ABSTRACTS OF TALKS

Camillo De Lellis, IAS. *A critical Hoelder exponent for isometric embeddings*

ABSTRACT. Consider a smooth connected closed two-dimensional Riemannian manifold Σ with positive Gauss curvature. If u is a C^2 isometric embedding of Σ in $\mathbb{R}^3$, then $u(\Sigma)$ is convex. In the fifties Nash and Kuiper showed, astonishingly, that this is not necessarily true when the map is C^1. It is expected that the threshold at which isometric embeddings "change nature" is the $\frac{1}{2}$-Hoelder continuity of their derivatives, a conjecture which has a striking similarity with a (recently solved) problem in the theory of fully developed turbulence. In my talk I will review several plausible reasons for the threshold and a recent work, joint with Dominik Inauen, which indeed shows a suitably weakened form of the conjecture.

Simon K. Donaldson, Imperial College & Simons Center. *Associative submanifolds and gradient line graphs.*

ABSTRACT. We will discuss a construction, in part conjectural, of associative submanifolds in 7-manifolds with G_2 structures having co-associative fibrations. The construction bears on the "adiabatic" regime, where the fibres are very small. The input for the construction is a graph in the base whose edges are integral curves of certain vector fields, which are locally gradient vector fields. We will explain how the fundamental transition phenomena, where co-asociative submanifolds develop singularities, are visible in the adiabatic picture. This is in part joint work with C. Scaduto.

Richard Hamilton, Columbia University. *TBA*

H. Blaine Lawson Jr., Stony Brook. *The Inhomogeneous Dirichlet Problem for Natural Operators on Manifolds.*

ABSTRACT. We shall discuss the inhomogeneous Dirichlet problem:

$$f(x, u, Du, D^2u) \;=\; \psi(x)$$

where f is a "natural" differential operator, with a restricted domain F, on a manifold X. By "natural" we mean operators that arise intrinsically from a given geometry on X. An important point is that the equation need not be convex and can be highly degenerate. Furthermore, the inhomogeneous term can take values at the boundary of the restricted domain F of the operator f.

A basic example is the real Monge-Ampère operator $\det(\mathrm{Hess}\, u) \;=\; \psi(x)$ on a riemannian manifold X, where Hess is the riemannian Hessian, the restricted domain is $F = \{\mathrm{Hess} \geq 0\}$, and ψ is continuous with $\psi \geq 0$ (allowed to vanish).

A main new tool is the idea of local jet-equivalence, which gives rise to local weak comparison, and then to comparison under a natural and necessary global assumption.

The main theorem applies to pairs (F, f), which are locally jet-equivalent to a given constant coefficient pair $(\mathbf{F}, \mathbf{f})$. This covers a large family of geometric equations on manifolds: orthogonally invariant operators on a riemannian manifold, G-invariant operators on manifolds with G-structure, operators on almost complex manifolds, and operators, such as the Lagrangian Monge-Ampère operator, on symplectic manifolds. It also applies to all branches of these operators. Complete existence and uniqueness results are established with existence requiring the same boundary assumptions as in the homogeneous case.

There are also results where the inhomogeneous term ψ is a delta function.

Si Li, Tsinghua University (Beijing) *Open-closed topological B-model on Calabi-Yau geometry.*

ABSTRACT. We describe a theory of open-closed B-model topological string field theory on Calabi-Yau geometry. This is formulated in terms of coupling holomorphic Chern-Simons theory with Kodaira-Spencer gravity. As an application, we show how large N matrices can be used to recover Calabi-Yau deformation theory and explain an analogue of Green-Schwarz mechanism for anomaly cancellation in topological string.

Fernando C. Marques, Princeton University. *Morse theory and the volume spectrum.*

ABSTRACT. In this talk I will survey recent developments on the existence theory of closed minimal hypersurfaces in Riemannian manifolds, including a Morse-theoretic existence result for the generic case.

Richard Schoen, University of California, Irvine. *The problem of quasi-local mass in general relativity*

ABSTRACT. This will be a talk about various quasi-local mass quantities which have been useful in general relativity. We will talk about comparisons between different masses. We will also discuss comparison-type theorems for polyhedral surfaces and how these relate to quasi-local masses.

Penny Smith, Lehigh University. *Quantum Geometry and Covariant Loop Quantum Gravity.*

ABSTRACT. In an attempt to quantize general relativity in a non-perturbational way, physicists have studied a discretization of its geometry and Action Functional called Regge Calculus. This is based on a piecewise flat triangularization of a space-time manifold with curvature concentrated at vertices. After much heuristic work by physicists, convergence of discrete geometric curvatures (Lipschitz-Killing curvatures) associated with a sequence of such triangularizations was proved in a brilliant 1984 paper of Jeff Cheeger.

However, as a method of quantizing general relativity the Regge calculus approach has--in common with most Quantum Field Theories--problems with infinities arising on small scales. This is very serious in general relativity, as general relativity is a non-renormalizable theory.

Another approach to quantizing general relativity in a non-perturbational way is Covariant Loop Quantum Gravity, which uses a triangularization (and a certain dual triangularization) with Lie Algebra and Lie Group representations associated to sub complexes. This is known as a spin foam. Convergence of this model has been studied by physicists in very special cases using perturbations based on minimal uncertainty states. This is not satisfying because we are trying to study a non-perturbative model.

In Covariant Loop Quantum Gravity: lengths, areas, volumes, and dihedral angles of the triangularization are determined by the eigenvalues of left invariant vector fields satisfying an angular momentum algebra. Thus, there is a natural smallest length scale (corresponding to smallest non-zero eigenvalues) and no infinities arise from smaller scales. However, Space-Time should be a non-compact space, and so convergence of the triangularizations on large length scales is important.

This gives rise to a quantum geometry--for example, the metric geometry of a non-regular tetrahedron in three space is determined by six numbers (in Regge Calculus, by the lengths of its sides) in the LQG setting by the four face areas, its volume, and by a dihedral angle. All of which are quantum objects as described above.

We extend Cheeger's convergence method in the setting of LPG to provide a geometric analysis approach to convergence without resorting to perturbation methods.

In our talk, we describe LPG and its spin foams in the recent formulation of C. Rovelli, and sketch the above convergence.