ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 7:

- Causality and Differential Equations 3.4

DYNAMICS

Dynamic behavior of well-posed model with energy storage elements

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DIFFERENTIAL EQUATION

↙ ↘

Analytical Solution Numerical Solution

Approach: Each independent energy storage element

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One first-order differential equation

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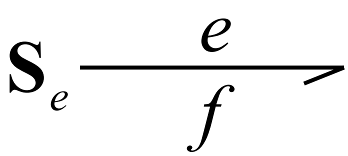
STATE VARIABLE REPRESENTATION

CAUSALITY OF EFFORT SOURCES

Effort Source:



$$e = e(t), e \neq e(f)$$

Effort is imposed by the source

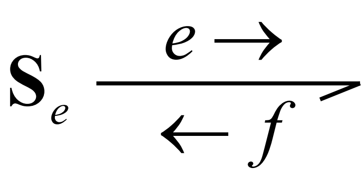
The effort e is CAUSED by action of S_e

Flow is imposed by…? Whatever system is attached to the bond

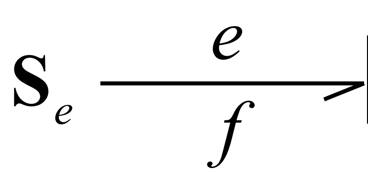
The flow f is CAUSED by system reaction

CAUSALITY OF EFFORT SOURCES

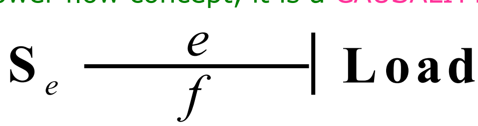
This Bilateral CAUSALITY can be indicated as:



or



This is not a power flow concept, it is a CAUSALITY concept

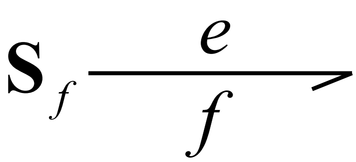


$\mathbf{S}_e$ causes e **Load** causes f

$$f = f_{Load}(e)$$

CAUSALITY OF FLOW SOURCES

Flow Source:



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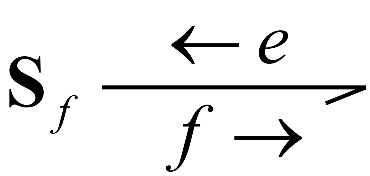
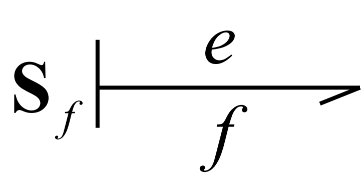


or



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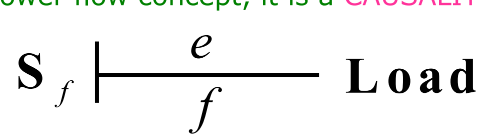
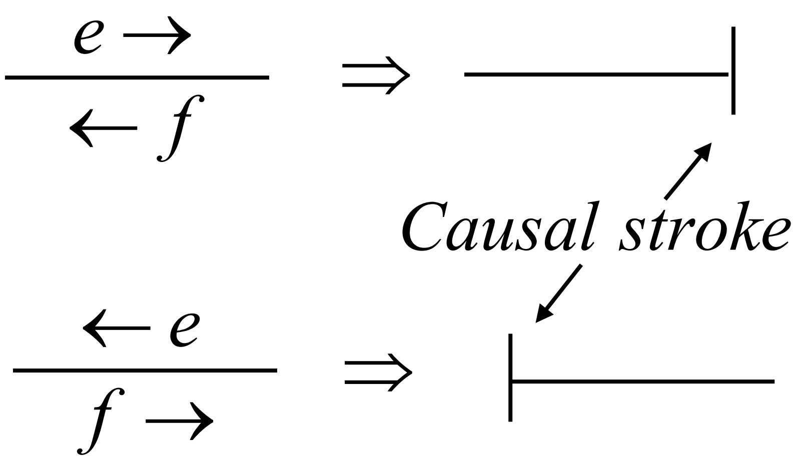


$\mathbf{S}_f$ causes f **Load** causes e

$$e = e_{Load}(f)$$

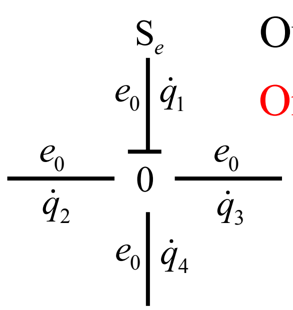
CAUSALITY

The CAUSALITY is Bilateral



JUNCTIONS WITH UNCOUPLED BEHAVIORS

S_e and the 0 junction:



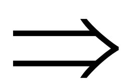
Output effort from $\mathbf{S}_e$ is input to 0

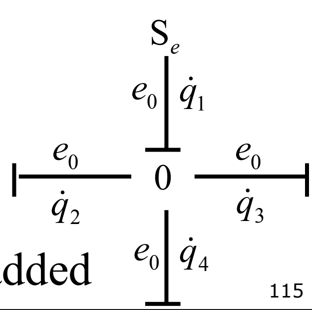
Only one effort input to 0 is possible.

