

Math 21 Hand-in Homework Problems

Due Friday January 21

1. Evaluate $\sin(7\pi/6)$ and $\tan(-\pi/4)$.
2. If $\sin(\theta) = \frac{1}{4}$ and $\frac{\pi}{2} < \theta < \pi$, what are the values of $\cos(\theta)$ and $\tan(\theta)$?
3. Sketch the graph of $f(x) = 1 + \sin(\frac{x}{2})$.
4. A flagpole is on top of a building which is 40 feet high. From a point 50 feet away from the building, the flagpole subtends an angle whose tangent is $1/5$. How many feet tall is the flagpole?
5. Find the domain of the function $g(x) = 1/(e^{\cos x} - \sqrt{e})$.
6. Sketch the graph of $f(x) = e^{x-1} - 1$.
7. If $g(x) = 2^x$, show that $g(x+h) - g(x) = 2^x(2^h - 1)$.

Due Wednesday January 26

1. Determine whether the function $f(x) = (1 + 3^x)/3^x$ is one-to-one and, if so, find a formula for the inverse function.
2. Solve the equation $\ln(x) + \ln(x - 2) = 1$.
3. Sketch the graph of $y = \ln |2x - 3|$.
4. Simplify the expression $\tan(2 \arcsin x)$.
5. For the curve $y = x^2$, let P be the point on the curve where $x = 2$. Let Q_1 , Q_2 , and Q_3 denote the points on the curve where $x = 3$, 2.1, and 2.01. Find the slope of the secant lines PQ_1 , PQ_2 , and PQ_3 .
6. If the height at time t of a ball thrown up is given by $y = 50t - 16t^2$, find the average velocity of the ball during the interval beginning at $t = 1$ and lasting (a) 0.1 sec, (b) 0.01 sec.

Due Monday January 31

1. Sketch the graph of an example of a function f satisfying $\lim_{x \rightarrow 2^+} f(x) = 1$, $\lim_{x \rightarrow 2^-} f(x) = 3$, $f(2) = 2$, $\lim_{x \rightarrow 3^-} f(x) = 4$, and $f(3) = 5$.
2. Determine the infinite limits $\lim_{x \rightarrow 1^+} \frac{x-2}{x-1}$ and $\lim_{x \rightarrow 1^-} \frac{x-2}{x-1}$.
3. Let $f(x) = \frac{x^2-3x+2}{x^2-1}$. Evaluate (a) $\lim_{x \rightarrow 1} f(x)$, (b) $\lim_{x \rightarrow -1} f(x)$, and (c) $\lim_{x \rightarrow -1^+} f(x)$.
4. Evaluate $\lim_{x \rightarrow 2} \frac{\frac{2}{x} - 1}{x - 2}$.
5. If $f(x) = \frac{1}{x}$, evaluate $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$.

6. Evaluate $\lim_{x \rightarrow 2} f(x)$ if $f(x) = \begin{cases} \frac{x^2-1}{x-1} & x > 2 \\ 5-x & x < 2 \end{cases}$.

Due Friday February 4

1. Prove $\lim_{x \rightarrow 1^+} \sqrt{x-1} = 0$ using the ϵ, δ definition of limit.
2. Prove $\lim_{x \rightarrow 1} \frac{x^2-3x+2}{x-1} = -1$ using the ϵ, δ definition of limit.
3. Prove $\lim_{x \rightarrow 1^+} \frac{1}{\sqrt[3]{x-1}} = \infty$ using the precise definition of infinite limit.
4. Give a specific example (formula, not just graph) of a function that has a removable discontinuity at $x = 1$ and a jump discontinuity at $x = 3$, but is continuous elsewhere.
5. Let $f(x) = \begin{cases} \frac{x^2-4}{x-2} & x < 2 \\ 2x + \ln(x) & x \geq 2 \end{cases}$. Determine whether f is continuous at $x = 2$.
6. Show that the equation $\sin x = x/2$ has a solution on the interval $[\frac{\pi}{2}, \pi]$.

Due Wednesday February 9

In this assignment, you may only use the definition of the derivative, and not the formulas derived in Chapter 3.

