LIFE-CYCLE OF STRUCTURES UNDER UNCERTAINTY

Emphasis on Fatigue-Sensitive Civil and Marine Structures

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Preface

The performance, functionality and safety of civil and marine structures have been deteriorating due to age, increase in external loadings, corrosion, fatigue and other physical and chemical mechanisms. According to the various reports of the American Society of Civil Engineers published during the past three decades, the amount of deterioration in civil infrastructure has increased, the overall grade for the civil infrastructure in the United States has been degraded from C (fair) in 1988 to D (poor) in 2017, and the budget required to eliminate all existing and future structural deficiencies has grown substantially. As the age of a deteriorating structure increases and its service life reaches its design threshold life, there is a mounting risk associated with unsatisfactory performance under both normal and extreme loading conditions. The unexpected loss of functionality or failure of civil and marine structures can lead to severe economic, social and environmental impacts. For this reason, natural and financial resources should be allocated consistently and rationally to maintain the structural performance of deteriorating civil and marine structures above certain threshold levels.

Life-cycle analysis is a systematic tool for efficient and effective service life management of deteriorating structures. In the last few decades, theoretical and practical approaches for life-cycle performance and cost analysis have been developed extensively due to increased demand on structural safety and service life extension. This book presents the state-
of-the-art in life-cycle analysis and maintenance optimization for fatigue-sensitive structures. Both theoretical background and practical applications have been provided for academics, engineers and researchers.

The primary topics covered in this book include (a) probabilistic concepts of life-cycle performance and cost analysis, (b) inspection and monitoring in life-cycle analysis, (c) fatigue crack detection under uncertainty, (d) optimum inspection and monitoring planning, (e) multi-objective life-cycle optimization, and (f) decision making in life-cycle analysis. For illustrative purposes, these topics have been applied to fatigue-sensitive details of bridges and ships.

This book includes eight chapters.

Chapter 1 provides the fundamental concepts of life-cycle analysis under uncertainty. Structural performance deterioration mechanisms such as corrosion and fatigue, the effects of maintenance actions on structural performance, cost and service life, and structural performance indicators related to structural condition, safety, tolerance to damage and cost are described. Furthermore, recent investigations on life-cycle optimization for fatigue-sensitive structures are reviewed.

Chapter 2 presents the role of inspection and monitoring in life-cycle analysis. This chapter covers the representative inspection, monitoring and maintenance methods for fatigue-sensitive structures, the effects of inspection and monitoring on life-cycle performance and cost under uncertainty, and statistical and probabilistic concepts associated with the efficient use of inspection and monitoring data including availability of monitoring data, loss function, Bayesian updating, and probabilistic importance indicators.

Chapter 3 describes the probabilistic concepts and methods related to fatigue crack damage detection, where the time-dependent fatigue crack propagation, probability of damage detection under multiple inspections, expected damage detection delay, and damage detection time-based probability of failure are provided. The concepts and approaches presented in this chapter are used for probabilistic optimum service life management in Chapters 4, 5 and 6.
In Chapter 4, the optimum inspection and monitoring planning for fatigue-sensitive structures is addressed using the fatigue crack damage detection-based objectives. The associated objectives are to maximize the lifetime probability of fatigue crack damage detection, minimize the expected fatigue crack damage detection delay, and minimize the fatigue crack damage detection time-based probability of failure. The formulations of these objectives are based on the probabilistic concepts provided in Chapter 3.

Chapter 5 deals with the optimum inspection and monitoring planning, considering the effects of inspection, monitoring and maintenance on service life extension and life-cycle cost. The objectives used in this chapter include minimizing the expected maintenance delay, maximizing the expected extended service life, and minimizing the expected life-cycle cost. The relationships among the number of inspections and monitorings, expected maintenance delay, expected extended service life and expected life-cycle cost are investigated.

Chapter 6 presents the multi-objective probabilistic optimum inspection and monitoring planning for fatigue-sensitive structures. The bi-, tri- and quad-objective optimization problems are investigated using the objective functions formulated in Chapters 4 and 5. Furthermore, the six objectives for optimum inspection planning and five objectives for optimum monitoring planning are used simultaneously to investigate the multi-objective optimization.

