ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 6:

- Junctions 3.3

HOMEWORK 2: Due on 2/4/05

- Problem 3.1
- Problem 3.4
- Problem 3.9
- Problem 3.11
- Problem 3.14
- Problem 3.15
- Problem 3.16
- Problem 3.29

JUNCTIONS

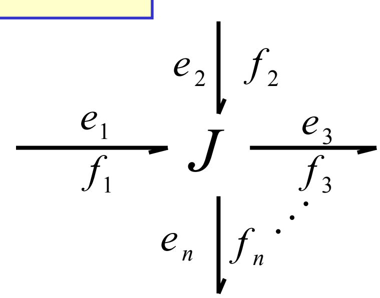
Elements introduced so far $\rightarrow$ ONE PORT

$$S, S_e, S_f, R, C, I$$

At termination (beginning or end)

JUNCTIONS:
- Branching
- Constraints

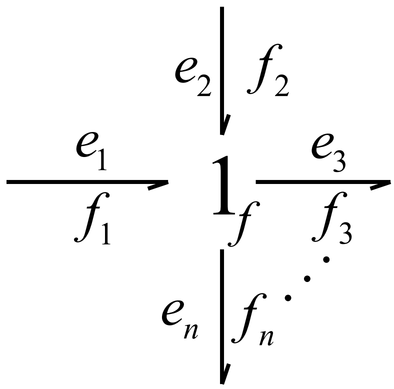
POWER CONSTRAINT: The junction is IDEAL, neither storing, creating, nor dissipating energy



$$\sum P = 0 \Rightarrow \sum P_{in} = \sum P_{out}$$

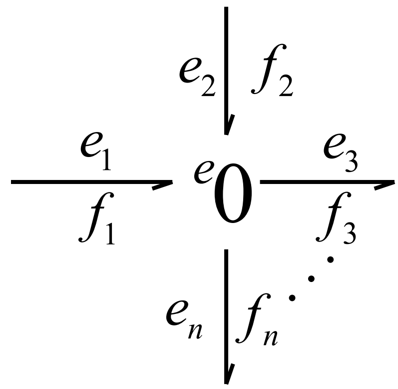
$$e_1 f_1 + e_2 f_2 = e_3 f_3 + \cdots + e_n f_n$$

JUNCTIONS: TYPES



1 - junction

The flows on all bonds are equal



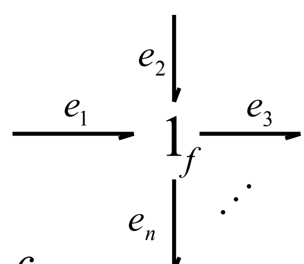
0 - junction

The efforts on all bonds are equal

JUNCTIONS: 1-JUNCTION

COMMON FLOW CONSTRAINT:

$$f_1 = f_2 = f_3 = \cdots = f_n = f$$



POWER CONSTRAINT:

$$e_1 f_1 + e_2 f_2 = e_3 f_3 + \cdots + e_n f_n$$

Then, we have

$$e_1 + e_2 = e_3 + \cdots + e_n \quad \Rightarrow \quad \sum e_{in} = \sum e_{out}$$

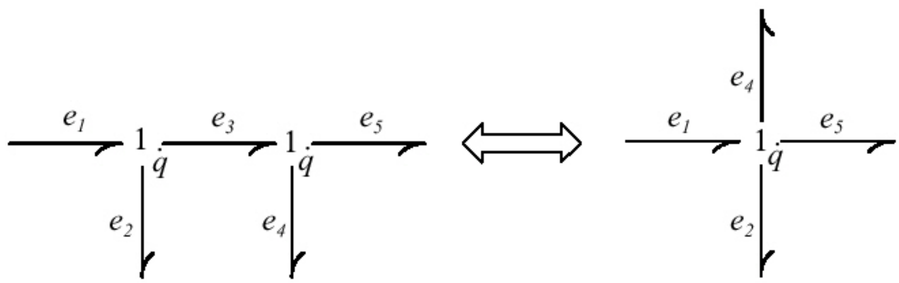
or

$$\sum e = 0$$

The **common flow – sum of effort** junction

JUNCTIONS: 1-JUNCTION

A 1-junction equivalence:

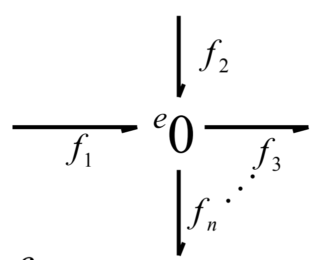


Two junctions of the SAME type connected by a bond can be collapsed into a single junction, with the bond vanishing

