristics

COMPLIANCE - CAUSALITY

Integral



Derivative



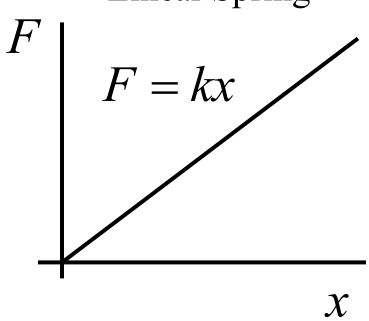
based on

$$q = q(e)$$

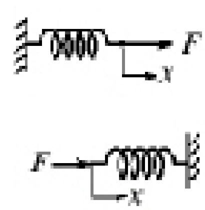
NONLINEAR COMPLIANCES - EXAMPLES

Mechanical Springs

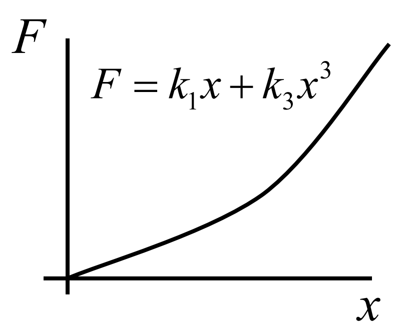
Linear Spring



k is the spring rate,
spring constant
spring stiffness

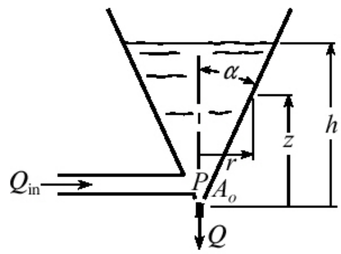


A Hardening Spring Model



NONLINEAR COMPLIANCES - EXAMPLES

A conical-shaped tank ends in a small orifice of area A_0. Determine its fluid compliance as a function of the volume V of liquid in the tank. Then, give a fully annotated bond-graph model, and write a solvable differential equation in terms of V, with Q_{in} treated as an input.



need V as a function of h
need h as a function of P

NONLINEAR COMPLIANCES - EXAMPLES

Solution: The compliance relation can be found as follows:

$$V = \int_0^h \pi r^2 \, dz = \int_0^h \pi z^2 \tan^2 \alpha \, dz = \frac{\pi \tan^2 \alpha \, h^3}{3} = \frac{\pi \tan^2 \alpha}{3} \left(\frac{P}{\rho g} \right)^3 .$$

This result is inverted to get the desired form $P = P(V)$:

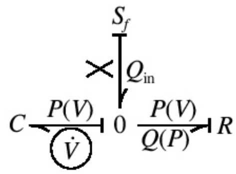
$$P = \rho g \left(\frac{3V}{\pi \tan^2 \alpha} \right)^{1/3} .$$

The expression $P = V/C$ still applies, although at this point finding C is an unnecessary extra step:

$$C(V) = \frac{V}{P(V)} = \frac{1}{\rho g} \left(\frac{\pi \tan^2 \alpha \, V^2}{3} \right)^{1/3} .$$

NONLINEAR COMPLIANCES - EXAMPLES

The fully annotated bond graph below includes the effort $P(V)$ above, and the orifice flow resistance relation $Q(P)$.



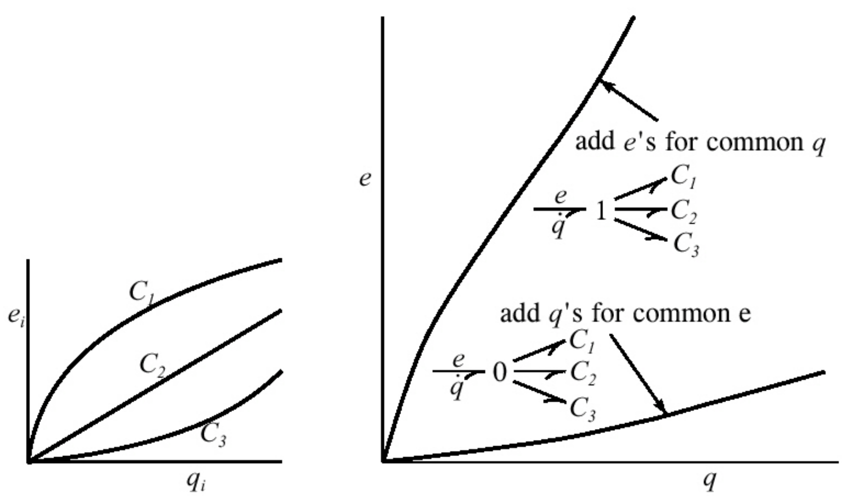
Using Bernoulli's equation for the nonlinear resistance that gives $Q(P)$,

$$\frac{dV}{dt} = Q_{in} - Q(P) = Q_{in} - A_0\sqrt{\frac{2}{\rho}P} = Q_{in} - A_0\sqrt{2g\left(\frac{3V}{\pi \tan^2 \alpha}\right)^{1/3}},$$

This equation is solvable, since the only unknowns on its right side are the given excitation Q_{in} and the state variable V.