Chapter 7 addresses the decision making framework for optimum inspection and monitoring planning in order to deal with a large number of objectives efficiently, and to select the best single optimum inspection and monitoring plan for practical applications. In this framework, there are two decision alternatives such as decision making before and after solving multi-objective life-cycle optimization.

Chapter 8 serves as conclusions of this book. A summary of the book and future directions of the field of life-cycle performance and cost analysis and optimization for civil and marine structures under fatigue are also provided.

This book will help engineers engaged in civil and marine structures including students, researchers and practitioners with reliable and cost-
Life-Cycle of Structures Under Uncertainty

effective maintenance planning of fatigue-sensitive structures, and to
develop more advanced approaches and techniques in the field of life-cycle
maintenance optimization and safety of structures under various aging and
deteriorating conditions. Since the book is self-contained it can be used
by all concerned with civil and marine structures, and probability and
optimization concepts for fatigue-sensitive structures, including students,
researchers and practitioners from all areas of engineering and industry. It
can also be used for an advanced undergraduate course or a graduate course
on life-cycle performance and cost of structures under uncertainty with
emphasis on fatigue-sensitive structures. The areas to which the concepts
and approaches presented in this book can be applied include not only civil
structures, such as buildings, bridges, roads, railways, dams, and ports, and
marine structures, such as naval vessels, offshore structures, submarines,
submersibles, pipelines, and subsea systems, but also aerospace structures,
nuclear power plants, and automotive structures.

The cover photo is the San Francisco-Oakland Bay Bridge located in
San Francisco, California, over the San Francisco Bay. All the bridge
components are constructed of steel. The authors would like to thank
Dr. Man-Chung Tang, T.Y. Lin International’s Chairman of the Board,
designer of more than 100 major bridges across the globe. The photo was
taken by Engel Cheng/Shutterstock.com.

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Chapter 1

Concepts of Life-Cycle Analysis Under Uncertainty

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ABSTRACT

Structural performance should be maintained above a certain threshold during the service life of a deteriorating structure. Financial resources for inspection, monitoring and maintenance have to be allocated consistently and rationally. Life-cycle performance and cost analysis can provide a practical solution to addressing the issues associated with the management of deteriorating structures under uncertainty. Chapter 1 presents the probabilistic concepts of life-cycle performance and cost analysis. This analysis requires an understanding of (a) structural performance under various deterioration mechanisms, (b) effects of maintenance actions on structural performance, cost and service life under uncertainty, and (c) structural performance indicators. The representative structural performance indicators associated with condition, safety, tolerance to damage and cost are described. Furthermore, the life-cycle optimization for service life management is also discussed. The objectives of life-cycle optimization are formulated based on the life-cycle performance and cost analysis. Finally, recent investigations into life-cycle optimization for fatigue-sensitive structures are addressed.

1.1 Introduction

Structural performance deteriorates over time due to aging, an increase in external loadings, corrosion, fatigue, and other physical and chemical mechanisms. As the age of a deteriorating structure increases and its service life reaches its design life, the risks of unserviceability and failure under normal and extreme loading conditions increase (Decó and Frangopol 2011, 2013; Ellingwood 2006). For this reason, structural performance should be maintained over certain thresholds during the service life of a deteriorating structure. Investments for inspection and maintenance have to be allocated consistently and rationally (ASCE 2017). The life-cycle performance and cost analysis can provide a rational and practical solution to address issues associated with the management of deteriorating
This chapter presents the probabilistic concepts of life-cycle performance and cost analysis. These concepts require an understanding of the (a) structural performance under various deterioration mechanisms (e.g., corrosion and fatigue), (b) effects of maintenance actions on structural performance, cost and service life, and (c) structural performance indicators under uncertainty. The structural performance indicators based on condition, safety, tolerance to damage and cost are reviewed. Furthermore, the life-cycle optimization for service life management is presented. The objectives of the life-cycle optimization are based on the life-cycle performance and cost analysis. Recent investigations on life-cycle optimization for fatigue-sensitive structures are addressed.

1.2 Life-Cycle Performance and Cost Analysis Under Uncertainty

Probabilistic life-cycle analysis leads to a systematic and rational evaluation of structural performance and cost. It serves as a basis for establishing an optimum service life management (Kim et al. 2013). Figure 1.1 shows the schematic for life-cycle analysis under uncertainty. The prediction of structural performance and cost under uncertainty is used to formulate the single- or multi-objective optimization process. Through this process, effective and efficient inspection and maintenance management can be achieved (Frangopol et al. 2012).