JUNCTIONS: 0-JUNCTION

COMMON EFFORT CONSTRAINT:

$$e_1 = e_2 = e_3 = \cdots = e_n = e$$



POWER CONSTRAINT:

$$e_1 f_1 + e_2 f_2 = e_3 f_3 + \cdots + e_n f_n$$

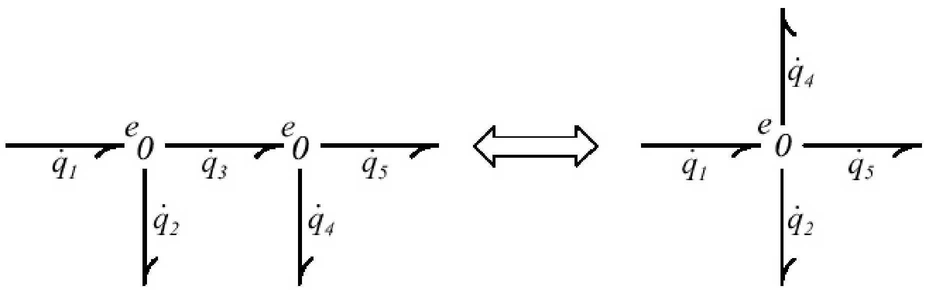
Then, we have

$$f_1 + f_2 = f_3 + \cdots + f_n \quad \Rightarrow \quad \sum f_{in} = \sum f_{out} \quad \textbf{or} \quad \sum f = 0$$

The **common effort – sum of flow** junction

JUNCTIONS: 0-JUNCTION

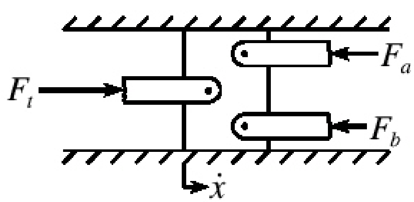
A 0-junction equivalence:

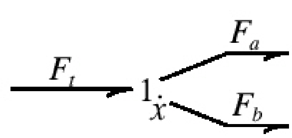


Two junctions of the SAME type connected by a bond can be collapsed into a single junction, with the bond vanishing

1-JUNCTION: MECHANICAL EXAMPLE 1

Push-rods:





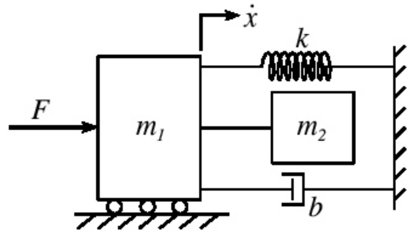
common flow? $\Rightarrow$ $\dot{x}$

$$\sum P_{in} = \sum P_{out} \quad \Rightarrow \quad F_t\dot{x} = F_a\dot{x} + F_b\dot{x}$$

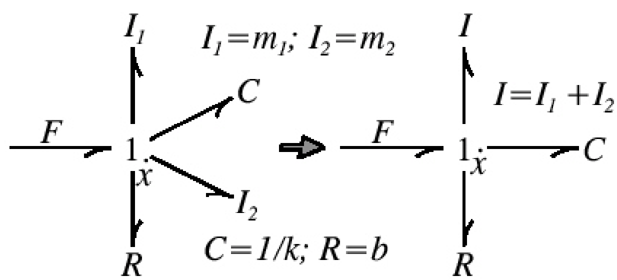
$$\Downarrow$$

$$\sum e_{in} = \sum e_{out} \quad \Leftarrow \quad F_t = F_a + F_b$$

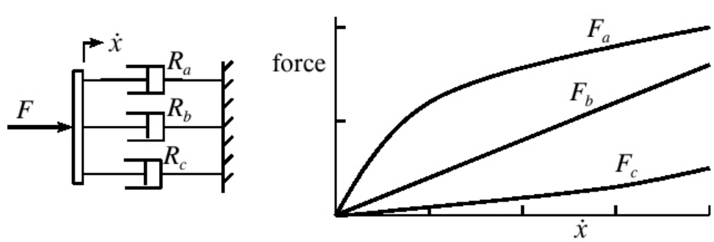
1-JUNCTION: MECHANICAL EXAMPLE 2



common flow? $\Rightarrow \quad \dot{x}$

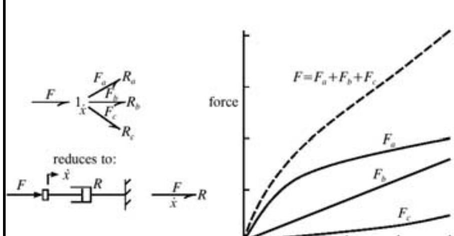


1-JUNCTION: MECHANICAL EXAMPLE 3



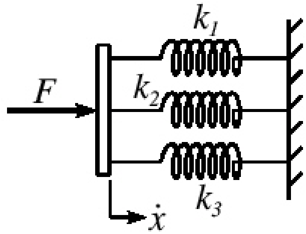
common flow? $\Rightarrow \quad \dot{x}$

