ME242 – MECHANICAL ENGINEERING SYSTEMS

LECTURE 4:

- Compliance Energy Storage 3.1

Dynamic Systems

So far, we have introduced:

SOURCES	Emanate Energy	Steady or Equilibrium Systems
RESISTANCES	Dissipate Energy	

We need new players to be able to represent dynamic systems

COMPLIANCES	Store Energy	Unsteady or Dynamic Systems
INERTANCES	Store Energy	

- Dynamic physical systems contain mechanisms that store energy temporarily, for later release.

- The dynamics can be thought of as a sloshing of energy between different energy storage mechanisms, and/or a gradual dissipation of energy in resistances.

Generalized Variables

Generalized Velocity or Flow: f

Generalized Displacement: q

$$f = \dot{q}, \text{ or } q = \int f dt$$

Generalized Force or Effort: e

Generalized Momentum: p

$$e = \dot{p}, \text{ or } p = \int e dt$$

$$\boxed{\mathrm{P} = ef = e\dot{q} = \dot{p}f}$$

ENERGY STORAGE: COMPLIANCE & INERTANCE

Power: $P = ef$

Energy: $E = \int P dt = \int ef\, dt$

Energy Storage Mechanisms

Compliance

Store energy by virtue of a generalized displacement

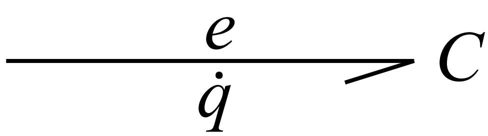
$$E = \int ef\, dt = \int e\dot{q} dt = \int e \frac{dq}{dt} dt = \int e dq$$

Inertance

Store energy by virtue of a generalized momentum

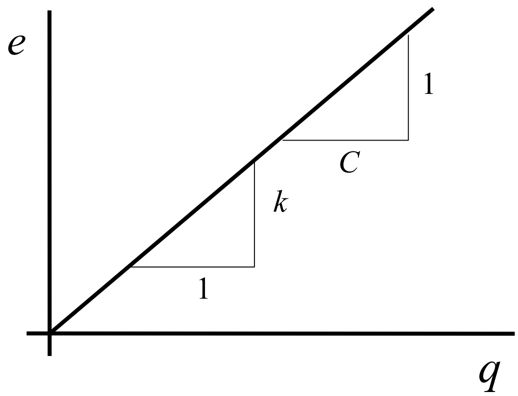
$$E = \int ef\, dt = \int \dot{p} f\, dt = \int \frac{dp}{dt} f\, dt = \int f\, dp$$

ENERGY STORAGE: LINEAR COMPLIANCE



$$e = \frac{1}{C} q$$

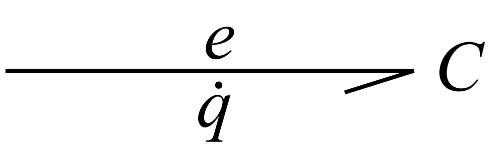
Constitutive Relationship



C Compliance

$k = \frac{1}{C}$ Stiffness

ENERGY STORAGE: LINEAR COMPLIANCE



$$e = \frac{1}{C} q$$

Energy Storage:

Work from point 1 to point 2:

$$W_{1\to 2} = \int_1^2 e\dot{q}dt = \int_1^2 e\frac{dq}{dt}dt = \int_{q_1}^{q_2} edq = \int_{q_1}^{q_2} \frac{1}{C} qdq = \frac{1}{2C} q^2 \Big|_{q_1}^{q_2} = \frac{1}{2C}\left(q_2^2 - q_1^2\right)$$

Work from point 2 to point 1:

$$W_{2\to 1} = \frac{1}{2C}\left(q_1^2 - q_2^2\right) = -W_{1\to 2}$$

The energy is conserved!

Potential Energy:

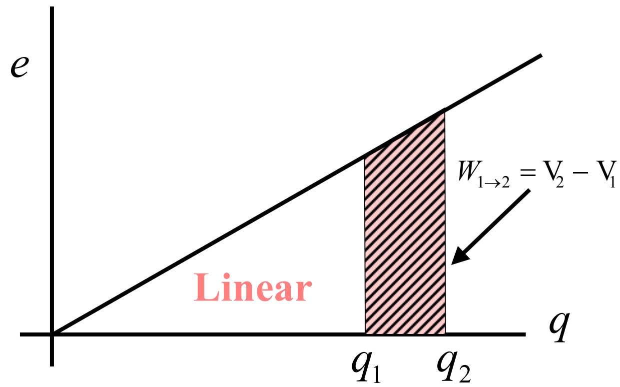
$$V = \frac{1}{2C} q^2 = \frac{1}{2} Ce^2 \quad \Rightarrow \quad W_{1\to 2} = V_2 - V_1$$

ENERGY STORAGE: LINEAR COMPLIANCE

Energy Storage:

Work from point 1 to point 2:

$$W_{1\to 2} = \int_1^2 e\dot{q}dt = \int_1^2 e\frac{dq}{dt}dt = \int_{q_1}^{q_2} edq = \int_{q_1}^{q_2} \frac{1}{C}qdq = \frac{1}{2C}q^2\Big|_{q_1}^{q_2} = \frac{1}{2C}\left(q_2^2 - q_1^2\right)$$