1.2.1 Structural Performance Deterioration

Structural performance prediction is one of the most significant phases in life-cycle analysis. Accurate prediction of the structural performance can result in the reliable service life management of deteriorating structures (Frangopol 2011, 2018). Structural performance deteriorates over time due
to the combined effects of mechanical stressors, harsh environment, and extreme events. Gradual deterioration of the structural performance can be induced by corrosion and/or fatigue. These are the most common causes of resistance reduction in reinforced concrete (RC) and steel structures. Seismic events, hurricanes, floods and/or other extreme events may lead to a sudden drop in structural performance (Frangopol and Soliman 2016). During the past three decades, significant research towards predicting structural performance accurately has focused on development of time-dependent models of corrosion and fatigue initiation and propagation.

Corrosion has been considered one of the main factors causing deterioration of RC structures (NCHRP 2005, 2006). The corrosion process generally consists of corrosion initiation and propagation. Corrosion initiation can be defined as the time when the chloride concentration in the reinforcement
Concepts of Life-Cycle Analysis Under Uncertainty

The steel surface reaches a predefined threshold (Arora et al. 1997; Stewart 2004; Zhang and Lounis 2006). Corrosion propagation produces damage such as cracking, spalling and reduction of reinforcement steel. The reduction of the reinforcement area can be represented by uniform and pitting corrosion models (Marsh and Frangopol 2008; Frangopol and Kim 2014a). As shown in Figure 1.2(a), the uniform corrosion model is based on the assumption that the cross sectional area of reinforcement steel is reduced uniformly. The pitting corrosion considers the highly localized corroded steel area as shown in Figure 1.2(b). Gonzalez et al. (1995), Stewart (2004) and Torres-Acosta and Martinez-Madrid (2003) conclude that pitting corrosion is more general for RC bridge decks, and results in a higher probability of failure and reduced service life than uniform corrosion when the same cross sectional area is considered.

Fatigue is one of the most common deterioration mechanisms in steel structures (Kwon and Frangopol 2011; Kwon et al. 2013). Under repeated loading and unloading, fatigue crack damage may initiate and accumulate at regions with initial flaws in the material, welding process or fabrication. When the number of cycles and stress ranges exceed their prescribed thresholds, the fatigue cracks can result in the fracture of the steel component and structural system failure. Details on fatigue damage propagation are addressed in Section 3.2.

Figure 1.2 Cross sectional area reduction of steel in concrete due to corrosion: (a) uniform corrosion model; (b) pitting corrosion model.
1.2.2 Structural Performance and Cost with Maintenance

The structural performance of deteriorating structures can be maintained and improved by maintenance and replacement. As shown in Figure 1.3(a), the service life $t_{\text{life}}$, which is defined as the time when the structural performance reaches a predefined threshold $P_{\text{th}}$, can be extended by applying maintenance and replacement. The preventive maintenance (PM) is applied to maintain the performance above the required level (Kong and Frangopol 2003a, 2003b, 2005; Neves et al. 2006; Bocchini and Frangopol 2011; Frangopol and Kim 2014a). The PM application time $t_{\text{pm}}$ can be predetermined considering the relation between improvement of the structural performance after the PM and cost required for the PM.
Concepts of Life-Cycle Analysis Under Uncertainty

(Okasha and Frangopol 2010a). Service life can be extended from the initial service life $t_{\text{life},0}$ to $t_{\text{life},\text{pm}}$ by applying PM. At time $t_{\text{em}}$, when the structural performance reaches the threshold $P_{\text{th}}$, the essential maintenance (EM) is applied so that the service life can be extended from $t_{\text{life},\text{pm}}$ to $t_{\text{life},\text{em}}$. The structural performance returns to its initial state $P_{\text{in}}$ by replacement (RP). Figure 1.3(b) shows the cumulative maintenance cost when the PM, EM and RP are applied at times $t_{\text{pm}}$, $t_{\text{em}}$, and $t_{\text{rp}}$, respectively. The PM can result in a relatively small improvement of structural performance. PM is associated with a much lower maintenance cost $C_{\text{pm}}$ than those associated with EM and RP. It is relevant to note that PM can also result in no improvement of $P$ but can delay the deteriorating process (e.g., painting of steel components). The RP leads to higher improvement of the structural performance $P$ and larger cost $C_{\text{rp}}$ than those associated with PM and EM.

