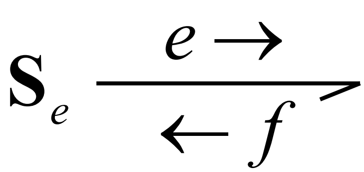
ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 8:

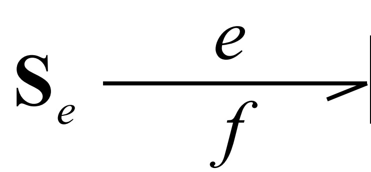
- Causality and Differential Equations 3.4

CAUSALITY OF EFFORT SOURCES

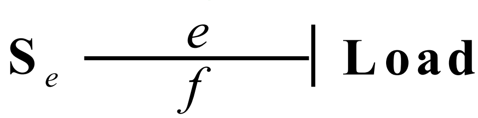
This Bilateral CAUSALITY can be indicated as:



or



This is not a power flow concept, it is a CAUSALITY concept

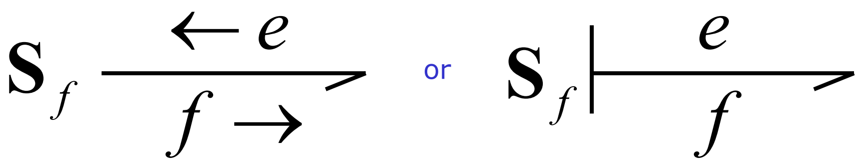


$\mathbf{S}_e$ causes e **Load** causes f

$$f = f_{Load}(e)$$

CAUSALITY OF FLOW SOURCES

This Bilateral CAUSALITY can be indicated as:



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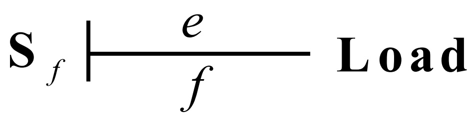
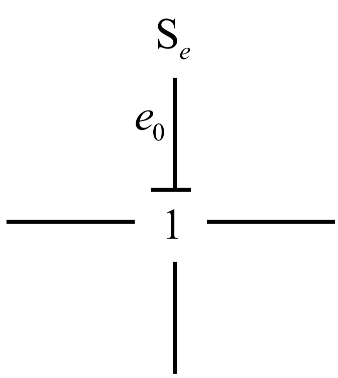


$\mathbf{S}_f$ causes f **Load** causes e

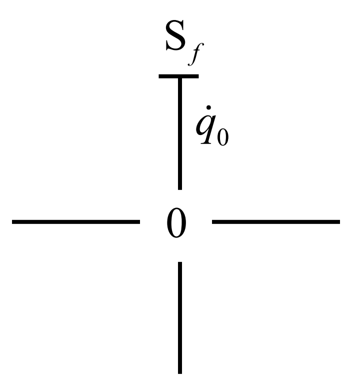
$$e = e_{Load}(f)$$

JUNCTIONS WITH COUPLED BEHAVIORS

S_e and the 1 junction:



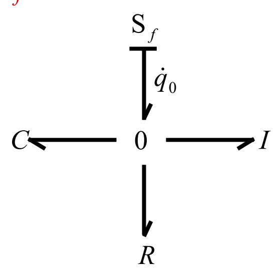
S_f and the 0 junction:

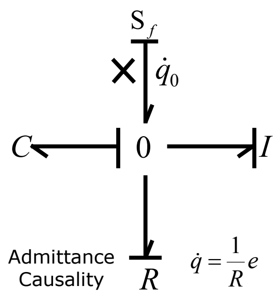


The source does NOT directly determine either the efforts or the flows on any bonds other than the source bond itself

JUNCTIONS WITH COUPLED BEHAVIORS

S_f and the 0 junction:





Admittance Causality $\dot{q} = \frac{1}{R} e$

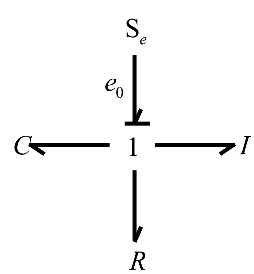
- In this case, the source does **NOT** directly determine the effort associated with the 0-Junction
- Who does? It must be one of the attached bonds!
- How? One bond will have its causal stroke adjacent to the junction. It will imposed flow to the attached element and will imposed effort to the junction as reaction.
- Due to the properties of the 0-Junction, the other bonds will have the causal strokes placed at the outer end. The effort is imposed by the junction to these bonds.
- We have, in this case, three possible patterns!
- Which one do we use? We use INTEGRAL CAUSALITY! C or I with stroke adjacent to junction?