LINEAR COMPLIANCE

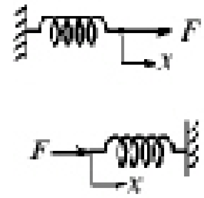
Examples:

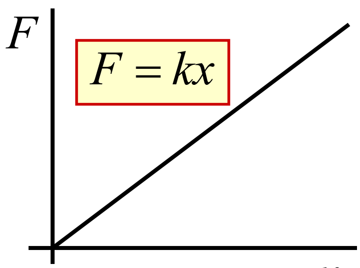
- Mechanical springs
- Electrical capacitor
- Fluid compliance due to gravity (tank, reservoir)
- Fluid compliance due to compressibility

MECHANICAL SPRING

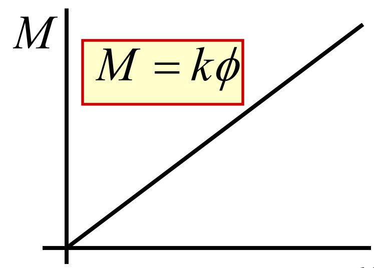
$$\frac{F(M)}{\dot{x}(\dot{\phi})} \diagup \, C$$

Spring





Linear Spring



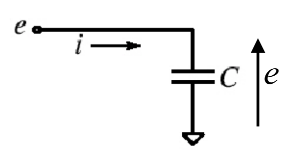
Rotational Spring

k is the spring rate, constant or stiffness

ELECTRICAL CAPACITOR

$$\frac{e\,(\text{voltage})}{i\,(\text{current})} \diagup \, C$$

Capacitor



$$\boxed{q = Ce} \Rightarrow \frac{dq}{dt} = C\frac{de}{dt} \Rightarrow i = C\frac{de}{dt}$$

Charge or electrical displacement

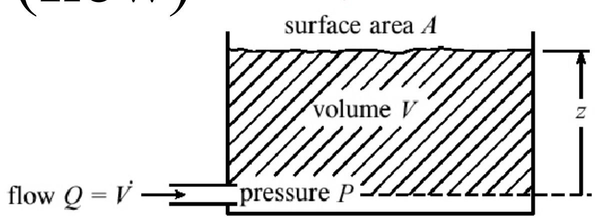
Recall: $i = \dfrac{dq}{dt} = \dot{q}$

C stands for both Compliance and Capacitance

TANKS, RESERVOIRS

$$\xrightarrow[\dot{V}\text{ (flow)}]{p\text{ (pressure)}} C$$

Tank, Reservoir



$$V = Cp$$

Depth: $V = zA \Rightarrow z = \frac{V}{A}$

Pressure: $p = \rho g z$

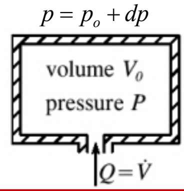
$$C = \frac{q}{e} = \frac{V}{p} = \frac{V}{\rho g V/A} = \frac{A}{\rho g}$$

COMPRESSIBLE FLUIDS

$$V = V_o + dV$$
$$p = p_o + dp$$

$$\xrightarrow[\dot{V}]{p} C$$

Compressible Chamber



$$C = \frac{q}{e} = \frac{dV}{dp}$$

Ideal Gas:
(No heat transfer)

$$pV^{\gamma} = \text{constant}, \quad \gamma = \frac{c_p}{c_v}$$

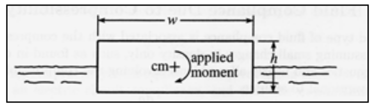
Logarithm: $\ln(p) + \gamma \ln(V) = \ln(\text{constant})$

Differentiation: $\frac{dp}{p} + \gamma \frac{dV}{V} = 0 \quad \Rightarrow C = \frac{dV}{dp} = -\frac{V}{\gamma p} \approx -\frac{V_o}{\gamma p_o}$

LINEAR COMPLIANCE

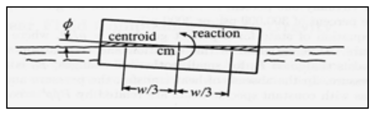
Example: Example 3.1, page 96

A uniform rectangular block has half the mass density as that of the water in which it floats. Find the compliance associated with small rotations induced by an applied moment about an axis normal to the page.



Buoyancy Force

Solution:

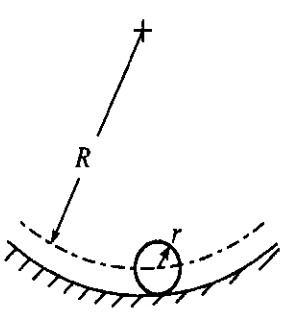


$$C = \frac{12l}{\rho g w^3}$$

LINEAR COMPLIANCE

Example: Problem 3.5, page 100

a- Find the potential energy of the disk in terms of this displacement. Previously, define a variable to describe the displacement of the disk from its equilibrium position.
b- Find the compliance of the disk with respect to the displacement assuming it has only small values.



Solution:

$$C = \frac{1}{mgR}$$