ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 15:

- State Variable Representation – Transfer Function 5.3

MODEL REPRESENTATION

Scalar Differential Equation

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^n} + \cdots + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m u}{dt^m} + b_{m-1} \frac{d^{m-1} u}{dt^{m-1}} + \cdots + b_1 \frac{du}{dt} + b_0 u$$

Transfer Function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

State Variable Representation

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + Bu$$

$$y = C\mathbf{x} + Du$$

Transfer Function

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1} B + D$$

STATE VARIABLE → SCALAR FORM

Example:

$$\frac{dp}{dt} = -2p + 2q + 24u(t)$$
$$\frac{dq}{dt} = 4p - 9q + 4\frac{du(t)}{dt} + 2u(t)$$

Find: - Transfer function between $q(t)$ and $u(t)$
- Scalar ODE for $q(t)$

SCALAR FORM → STATE VARIABLE

Example:

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = 2\frac{du}{dt} + u$$

Find: - State variable representation

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} = \frac{Y(s)}{X(s)}\frac{X(s)}{U(s)}$$

$$\frac{X(s)}{U(s)} = \frac{1}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \qquad \frac{Y(s)}{X(s)} = b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0$$

Choosing $x_1 = x^{(n-1)}, x_2 = x^{(n-2)}, \ldots, x_{n-1} = x^{(1)}, x_n = x$

$$A = \begin{bmatrix} -a_{n-1} & -a_{n-2} & \cdots & -a_1 & -a_0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ . & . & \cdots & . & . \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 \end{bmatrix}, D = 0$$