1. Evaluate $\lim_{x \rightarrow \infty} (\sqrt{x^2+x} - x)$.
2. Evaluate $\lim_{x \rightarrow \infty} e^{-x} \sin x$.
3. Find the horizontal and vertical asymptotes of the curve $y = \frac{x^2-3x+2}{x^2-4}$.
4. Find an equation of the tangent line to the curve $y = 1/x$ at the point $(2, \frac{1}{2})$.
5. Find the derivative function for $f(x) = \sqrt{x-1}$.
6. Let $f(x) = \begin{cases} ax+b & x \leq 2 \\ x^2+x & x > 2 \end{cases}$. Find the values of the constants a and b which will make f differentiable at $x = 2$.

Due Monday February 14

1. Evaluate (a) $f'(x)$ for $f(x) = x^4 + \sqrt[3]{x^4}$; (b) $\left. \frac{d}{dt}(t^2 + \frac{1}{t}) \right|_{t=2}$; (c) $g''(4)$ for $g(x) = x^{1/2}$.
2. Find an equation of the tangent line to the curve $y = x^3 + \sqrt{x}$ at the point where $x = 9$.
3. Find all the points on the curve $y = 2x^3 - 3x^2 + 4$ where there is a horizontal tangent.
4. Evaluate (a) $f'(1)$ for $f(x) = e^x(x^2 + 1)$, and (b) $\frac{d}{dx} \left(\frac{e^x}{x + \frac{1}{x}} \right)$.
5. If f is differentiable and $g(x) = \frac{f(x)}{x-1}$, give a formula for $g'(x)$ in terms of f and f' .
6. Suppose f is differentiable, $(1, 2)$ is on the graph of f , and $(1, 3)$ is on the graph of f' . Evaluate $\lim_{x \rightarrow 1} \frac{xf(x)-2}{x-1}$.

Due Wednesday February 23

1. Evaluate $\frac{d}{dx} \left(\frac{\sin x}{1 + \cos x} \right) \Big|_{x=\pi/3}$.
2. Find all points where the curve $y = x - 2 \cos x$ has a horizontal tangent.
3. Evaluate $\lim_{t \rightarrow 0} \frac{\sin^2(2t)}{t^2}$.
4. Evaluate (a) $\frac{d}{dt} \sqrt{1 + \tan(2t)}$ and (b) $f'(2)$ if $f(x) = \left(\frac{x+1}{x^2}\right)^4$.
5. Find the second derivative of $f(x) = e^{2x} \cos(3x)$.
6. If f is a differentiable function, find the derivatives of (a) $f(x^2)$, (b) $(f(x))^2$, and (c) $f(f(x))$.

Due Monday February 28

1. Find $\frac{dy}{dx}$ for (a) $\sqrt{x} + \sqrt{2y} = 1$ and (b) $x = \sin(xy^3)$.
2. Find the equation of the tangent line to the curve $x^2 - xy + y^2 = 7$ at the point $(3, 1)$.
3. Find y'' for the curve $9x^2 + y^2 = 1$.
4. Differentiate (a) $y = \ln(x + \sqrt{x^2 + 1})$ and (b) $f(x) = (\ln(1 + e^x))^3$.
5. Find $\frac{dy}{dx}$ if $y = x^{\cos(2x)}$.
6. Use the definition of derivative to prove $\lim_{x \rightarrow 0} \frac{\ln(1+2x)}{x} = 2$.

Due Friday March 4

1. A particle moves according to $s = t^4 - 2t^3$ for $t \geq 0$, t in seconds.
 - a. Find the velocity and acceleration at time t .
 - b. When is the particle at rest?
 - c. When is the particle moving in the positive direction?
 - d. When is the particle speeding up? When is it slowing down?
 - e. Draw a diagram similar to Figure 2 on page 222 illustrating the motion of the particle.
 - f. Find the total distance traveled during during the first 3 seconds.
2. A ball thrown upward has height $s = 40t - 16t^2$ feet after t seconds, until the ball hits the ground. What is the ball's initial velocity? What is the maximum height reached by the ball?
3. If a tank holds 2000 gallons of water, which drains from the bottom of the tank in 20 minutes, the volume of water remaining in the tank after t minutes is $V = 2000\left(1 - \frac{t}{20}\right)^2$ for $0 \leq t \leq 20$. Find the rate at which water is draining from the tank after 5, 10, 15, and 20 minutes. When is it flowing out the fastest? The slowest?