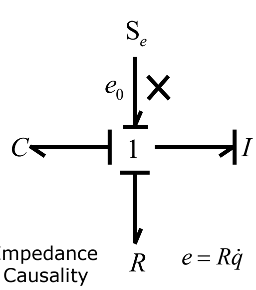
$$C: \quad q = Ce \Rightarrow \dot{q} = C\frac{de}{dt} \quad \text{or} \quad e = \frac{1}{C}\int \dot{q}dt$$

$$I: \quad p = I\dot{q} \Rightarrow e = \dot{p} = I\frac{d\dot{q}}{dt} \quad \text{or} \quad \dot{q} = \frac{1}{I}\int edt$$

JUNCTIONS WITH COUPLED BEHAVIORS

S_e and the 1 junction:





Impedance Causality $e = R\dot{q}$

- In this case, the source does **NOT** directly determine the flow associated with the 1-Junction
- Who does? It must be one of the attached bonds!
- How? One bond will have its causal stroke at the outer end of the junction. It will imposed effort to the attached element and will imposed flow to the junction as reaction.
- Due to the properties of the 1-Junction, the other bonds will have the causal strokes adjacent to the junction. The flow is imposed by the junction to these bonds.
- We have, in this case, three possible patterns!
- Which one do we use? We use INTEGRAL CAUSALITY! C or I with stroke adjacent to junction?

$$C: \quad q = Ce \Rightarrow \dot{q} = C\frac{de}{dt} \quad \text{or} \quad e = \frac{1}{C}\int \dot{q}dt$$

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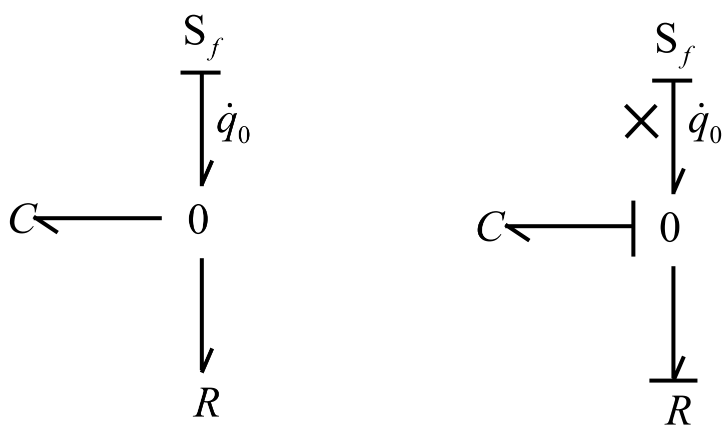
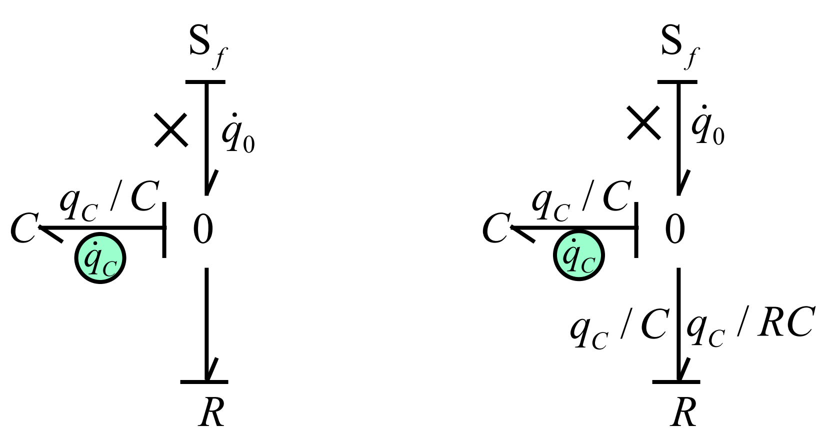
S_f AND THE 0 JUNCTION

First Order Problem: RC Model



(a) model (b) integral causality added

S_f AND THE 0 JUNCTION



(c) annotation of causal bonds (d) annotation completion

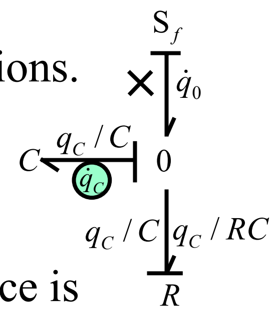
S_f AND THE 0 JUNCTION

The state of the capacitance is determined by q_C.

$$\dot{q}_C = \dot{q}_0 - q_C / RC$$

There is one state differential equations.

The order of the system is one.