Based on Figure 1.3, structural performance over time and extended service life under uncertainty are illustrated in Figure 1.4. The uncertainties associated with the improvement of $P$ after PM, EM and RP can be represented by the associated probability density functions (PDFs). Furthermore, the PDFs of the extended service life by applying maintenance can be obtained using probabilistic concepts and methods. As mentioned previously, the probabilistic life-cycle analysis shown in Figure 1.4 is used for optimum service life management.

![Figure 1.4 Life-cycle analysis considering maintenance actions under uncertainty.](image-url)
1.3 Structural Performance Indicators

The structural condition, safety, tolerance to damage and cost can be represented by structural indicators (Zhu and Frangopol 2012; Frangopol and Saydam 2014; Ghosn et al. 2016a, 2016b; Biondini and Frangopol 2016). Condition rating is generally estimated based on visual inspection. In the United States, two representative condition rating methods are used for bridges. The first condition rating method is the National Bridge Inventory (NBI) condition rating, where the conditions of bridge components (e.g., deck, superstructures, substructures) are estimated using a value ranging from 0 to 9. The condition rating value of 0 is associated with the failed condition, and condition rating value of 9 indicates excellent condition. The second condition rating method is Pontis (Cambridge Systematics, Inc 2009), where the rating value ranges from 1 (i.e., no evidence of damage in a bridge component) to 5 (i.e., severe damage affecting the strength and serviceability of a bridge component). Table 1.1 describes the condition states of NBI and Pontis. More details on NBI and Pontis condition rating methods can be found in FHWA (1995), CDOT (1998), Estes and Frangopol (2003) and Saydam et al. (2013).

The representative safety-based structural performance indicators are the probability of failure $P_f$ and reliability index $\beta$, which are expressed as

$$P_f = 1 - P_s = \int_{g(X) > 0} f_X(x) \, dx$$

(1.1a)

$$\beta = \Phi^{-1} \left( 1 - P_f \right)$$

(1.1b)

where $X$ is a vector of random variables, $f_X(x)$ is the joint PDF of random variables $X$, and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution function (CDF). The state of the structure can be determined as follows: $[g(X) > 0] = $ “safe state”; $[g(X) < 0] = $ “failure state”; and $[g(X) = 0] = $ “limit state”. If the random variables $X$ of the state function $g(X)$ consist of the uncorrelated normal variables $X_i$ and $X_j$, the state of the structure can be illustrated as shown in Figure 1.5. The random variable $X'_{ij}$ in the standard normal space of Figure 1.5 is defined as $(X_i - \mu_i)/\sigma_i$, where $\mu_i$ and $\sigma_i$ are the mean and standard deviation of $X_i$, respectively. In the standard
### Table 1.1 NBI and Pontis condition ratings.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Conditions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A</td>
<td>Not applicable</td>
<td>Not applicable</td>
</tr>
<tr>
<td>9</td>
<td>Excellent</td>
<td>No problem noted</td>
</tr>
<tr>
<td>8</td>
<td>Very good</td>
<td>No problem noted</td>
</tr>
<tr>
<td>7</td>
<td>Good</td>
<td>Some minor problems</td>
</tr>
<tr>
<td>6</td>
<td>Satisfactory</td>
<td>Structural elements show some minor deterioration</td>
</tr>
<tr>
<td></td>
<td></td>
<td>All primary structural elements are sound but may have some minor section loss, cracking, spalling or scour</td>
</tr>
<tr>
<td>5</td>
<td>Fair</td>
<td>Advanced section loss, deterioration, spalling or scour</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Loss of section, deterioration, spalling or scour have seriously affected primary structural components. Local failures are possible. Fatigue cracks in steel or shear cracks in concrete may be present</td>
</tr>
<tr>
<td>4</td>
<td>Poor</td>
<td>Advanced deterioration of primary structural elements. Fatigue cracks in steel or shear cracks in concrete may be present or scour may have removed substructure support</td>
</tr>
<tr>
<td>3</td>
<td>Serious</td>
<td>Major deterioration or section loss present in critical structural components or obvious vertical or horizontal movement affecting structure stability</td>
</tr>
<tr>
<td>2</td>
<td>Critical</td>
<td>Out of service—beyond corrective action</td>
</tr>
<tr>
<td>1</td>
<td>Imminent failure</td>
<td>There is no evidence of active corrosion and the paint system is sound and functioning as intended to protect the metal surface</td>
</tr>
<tr>
<td></td>
<td></td>
<td>There is little or no active corrosion. Surface or freckled rust has formed or is forming. The paint system may be chalking, peeling, curling or showing other early evidence of paint system distress but there is no exposure of metal</td>
</tr>
</tbody>
</table>