numerically

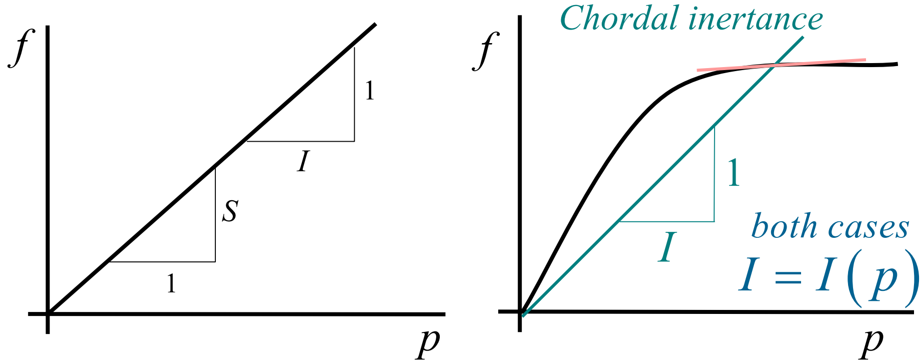
EQUIVALENT COMPLIANCES



(a) individual characteristics (b) combined characteristics

LINEAR vs. NONLINEAR INERTANCE

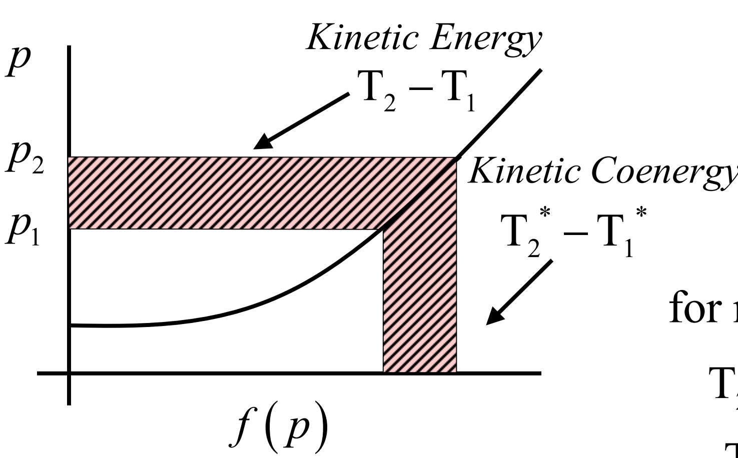
Linear | **Nonlinear**



Tangential inertance $\frac{1}{I} = \frac{\partial f(p)}{\partial p}$

KINETIC ENERGY STORAGE

$$Kinetic\ Energy = \int f\ dp = \int f(p)\, dp = \mathrm{T}_2 - \mathrm{T}_1$$



for nonlinear

$\mathrm{T}_2 - \mathrm{T}_1 \neq \mathrm{T}_2^* - \mathrm{T}_1^*$

CHARACTERISTIC

$$\text{Given: } T = T(p)$$

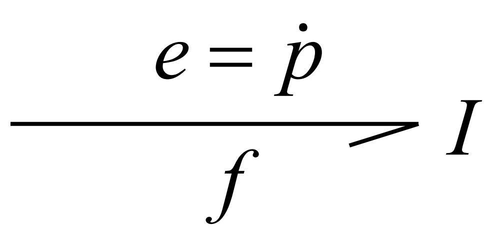
$$\text{Find: } \textit{characteristic}$$

$$\text{Recall: } T = \int f(p)\,dp$$

$$\text{Thus: } \frac{\partial T}{\partial p} = f(p)$$

Governing way to obtain $f(p)$

INERTANCE ENERGY STORAGE

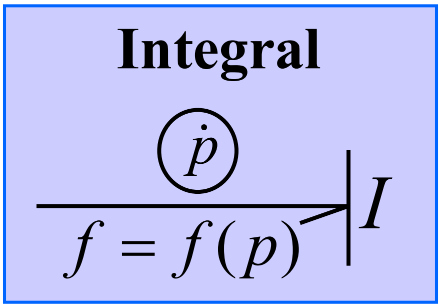


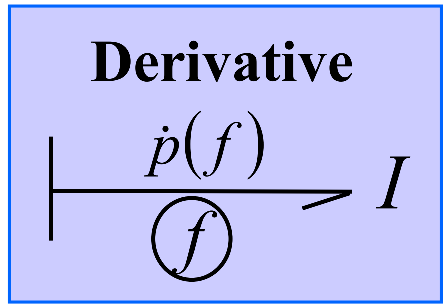
where

$$p = p(f) \quad \textit{and} \quad f = f(p)$$

alternative description of inertance characteristics

INERTANCE - CAUSALITY



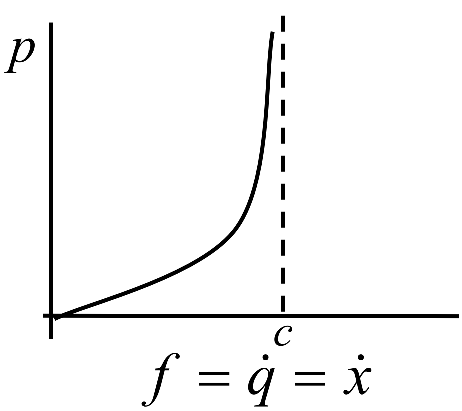


based on

$p = p(f)$

NONLINEAR INERTANCES - EXAMPLES

relativistic mechanics



magnetic saturation