All efforts are equal.

All other efforts are outputs

(a) model

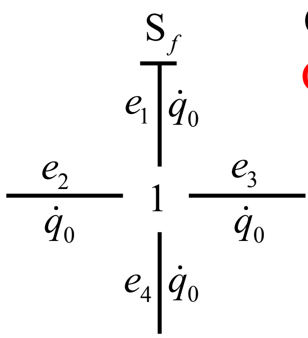




(b) causality added

JUNCTIONS WITH UNCOUPLED BEHAVIORS

S_f and the 1 junction:



(a) model

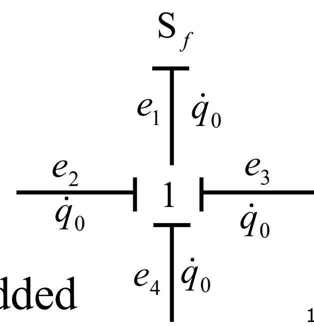
Output flow from S_f is input to 1

Only one flow input to 1 is possible.

All flows are equal.

All other flows are outputs

$\Rightarrow$

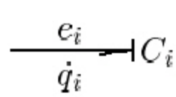


(b) causality added

CAUSALITY TYPES FOR C AND I

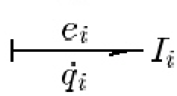
Differential Causality:

compliance :



$$\dot{q}_i = C_i \frac{de_i}{dt},$$

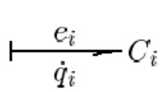
inertance :



$$e_i = I_i \frac{d\dot{q}}{dt}$$

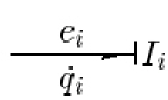
Integral Causality:

compliance :



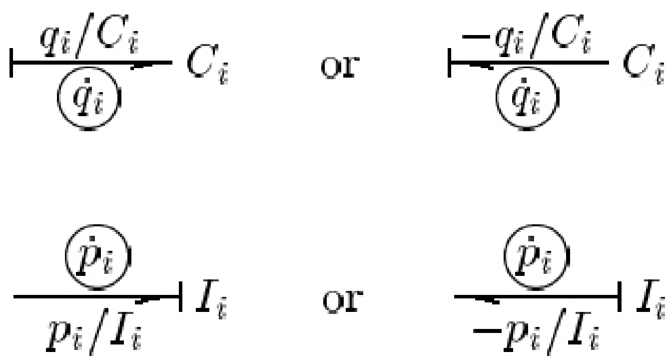
$$e_i = e_i(q_i) = \frac{q_i}{C_i} = \frac{1}{C_i} \int \dot{q}_i \, dt,$$

inertance :



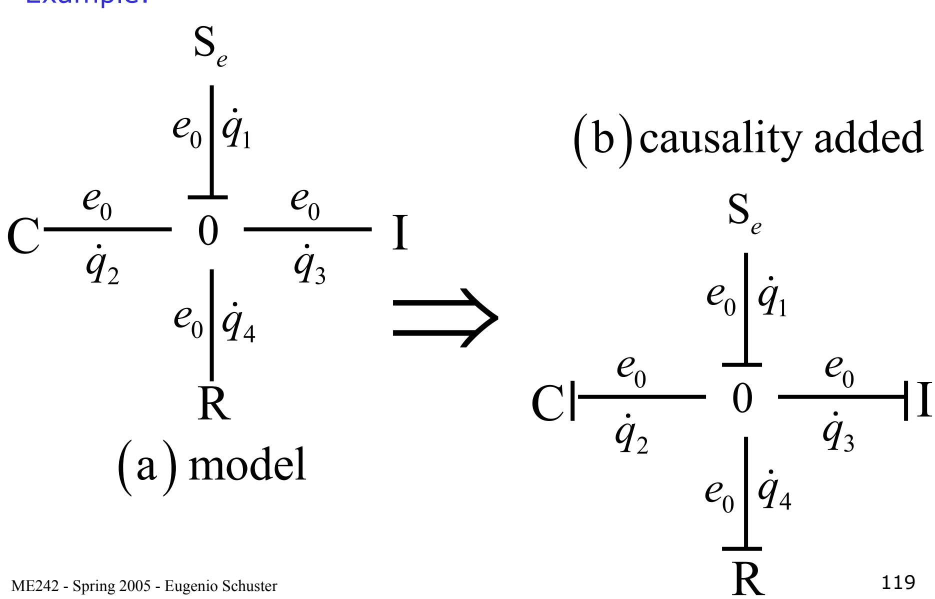
$$\dot{q}_i = \dot{q}(p_i) = \frac{p_i}{I_i} = \frac{1}{I_i} \int e_i \, dt$$

POWER CONVENTION



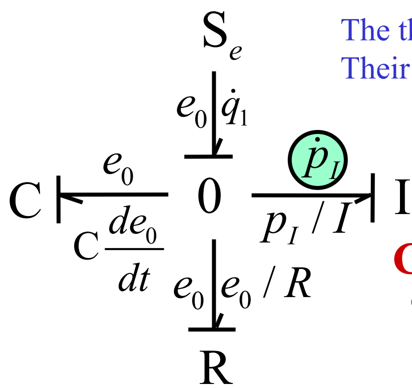
S_e AND THE 0 JUNCTION

Example:



S_e AND THE 0 JUNCTION

(c) annotation of causal bonds



The three elements have admittance causality
Their flow respond to an imposed effort

C has **differential** causality.