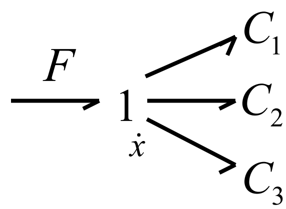
$$\sum e_{in} = \sum e_{out} \quad \Rightarrow \quad F = F_a + F_b + F_c$$
$$= R_a \dot{x} + R_b \dot{x} + R_c \dot{x} = R\dot{x}$$



$$\Rightarrow \quad \boxed{R = R_a + R_b + R_c}$$

1-JUNCTION: MECHANICAL EXAMPLE 4





$C_1 = 1/k_1$

$C_2 = 1/k_2$

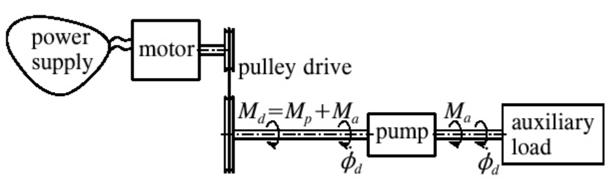
$C_3 = 1/k_3$

common flow? $\Rightarrow$ $\dot{x}$

$$\sum e_{in} = \sum e_{out} \quad \Rightarrow \quad F = F_1 + F_2 + F_3$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \quad \Leftarrow \quad = \frac{x}{C_1} + \frac{x}{C_2} + \frac{x}{C_3} = \frac{x}{C}$$

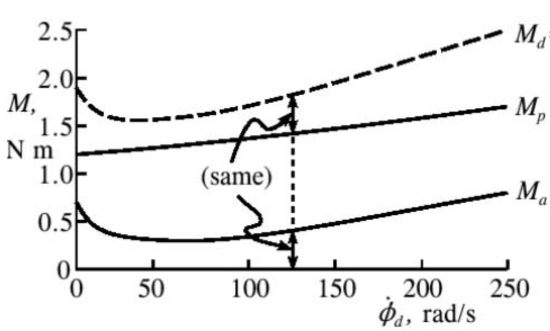
1-JUNCTION: MECHANICAL EXAMPLE 5



common flow?

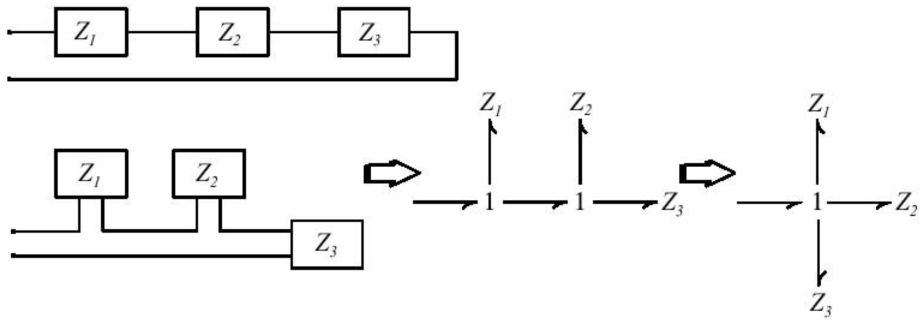
$\dot{\phi}_d$

$$\sum e_{in} = \sum e_{out} \quad \Rightarrow \quad M_d = M_p + M_a$$



1-JUNCTION: ELECTRICAL EXAMPLE

Common Flow ⇒ Common Current ⇒ Series Interconnection



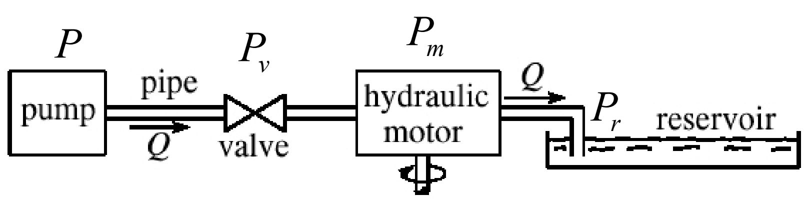
"The sum of the voltage drops (efforts) around a loop or mesh equals zero"

