ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 9:

- Causality and Differential Equations 3.4

HOMEWORK 3: Due on 2/11/05

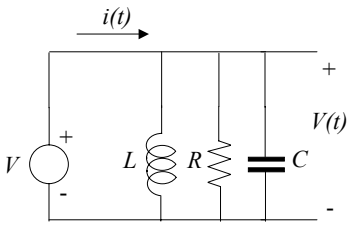
- Problem 3.17
- Problem 3.18
- Problem 3.20
- Problem 3.30
- Problem 3.32
- Problem 3.38
- Problem 3.39
- Problem 3.40

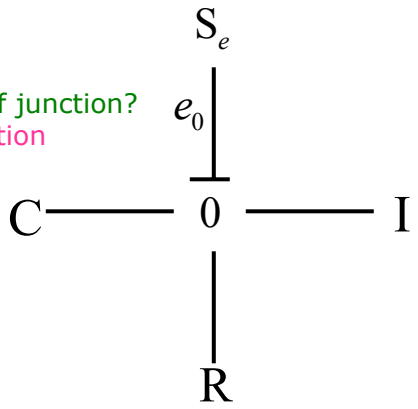
DYNAMIC MODELS

Example: RLC Parallel

Shared Variable?
Voltage (effort)

Type of junction?
0-Junction





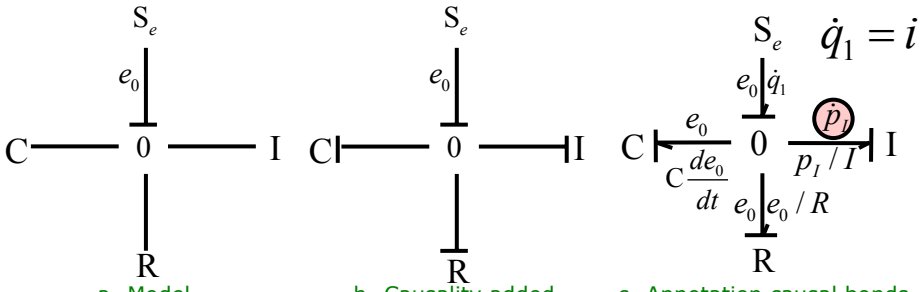
Type of behavior?
Uncoupled

$$C = C,\ R = R,\ I = L$$

$$e_o = V$$

DYNAMIC MODELS

Example: RLC Parallel



a- Model b- Causality added c- Annotation causal bonds

$$\dot{p}_I = e_o$$

$$\dot{q}_1 = C\frac{de_o}{dt} + \frac{e_o}{R} + \frac{p_I}{I}$$

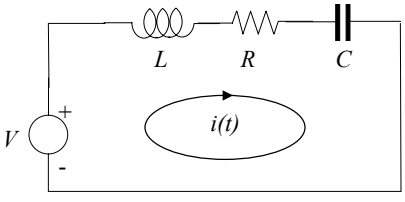
$$i = C\frac{dV}{dt} + \frac{V}{R} + \frac{1}{L}\int V dt$$

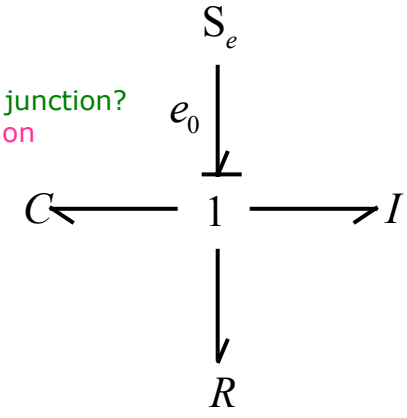
DYNAMIC MODELS

Example: RLC Series

Shared Variable?
Current (flow)