4. Bismuth-210 has a half-life of 5 days. A piece initially contains 1000 mg.
 - a. Write a formula for the mass remaining after t days.
 - b. Find the mass remaining after 20 days. (In this and the remainder of this assignment, give both exact answers and decimal approximations.)
 - c. When will the mass be reduced to 1 mg?
5. A thermometer is taken from a room where the temperature is 70° F to the outdoors where the temperature is 20° F. After one minute, the thermometer reads 50° F.
 - a. What will be the reading on the thermometer after one more minute?
 - b. When will the thermometer say 21° F?
6. \$1000 is deposited in a bank account with a 3% annual interest rate. Find the amount in the account after 20 years if the interest is compounded (a) annually, (b) monthly, and (c) continuously.

Due Wednesday March 16

1. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 2 meters higher than the bow of the boat. If the rope is pulled in at the rate of 1 m/s, how fast is the boat approaching the dock when it is 20 m from the dock? (See picture on page 246, if you wish.)
2. A kite 120 ft above the ground moves horizontally at a speed of 15 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 feet of string has been let out?
3. A Ferris wheel with a radius of 12 m is rotating at a rate of one revolution every $2\frac{1}{2}$ minutes. How fast is a rider rising when his seat is 10 m above the ground?
4. The minute hand on a clock is 5 inches long and the hour hand is 3 inches long. What is the rate of change of distance between the tips at 3:00?
5. Use a linear approximation (or differentials) to estimate (a) $\sqrt{99.7}$ and (b) $\sin(44^\circ)$.
6. The radius of a circular disk is given as 25 cm with a maximum error in measurement of 0.1 cm. Use differentials to estimate the maximum error in the calculated area of the disk. What is the relative error?

Due Monday March 21

1. Prove $\tanh(x + y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$.
2. Find the derivatives of (a) $f(t) = \sinh(\cosh t)$ and (b) $y = \arcsin(\sinh x)$.

3. Show that if $y = \frac{1}{k} \cosh(kx)$, then $\frac{d^2y}{dx^2} = k\sqrt{1 + (dy/dx)^2}$. (See Exercise 52 on page 260 for an application of this.)
4. Find the critical numbers of (a) $f(x) = |x^2 - 4|$ and $g(t) = \frac{t}{t^2 + 2t + 5}$.
5. Find the absolute maximum and minimum values for $y = x^3 - 6x + 2$ on $[-2, 3]$.
6. Find the absolute maximum and minimum values for $f(x) = x - \ln x$ on $[\frac{1}{3}, e]$.

Due Friday March 25

1. Suppose f is a differentiable function and has at least three roots. Show that f' has at least two roots.
2. Suppose $2 \leq f'(x) \leq 5$ for all values of x . Show that $18 \leq f(10) - f(1) \leq 45$.

For each of the following, find (a) the intervals on which the function is increasing or decreasing, (b) the local maximum and local minimum points, and (c) the intervals of concavity and the inflection points.

3. $f(x) = x^3 - 3x + 2$
4. $f(x) = 3x^{4/3} - 4x$
5. $f(x) = x + \cos x, 0 \leq x \leq 2\pi$
6. $f(x) = x + \frac{1}{x}$.

Due Monday April 4

1. Evaluate $\lim_{x \rightarrow 0^+} \frac{x}{\sin \sqrt{x}}$
2. Evaluate $\lim_{x \rightarrow 0} \frac{\tan \pi x}{e^x - 1}$
3. Evaluate $\lim_{x \rightarrow 0} x^{\sin x}$
4. Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{\ln(1+x)} - \frac{1}{x} \right)$
5. Evaluate $\lim_{x \rightarrow 0} x \ln |\sin x|$

For each of the following, sketch the graph of the function. Tell the intervals on which the function is increasing and decreasing, the local and absolute maximum and minimum points, the intervals of concavity and inflection points, the horizontal and vertical asymptotes, and x - and y -intercepts. Include these points and asymptotes in your sketch.