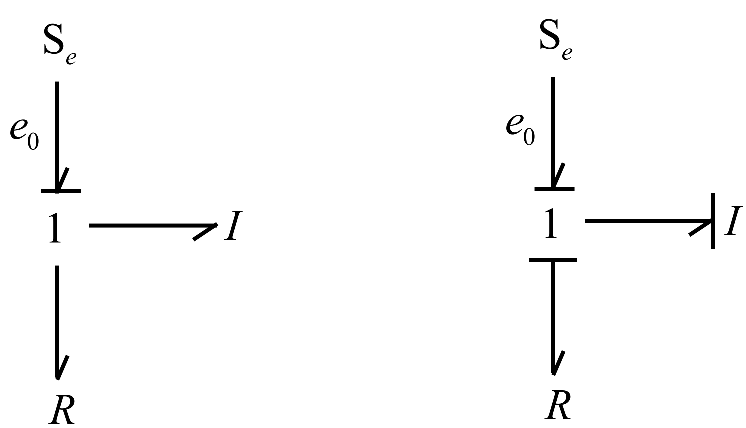


The effort required of the flow source is

$$e_f = q_C / C$$

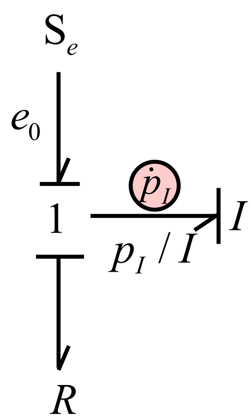
S_e AND THE 1 JUNCTION

First Order Problem: IR Model

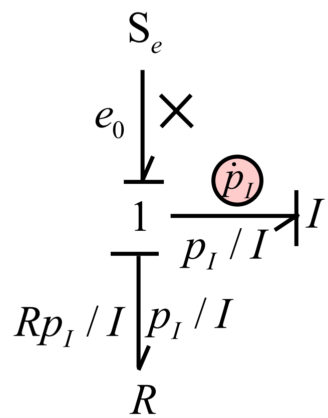


(a) model (b) integral causality added

S_e AND THE 1 JUNCTION



(c) annotation of causal bonds

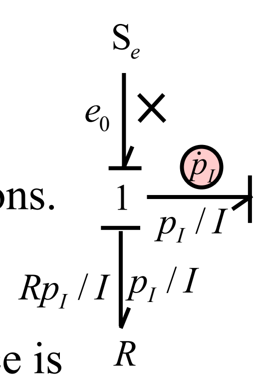


(d) annotation completion

S_e AND THE 1 JUNCTION

The state of the inertance is determined by p_I.

$$\dot{p}_I = e_0 - Rp_I / I$$



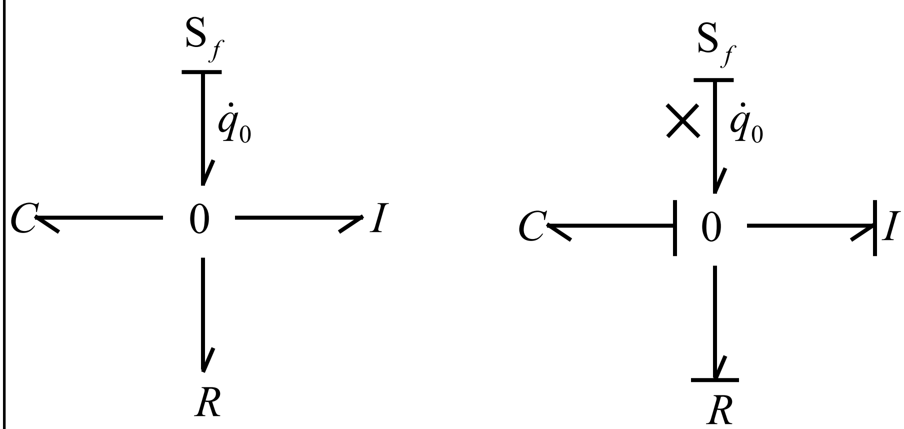
There is one state differential equations.
The order of the system is one.