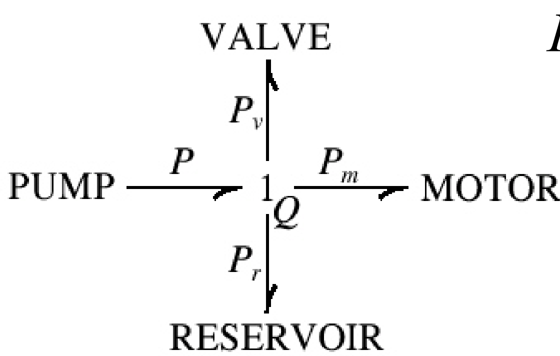
$$Z_i = R_i \Rightarrow R = R_1 + R_2 + R_3$$

$$Z_i = L_i \Rightarrow L = L_1 + L_2 + L_3$$

$$Z_i = C_i \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

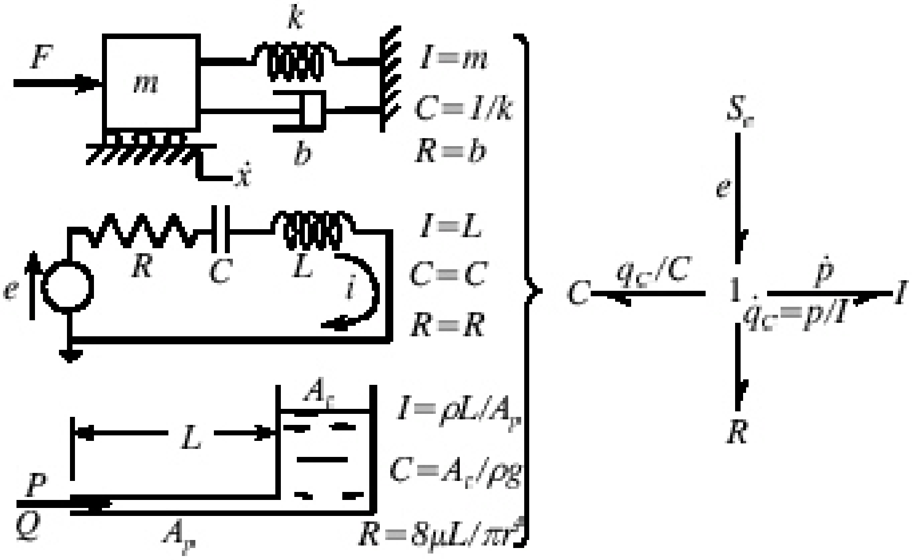
1-JUNCTION: FLUID EXAMPLE



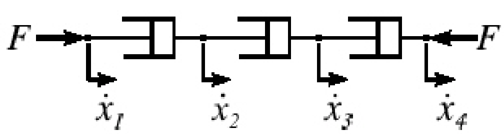


$$P = P_v + P_m + P_r$$

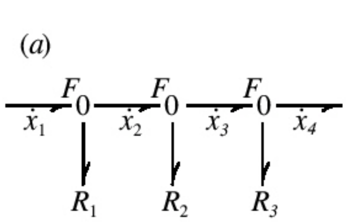
1-JUNCTION: SIMPLE IRC MODELS

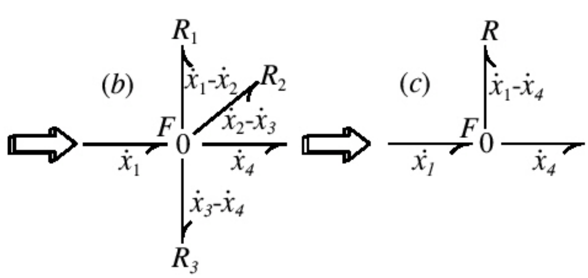


0-JUNCTION: MECHANICAL EXAMPLE 1



common effort? $\Rightarrow$ F

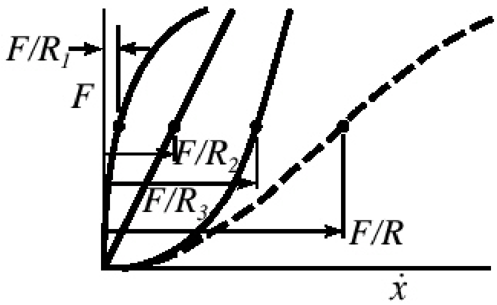




0-JUNCTION: MECHANICAL EXAMPLE 1

$$\sum f_{in} = \sum f_{out} \quad \Rightarrow \quad \dot{x}_1 - \dot{x}_4 = (\dot{x}_1 - \dot{x}_2) + (\dot{x}_2 - \dot{x}_3) + (\dot{x}_3 - \dot{x}_4)$$