6. $f(x) = x/(x^2 - 4)$
7. $f(x) = e^x/x$

Due Friday April 8

1. A rectangular box with an open top is to be constructed from a 3-by-5 rectangular piece of cardboard by cutting out a square from each of the four corners and bending up the sides. Find the largest volume the box can have.
2. An architect wants to design a window in the shape of a rectangle capped by a semi-circle. If the perimeter of the window is constrained to be 24 feet, what dimensions should the architect choose for the window in order to admit the greatest amount of light?
3. A highway department plans to construct a road between towns A and B . Town A lies on an abandoned road that runs east-west. Town B is 3 miles north of the point on that road that is 5 miles east of A . The engineering department proposes that the road be constructed by restoring a section of the old road from A up to a point P and joining it to a new road that connects P and B . If the cost of restoring the old road is \$200K per mile and the cost of the new road is \$400K per mile, how much of the old road should be restored in order to minimize the department's cost?
4. A piece of wire 10 m long is cut into two pieces. One is bent into a circle and the other into an equilateral triangle. How should the wire be cut so that the total area is (a) a maximum and (b) a minimum?
5. Find x_4 in the application of Newton's method with $x_1 = 2$ to find an approximate solution of $2 \sin x = x$. You may use a calculator for the arithmetic, but show your work.

Due Friday April 15

1. Find f if $f'(x) = x(2 - x)^2$ and $f(0) = 1$.
2. A particle moves with acceleration $a(t) = \cos t + 2 \sin t$. If its position and velocity at time 0 are 0 and 2, respectively, find its position function $s(t)$.
3. Estimate the area under the curve $y = \sqrt{x}$ between $x = 0$ and $x = 5$ and lying above the x -axis using five approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or overestimate?
4. For the definite integral $\int_0^2 4^x dx$, find the Riemann sum approximations with four intervals, using (a) left endpoints and (b) right endpoints. Use a picture to explain inequalities relating your estimates to the actual integral.

- Using a limit of Riemann sums with sample points given by left endpoints, evaluate $\int_0^5 (x^2 - 2x) dx$.
- Evaluate the integral $\int_0^8 |x - 4| dx$ by interpreting it as an area.
- Which is larger, $\int_0^1 \sqrt{1+x^3} dx$ or $\int_0^1 \sqrt{1+x} dx$? Explain. Do not try to evaluate these integrals explicitly.

Due Wednesday April 20

- Let $g(x) = \int_0^{2x} e^{-t^2} dt$. Find $g'(x)$.
- Evaluate $\int_1^2 x\sqrt{x} dx$.
- Evaluate $\int_1^{\sqrt{3}} 1/(x^2 + 1) dx$.
- Find c so that $\int_0^2 f(x) dx = 0$ if $f(x) = \begin{cases} x & 0 \leq x < 1 \\ -cx^2 & 1 \leq x \leq 2 \end{cases}$.
- Find the general indefinite integral $\int x(x+2)^2 dx$.
- Water flows out of a tank at a rate $r(t) = 50 - 2t$ liters per minute. Find the amount of water that flows from the tank during the first 10 minutes.

Due Monday April 25

- Evaluate $\int e^x \cos(e^x) dx$.
- Evaluate $\int \frac{2x+1}{x^2+1} dx$.
- Evaluate $\int_0^{\pi/2} \sin x \cos(\cos x) dx$.
- Find the area of the region bounded by the curves $y = x^3 - x$ and $y = 2x$.
- Find the area of the region bounded by the curves $y = |x|$ and $y = 2 - x^2$.
- Find the area of the region between the curves $y = \sin x$ and $y = \cos x$ which lies between the y -axis and the first point of intersection of the curves which has positive x .

Due Friday April 29

- Use the method of washers to find the volume of the solid obtained when the region between the curves $y = x$ and $y = \sqrt{x}$ is rotated about the (a) x -axis and (b) y -axis. (c) One is larger than the other; give an intuitive explanation of why this should be true.
- Use the method of washers to find the volume of the solid obtained when the region between the curve $y = 2 - x^2$ and the x -axis is rotated about the line $y = -2$.

3. The base of a solid is the triangle with vertices at $(0,0)$, $(1,0)$, and $(0,1)$. The cross-sections perpendicular to the x -axis are semicircles. What is its volume?
4. Work problem 1 by the method of cylindrical shells. (You don't have to answer the last part of it again.)
5. Work problem 2 by the method of cylindrical shells.
6. Find the volume of the portion of a sphere of radius 1 consisting of points whose distance from the y -axis is $\leq r$. Here r can be any number between 0 and 1, and your answer will be in terms of r .