Pontis condition rating for painted steel girder element (adopted from CDOT (1998))

<table>
<thead>
<tr>
<th>Rating</th>
<th>Conditions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Good</td>
<td>There is no evidence of active corrosion and the paint system is sound and functioning as intended to protect the metal surface</td>
</tr>
<tr>
<td></td>
<td></td>
<td>There is little or no active corrosion. Surface or freckled rust has formed or is forming. The paint system may be chalking, peeling, curling or showing other early evidence of paint system distress but there is no exposure of metal</td>
</tr>
<tr>
<td>2</td>
<td>Fair</td>
<td>There is no evidence of active corrosion and the paint system is sound and functioning as intended to protect the metal surface</td>
</tr>
<tr>
<td></td>
<td></td>
<td>There is little or no active corrosion. Surface or freckled rust has formed or is forming. The paint system may be chalking, peeling, curling or showing other early evidence of paint system distress but there is no exposure of metal</td>
</tr>
</tbody>
</table>

*Table 1.1 contd.*
normal space, the reliability index $\beta$ is the minimum distance from the origin to the limit state. The system reliability can be assessed through the appropriate system modeling. The probabilities of failure $P_f$ of a series system, parallel system and series-parallel system in Figure 1.6 are expressed as

$$P_f = P\left(\bigcup_{i=1}^{N} g_i(X) \leq 0\right)$$

for a series system \hspace{1cm} (1.2a)

$$P_f = P\left(\bigcap_{i=1}^{N} g_i(X) \leq 0\right)$$

for a parallel system \hspace{1cm} (1.2b)

$$P_f = P\left(\bigcup_{i=1}^{M} \bigcap_{j=1}^{N_i} g_{ij}(X) \leq 0\right)$$

for a series-parallel system \hspace{1cm} (1.2c)

where $N$ = number of components in a series system (see Figure 1.6(a)), or a parallel system (see Figure 1.6(b)); $M = number of parallel systems in a series-parallel system (see Figure 1.6(c)). The i\textsuperscript{th} parallel system consists of $N_i$ components. The probabilities of failure $P_f$ of a series system for a perfectly correlated case and statistically independent case are expressed as

---

### Table 1.1 contd.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Conditions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Paint failure</td>
<td>Surface or freckled rust is prevalent. There may be exposed metal but there is no active corrosion which is causing loss of section</td>
</tr>
<tr>
<td>4</td>
<td>Paint failure with steel corrosion</td>
<td>Corrosion may be present but any section loss due to active corrosion does not yet warrant structural analysis of either the element or the bridge</td>
</tr>
<tr>
<td>5</td>
<td>Major section loss</td>
<td>Corrosion has caused section loss and may be sufficient to warrant structural analysis to ascertain the impact on the ultimate strength and/or serviceability of either the element or the bridge</td>
</tr>
</tbody>
</table>
Figure 1.5 Safe domain, limit state and failure domain in the standard normal space.

Figure 1.6 System modeling: (a) series system; (b) parallel system; (c) series-parallel system.
Life-Cycle of Structures Under Uncertainty

\[ P_f = \max_{i=1}^N \{ P_{f,i} \} \quad \text{for perfectly correlated case} \quad (1.3a) \]

\[ P_f = 1 - \prod_{i=1}^N (1 - P_{f,i}) \quad \text{for statistically independent case} \quad (1.3b) \]

where \( P_{f,i} \) is the probability of failure for the \( i \)th component in a series system. Furthermore, the probabilities of failure \( P_f \) of a parallel system for a perfectly correlated case and statistically independent case are computed as

\[ P_f = \min_{i=1}^N \{ P_{f,i} \} \quad \text{for perfectly correlated case} \quad (1.4a) \]

\[ P_f = \prod_{i=1}^N P_{f,i} \quad \text{for statistically independent case} \quad (1.4b) \]