The flow required of the effort source is

$$f_e = p_I / I$$

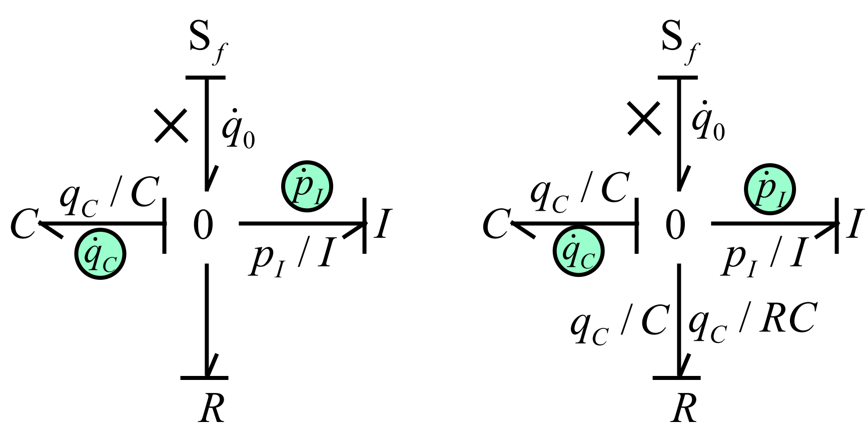
S_f AND THE 0 JUNCTION

Second Order Problem: IRC Model



(a) model   (b) integral causality added

S_f AND THE 0 JUNCTION



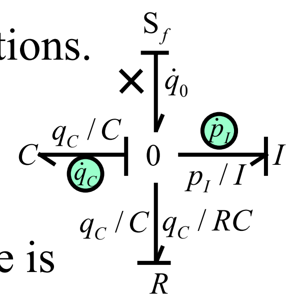
(c) annotation of causal bonds   (d) annotation completion

S_f AND THE 0 JUNCTION

The state of the inertance and the capacitance is determined by q_C and p_I respectively.

$$\dot{p}_I = q_C / C \qquad \dot{q}_C = \dot{q}_0 - p_I / I - q_C / RC$$

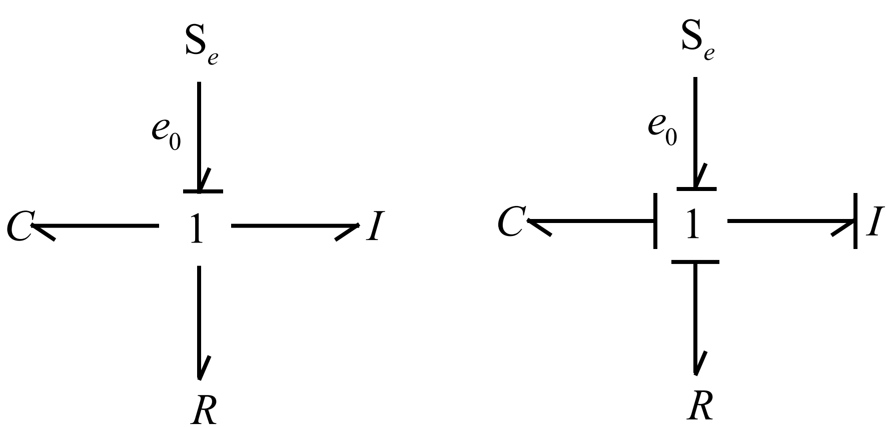
There are two state differential equations.
The order of the system is two.



The effort required of the flow source is

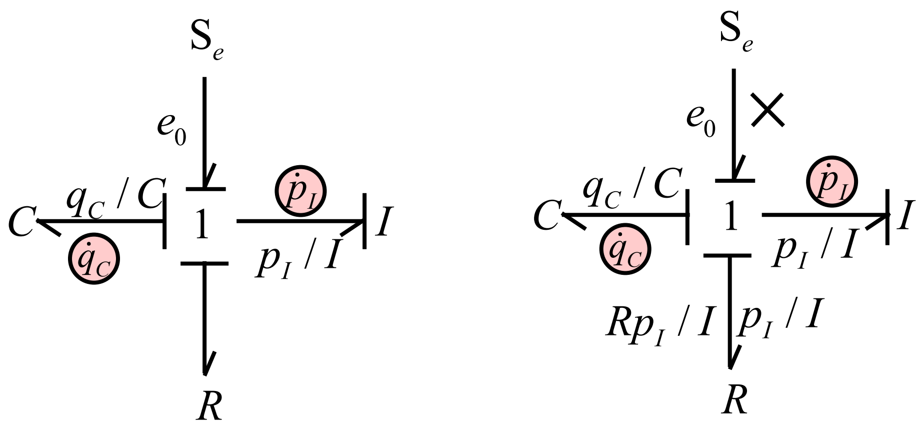
$$e_f = q_C / C$$

S_e AND THE 1 JUNCTION



(a) model (b) integral causality added

S_e AND THE 1 JUNCTION



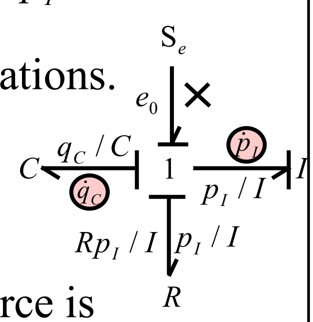
(c) annotation of causal bonds (d) annotation completion

S_e AND THE 1 JUNCTION

The state of the inertance and the capacitance is determined by q_C and p_I respectively.

$$\dot{p}_I = e_0 - q_C / C - Rp_I / I \qquad \dot{q}_C = p_I / I$$

There are two state differential equations.
The order of the system is two.



The flow required of the effort source is

$$f_e = p_I / I$$