$$= \frac{F}{R_1} + \frac{F}{R_2} + \frac{F}{R_3} = \frac{F}{R}$$

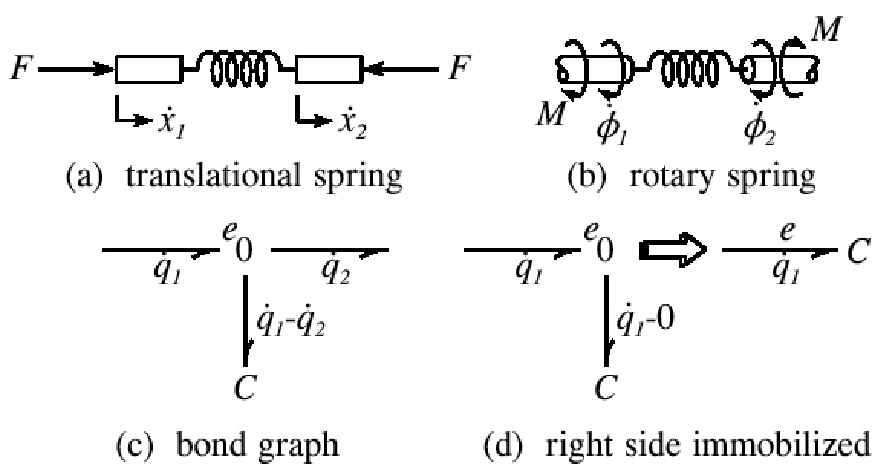


at each F

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

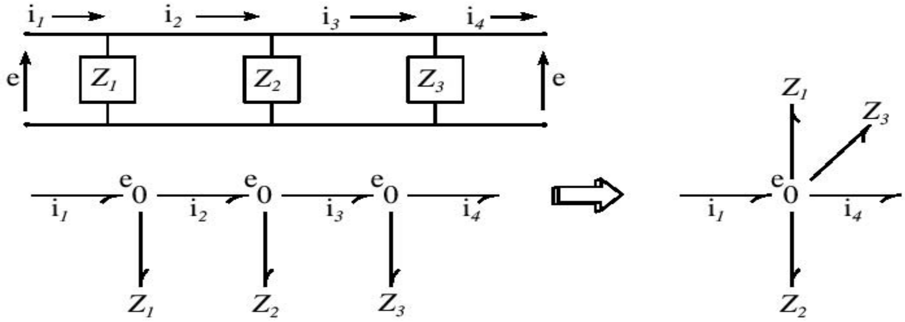
conductance=1/resistance

0-JUNCTION: MECHANICAL EXAMPLE 2



0-JUNCTION: ELECTRICAL EXAMPLE 1

Common Effort ⇒ Common Voltage ⇒ Parallel Interconnection



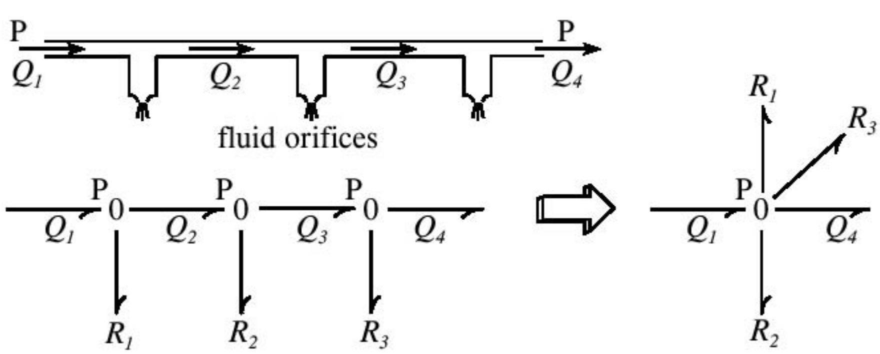
"The sum of the currents (flows) entering a node equals zero"

$$Z_i = R_i \Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$Z_i = L_i \Rightarrow \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$Z_i = C_i \Rightarrow C = C_1 + C_2 + C_3$$

0-JUNCTION: FLUID EXAMPLE



0-JUNCTION: SIMPLE IRC MODELS