Equations (1.3) and (1.4) are associated with the two extreme cases (i.e., perfectly correlated case and statistically independent case). Considering the correlation among the random variables and various types of PDFs for the random variables, the probability of failure of the system can be computed (Cornell 1967; Ditlevsen 1979; Ang and Tang 1984; Thoft-Christensen and Murotsu 1986). Structural redundancy, a measure of reserve capacity, has been used as a system level structural performance indicator for bridges and naval ships, among others (Frangopol and Curley 1987; Fu and Frangopol 1990; Frangopol and Nakib 1991; Hendawi and Frangopol 1994; Okasha and Frangopol 2010c; Ghosn et al. 2010; Decò et al. 2011, 2012; Zhu and Frangopol 2013c, 2015). Their structural redundancy can be quantified in several ways. Frangopol and Nakib (1991) expresses the structural redundancy index \( RI \) as

\[ RI = \frac{P_{f,dmg} - P_{f,sys}}{P_{f,sys}} \quad (1.5) \]

where \( P_{f,dmg} \) is the probability of damage occurrence to the systems, and \( P_{f,sys} \) is the probability of system failure. The difference between \( P_{f,sys} \) and \( P_{f,dmg} \) in Eq. (1.5) indicates the availability of system warning. If the damage occurrence to the system results in structural system failure (i.e., \( P_{f,dmg} = \)}
$P_{f,sys}$ in Eq. (1.5)), a structural system has no redundancy (i.e., $RI_I = 0$). Furthermore, the structural redundancy index $RI$ can be expressed in terms of the reliability index as (Frangopol and Curley 1987)

$$RI_I = \frac{\beta_{int}}{\beta_{int} - \beta_{dmg}}$$  \hspace{1cm} (1.6)

where $\beta_{int}$ = reliability index of the intact system; $\beta_{dmg}$ = reliability of the damaged system. When there is almost no change in reliability index $\beta$ due to damage (i.e., $\beta_{int} \approx \beta_{dmg}$ in Eq. (1.6)), a structural system will have a very large structural redundancy index (i.e., $RI_I \to \infty$).

Robustness defines the ability of a structural system to resist progressive collapse under localised damage and damage tolerance of a structural system (Frangopol and Saydam 2014). According to Maes et al. (2006), the robustness of a structural system is expressed as

$$RB = \min_i \frac{P_{f,0}}{P_{f,i}}$$  \hspace{1cm} (1.7)

where $P_{f,0}$ is the probability of failure of the undamaged system, and $P_{f,i}$ is the probability of failure of a system with the damaged component $i$. Furthermore, vulnerability can be used as the structural performance indicator to represent damage tolerance of a structure. Lind (1995) defines the vulnerability of a system $VL$ as

$$VL = \frac{P_f(S_d,Q)}{P_f(S_0,Q)}$$  \hspace{1cm} (1.8)

where $P_f = \text{probability of failure of the system}; S_d = \text{damage state } d; S_0 = \text{pristine system state}; Q = \text{prospective loading}$. Vulnerability $VL = 1$ indicates that the probability of failure of a pristine system is equal to the probability of failure of a damaged system.

Life-cycle cost is one of the most common cost-based structural performance indicators (Frangopol and Soliman 2016). The expected total life-cycle cost $C_{life}$ can be computed as (Frangopol et al. 1997)

$$C_{life} = C_{int} + C_{insp} + C_{ma} + C_{fail}$$  \hspace{1cm} (1.9)
where $C_{\text{int}}$ is the initial cost; $C_{\text{insp}}$ is the expected cost for inspections; $C_{\text{ma}}$ is the expected cost for maintenance and repair actions; and $C_{\text{fail}}$ is the expected failure cost. Risk, which is defined as multiplication of occurrence probability by the consequences (i.e., cost) of an event, can also be used to represent a structural performance. The formulation of the total risk is (Baker et al. 2008)

$$RS = P_{f,\text{com},i} \cdot C_{\text{dir},i} + P_{f,\text{com},i} \cdot P_{f,\text{subsys},\text{com},i} \cdot C_{\text{ind},i}$$