Causal Output = Derivative of Causal Input

I has **integral** causality.

Causal Output = Integral of Causal Input

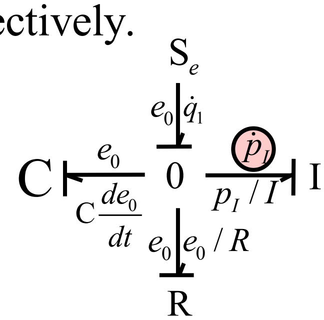
Element behavior is **uncoupled**.

S_e AND THE 0 JUNCTION

The state of the inertance and the capacitance is determined by q_C and p_I respectively.

$$\dot{p}_I = e_0$$

$$q_C = Ce_0 \Leftrightarrow \dot{q}_C = C\frac{de_0}{dt}$$



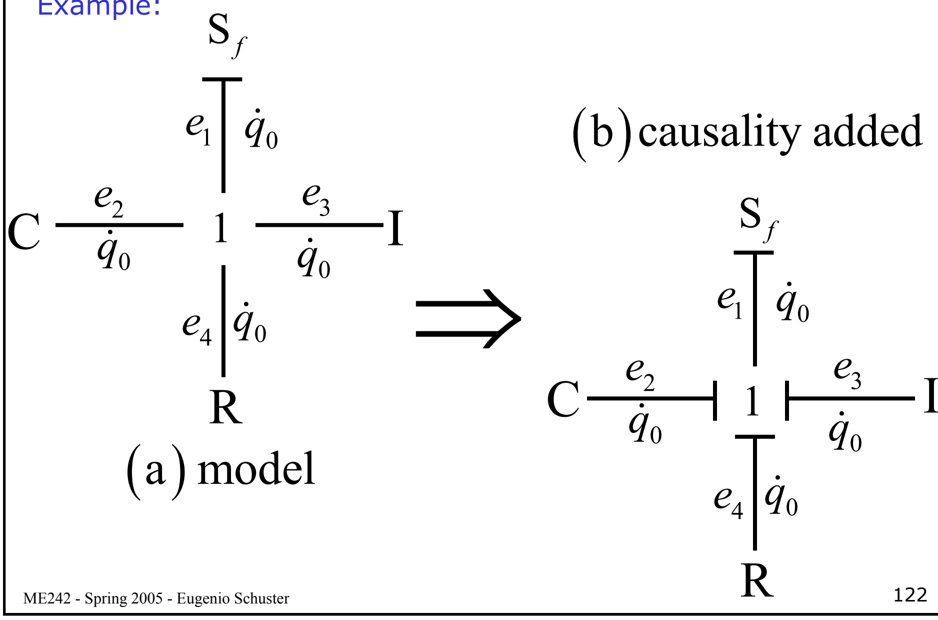
The order of the systems is two

The flow required of the effort source is

$$\dot{q}_1 = C\frac{de_0}{dt} + \frac{e_0}{R} + \frac{p_I}{I}$$

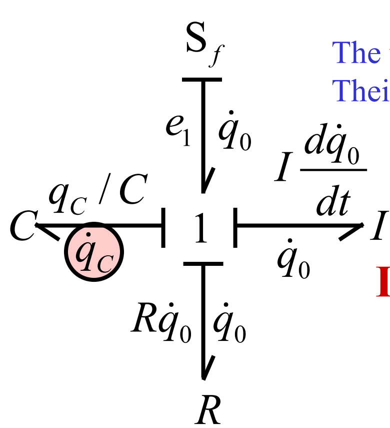
S_f AND THE 1 JUNCTION

Example:



S_f AND THE 1 JUNCTION

(c) annotation of causal bonds



The three elements have impedance causality
Their efforts respond to an imposed flow

I has **differential** causality.

Causal Output = Derivative of Causal Input

C has **integral** causality.

Causal Output = Integral of Causal Input

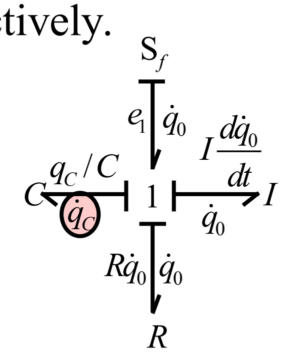
Element behavior is **uncoupled**.

S_f AND THE 1 JUNCTION

The state of the inertance and the capacitance is determined by q_C and p_I respectively.

$$\dot{q}_C = \dot{q}_0$$

$$\dot{p}_I = e_I = I\frac{d\dot{q}_0}{dt}$$



The order of the systems is two

The effort required of the flow source is

$$e_1 = I\frac{d\dot{q}_0}{dt} + R\dot{q}_0 + q_c / C$$