Type of junction?
1-Junction





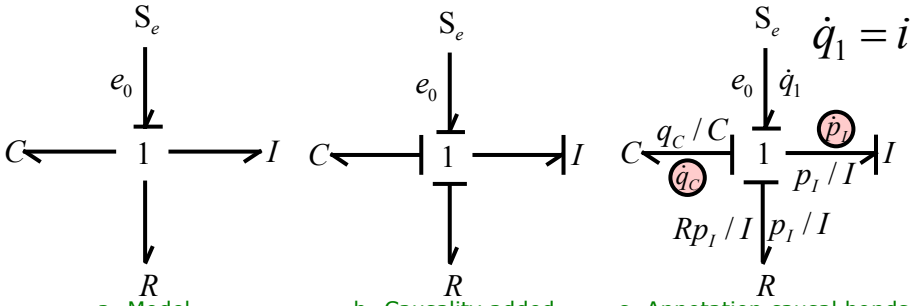
Type of behavior?
Coupled

$$\mathrm{C} = C,\ \mathrm{R} = R,\ \mathrm{I} = L$$

$$e_o = V$$

DYNAMIC MODELS

Example: RLC Series



a- Model b- Causality added c- Annotation causal bonds

$$\dot{q}_C = \frac{p_I}{I}$$

$$e_o = \frac{q_C}{C} + \dot{p}_I + R\frac{p_I}{I}$$

$$V = \frac{1}{CL}\int\left(\int V_L dt\right)dt + V_L + \frac{R}{L}\int V_L dt$$

$$V = V_C + LC\frac{d^2V_C}{dt^2} + RC\frac{dV_C}{dt}$$

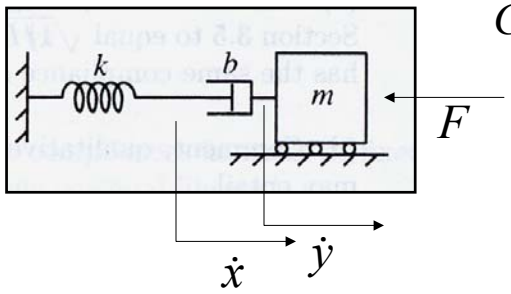
$$i = C\frac{dV_C}{dt} = \frac{1}{L}\int V_L dt$$

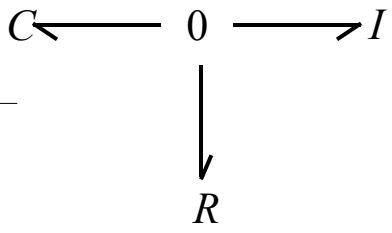
DYNAMIC MODELS

Example: Mechanical System I

Shared Variable?
Force (effort)

Type of junction?
0-Junction



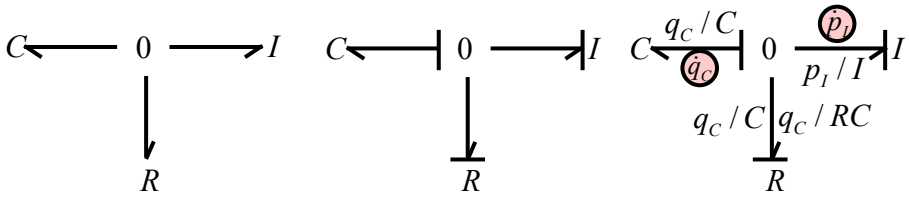


Type of behavior?
Coupled

$$\mathrm{C} = \frac{1}{k},\ \mathrm{R} = b,\ \mathrm{I} = m$$

DYNAMIC MODELS

Example: Mechanical System I



a- Model b- Causality added c- Annotation causal bonds

$$\dot{p}_I = \frac{q_C}{C}$$

$$0 = \dot{q}_C + \frac{q_C}{RC} + \frac{p_I}{I}$$

$$0 = \dot{x} + \frac{kx}{b} + \frac{k}{m}\int x dt$$

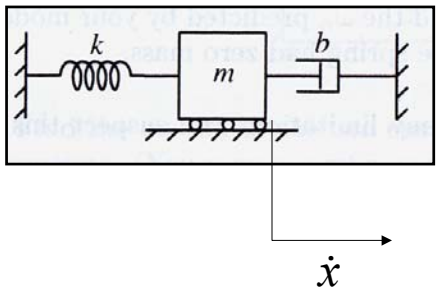
$$0 = m\ddot{x} + \frac{mk}{b}\dot{x} + kx$$