(1.10)

where $P_{f,\text{com},i}$ = probability of failure of component $i$ in a system; $P_{f,\text{subsys},\text{com},i}$ = probability of subsequent system failure to the failure of component $i$. $C_{\text{dir},i}$ and $C_{\text{ind},i}$ are the direct and indirect costs caused by the failure of component $i$. A system for which the indirect risk does not significantly affect the total risk is a robust system. Accordingly, the risk-based robustness index $RB_{rs}$ is expressed as (Baker et al. 2008)

$$RB_{rs} = \frac{P_{f,\text{com},i} \cdot C_{\text{dir},i}}{P_{f,\text{com},i} \cdot C_{\text{dir},i} + P_{f,\text{com},i} \cdot P_{f,\text{subsys},\text{com},i} \cdot C_{\text{ind},i}}$$

(1.11)

The risk-based robustness index $RB_{rs}$ ranges from 0 to 1. A larger value of $RB_{rs}$ represents a higher robustness. The relation among the structural performance indicators defined in Eqs. (1.1) to (1.11), and the effects of deterioration mechanisms, loading and extreme events on the structural performance indicators were investigated in Okasha and Frangopol (2010a, 2010b, 2010c), Saydam and Frangopol (2011, 2013), Barone and Frangopol (2014), Zhu and Frangopol (2012, 2013a), Decò and Frangopol (2011, 2013), Decò et al. (2012), Dong and Frangopol (2016), Ghosn et al. (2016a, 2016b), among others.

1.4 Life-Cycle Optimization

Life-cycle optimization is performed for efficient service life management as shown in Figure 1.1. Through the life-cycle optimization process considering various objectives based on the structural performance, cost, and service life, the optimum times and types of inspection and
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Maintenance applications are determined (Liu and Frangopol 2005b, 2006; Frangopol and Liu 2007; Frangopol and Kim 2014a). The relationship between expected structural performance during service life and present value of total expected life-cycle cost near the optimal region is illustrated in Figure 1.7. A larger expected structural performance can require a larger initial cost $C_{\text{int}}$, less expected failure cost $C_{\text{fail}}$, and less expected cost for inspection and maintenance $C_{\text{insp}} + C_{\text{ma}}$ during the service life of a structure. By applying the optimum inspection and maintenance strategy for the expected structural performance during the service life $P^*$, the expected total life-cycle cost $C_{\text{life}}$ defined in Eq. (1.9) can be minimized to be $C^*_{\text{life}}$ (Frangopol et al. 1997).

Optimization can be formulated with single and multiple objectives. It has been shown that as the number of objectives for the life-cycle optimization increases, more rational and well-balanced solutions can be obtained, and a more flexible decision can be made to select the best solution among the computed multiple solutions (Kim and Frangopol 2017, 2018a, 2018b). Figure 1.8 shows the feasible criterion space and the Pareto optimal set.
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when two objective functions $f_1$ and $f_2$ are considered simultaneously. The Pareto optimal set of the bi-objective optimization for minimizing both $f_1$ and $f_2$ is shown in Figure 1.8(a). When the bi-objective optimization is based on maximization of both $f_1$ and $f_2$, the Pareto optimal set is illustrated in Figure 1.8(b). Figure 1.8(c) shows the Pareto optimal set associated with minimization of $f_1$ and maximization of $f_2$. Finally, Figure 1.8(d) shows the Pareto optimal set associated with maximization of $f_1$ and minimization of $f_2$.

Recent investigations on life-cycle managements for deteriorating structures have adopted the multi-objective optimization. Table 1.2 summarizes recent investigations for fatigue-sensitive structures. The objectives used in the optimization are based on cost (e.g., expected life-cycle cost, maintenance cost, failure cost, inspection and monitoring cost),

![Figure 1.8 Pareto sets of bi-objective optimizations: (a) minimization of $f_1$ and $f_2$; (b) maximization of $f_1$ and $f_2$; (c) minimization of $f_1$ and maximization of $f_2$; (d) maximization of $f_1$ and minimization of $f_2$.](image-url)
damage detection (e.g., probability of damage detection, expected damage detection delay, and maintenance delay), reliability index (or probability of failure), and service life. Through the life-cycle optimization, optimum inspection times and types (or quality), monitoring starting times and duration, and maintenance times and types (or quality) can be computed. Additional life-cycle optimizations for fatigue-sensitive structures are addressed in Chapters 4, 5 and 6.

**Table 1.2** Recent investigations on life-cycle optimization for fatigue-sensitive structures.

<table>
<thead>
<tr>
<th>Applications</th>
<th>Objectives</th>
<th>Design variables</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bridges</td>
<td>• Minimize expected total life-cycle cost</td>
<td>Inspection times and quality, and repair quality</td>
<td>Lukić and Cremona (2001)</td>
</tr>
<tr>
<td></td>
<td>• Maximize reliability index</td>
<td>Retrofit length and height</td>
<td>Liu et al. (2010a)</td>
</tr>
<tr>
<td></td>
<td>• Minimize retrofit size</td>
<td>Maintenance application times and types</td>
<td>Orcesi et al. (2010)</td>
</tr>
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<td></td>
<td>• Minimize cumulative maintenance cost</td>
<td>Monitoring starting times and durations</td>
<td>Kim and Frangopol (2011b)</td>
</tr>
<tr>
<td></td>
<td>• Minimize expected failure cost</td>
<td>Maintenance time and types</td>
<td>Orcesi and Frangopol (2011a)</td>
</tr>
<tr>
<td></td>
<td>• Maximize probability of fatigue detection</td>
<td>Inspection times and types</td>
<td>Soliman et al. (2013a)</td>
</tr>
<tr>
<td></td>
<td>• Minimize inspection cost</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Maximize probability of fatigue detection</td>
<td>Inspection times</td>
<td>Kim and Frangopol (2018b)</td>
</tr>
<tr>
<td></td>
<td>• Minimize expected damage detection delay</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>• Minimize expected repair delay</td>
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<tr>
<td></td>
<td>• Minimize probability of failure</td>
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<td></td>
<td>• Maximize expected extended service life</td>
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<td></td>
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<tr>
<td></td>
<td>• Minimize expected total life-cycle cost</td>
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Table 1.2 contd. ...
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<thead>
<tr>
<th>Applications</th>
<th>Objectives</th>
<th>Design variables</th>
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<tbody>
<tr>
<td>Ships</td>
<td>Minimize expected total life-cycle cost</td>
<td>Inspection times and monitoring starting times</td>
<td>Kim and Frangopol (2011a)</td>
</tr>
<tr>
<td></td>
<td>Minimize expected damage detection delay</td>
<td>Inspection times and quality</td>
<td>Kim and Frangopol (2011d)</td>
</tr>
<tr>
<td></td>
<td>Minimize inspection cost</td>
<td>Inspection times and quality, monitoring times and durations, and combination of inspection and monitoring</td>
<td>Kim and Frangopol (2012)</td>
</tr>
<tr>
<td></td>
<td>Minimize expected damage detection delay</td>
<td>Inspection times and crack size for maintenance</td>
<td>Kim et al. (2013)</td>
</tr>
<tr>
<td></td>
<td>Minimize expected total life-cycle cost</td>
<td>Monitoring starting times and durations</td>
<td>Kim and Frangopol (2018a)</td>
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1.5 Conclusions

In this chapter, general concepts of life-cycle performance and cost analysis under uncertainty were presented. The life-cycle structural performance and cost estimation should consider inspection and maintenance. The
Concepts of Life-Cycle Analysis Under Uncertainty

objectives used for the optimum service life management can be formulated based on the life-cycle structural performance and cost prediction. Through the single- or multi-objective optimization process, effective and efficient inspection and maintenance management can be achieved. The practical application of life-cycle performance and cost analysis requires accurate prediction of (a) structural performance using information obtained from inspection and structural health monitoring (Catbas et al. 2008, 2013; Decò and Frangopol 2015; Frangopol 2011; Frangopol et al. 2012; Okasha et al. 2010, 2011; Soliman et al. 2015; Frangopol and Soliman 2016), (b) service life extension by maintenance actions (Kim et al. 2011, 2013), and (c) structural performance loss due to extreme events such as earthquakes, tsunamis, floods and hurricanes (Akiyama et al. 2011, 2013; Dong and Frangopol 2017; Mondoro et al. 2017; Mondoro and Frangopol 2018).