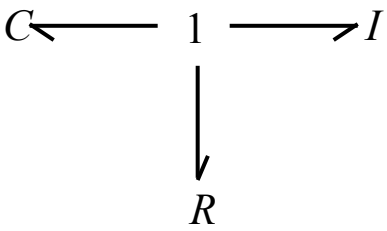
DYNAMIC MODELS

Example: Mechanical System II

Shared Variable?
Velocity (flow)

Type of junction?
1-Junction



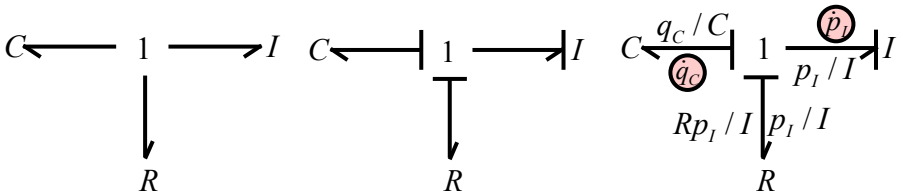


Type of behavior?
Coupled

$$C = \frac{1}{k},\ R = b,\ I = m$$

DYNAMIC MODELS

Example: Mechanical System II



a- Model b- Causality added c- Annotation causal bonds

$$\dot{q}_C = \frac{p_I}{I}$$

$$0 = kx + m\ddot{x} + b\dot{x}$$

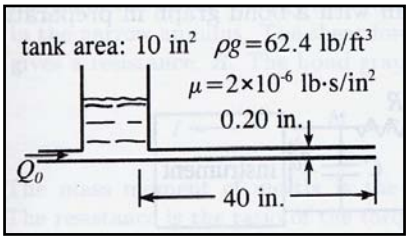
$$0 = \frac{q_C}{C} + \dot{p}_I + R\frac{p_I}{I}$$

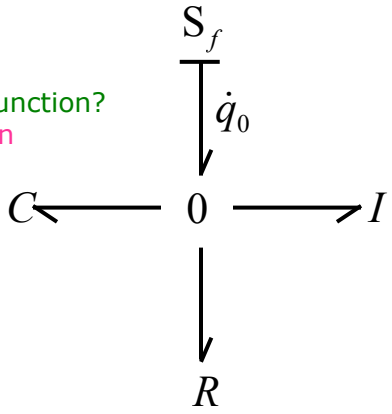
DYNAMIC MODELS

Example: Fluid System

Shared Variable?
Pressure (effort)

Type of junction?
0-Junction





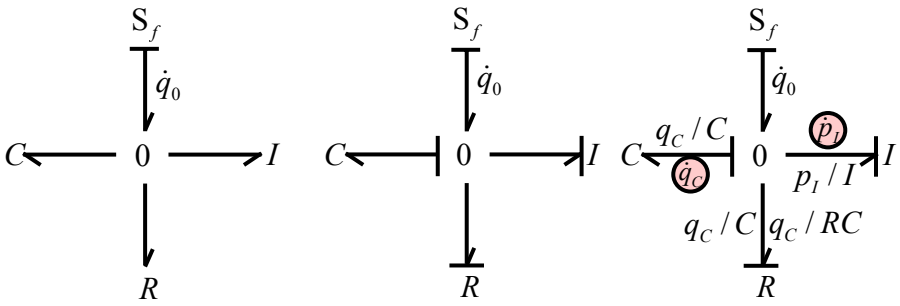
Type of behavior?
Coupled

$$C = \frac{A}{\rho g},\ R = \frac{8\mu L}{\pi r^4},\ I = \frac{\rho L}{\pi r^2}$$

$$\dot{q}_o = Q$$

DYNAMIC MODELS

Examples: Fluid System



a- Model b- Causality added c- Annotation causal bonds

$$\dot{p}_I = \frac{q_C}{C}$$

$$\dot{q}_o = \dot{q}_C + \frac{q_C}{RC} + \frac{p_I}{I}$$

$$Q = \frac{dV}{dt} + \frac{1}{RC}V + \frac{1}{IC}\int V dt$$

$$\dot{Q} = \frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{IC}V$$